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A PREDICTIVE CONTROL SCHEME FOR VAPOR COMPRESSION BASED COOLING MACHINES

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Abstract. *This paper presents a predictive control scheme for a cooling machine based on vapor compression. The predictive control algorithm allows the independent control of freezing power and superheating. The main objective is to keep comfortable environmental conditions under time varying thermal loads. The results are for cooling systems in which the temperature sensor is located far from the cooling machine. Simulation results are finally presented to illustrate the controller performance.*

Keywords: *Cooling Systems, Predictive Control, System Decoupling*

1. INTRODUCTION

Cooling machines are multi-input multi-output, cross-coupled, time-varying, nonlinear systems whose inputs present saturation and rate constrains. Classical control solutions such as On-Off and PID controllers are not usually adequate to deal with this type of plant.

Besides that, the attempt to improve energy efficiency and people's comfort leads to the placement of the temperature sensor somewhere inside the target environment and usually far from the freezing power source. Thus, the temperature sensor will recognize changes in the generated freezing power only sometime after they occur. This characterizes a time delay in the control loop that can not usually be compensated by classical control techniques. Under this scenario the independent control of superheating and freezing-power is under research and the development of new control schemes for cooling systems has become a challenging task.

This paper focus on the application of Model Based Predictive Control (MBPC) algorithms in the control of cooling machines. The research objective in the future is to extend the broadness of the current control algorithm to deal with the temperature control inside a target environment that may include several cooling machines. It is clear that this cooling system is also a MIMO one that includes strong cross coupling interaction among inputs and outputs and time delays.

2. THE COOLING MACHINE

This work is concerned with the control of the system constituted by the expansion valve, evaporator and compressor as shown in Fig. (1). The system inputs are the expansion valve opening position, which defines the mass flow rate (MFR) and the compressor speed, which controls the volume flow rate (VFR). The system outputs are the super heating, ***DT***, and the freezing power, Q_1. In this case, the system dynamics is defined by a matrix function of the form:

$$[Y(s)] = [G(s)]\ [U(s)] \tag{1}$$

or

$$\begin{bmatrix} \Delta T(s) \\ Q(s) \end{bmatrix} = \begin{bmatrix} G_{11}(s) & G_{12}(s) \\ G_{21}(s) & G_{22}(s) \end{bmatrix} \begin{bmatrix} \dot{a}(s) \\ \dot{q}(s) \end{bmatrix} \tag{2}$$

with

$$[Y(s)] = \begin{bmatrix} \Delta T(s) \\ Q(s) \end{bmatrix} \tag{3}$$

$$[U(s)] = \begin{bmatrix} \dot{a}(s) \\ \dot{q}(s) \end{bmatrix} \tag{4}$$

and

$$[G(s)] = \begin{bmatrix} G_{11}(s) & G_{12}(s) \\ G_{21}(s) & G_{22}(s) \end{bmatrix} \tag{5}$$

where

$\Delta T(s)$ = Superheating
$Q(s)$ = Freezing power
$\dot{a}(s)$ = Mass flow rate (Expansion valve opening)
$\dot{q}(s)$ = Volume flow rate (Compressor speed)

In the case that the freezing power is measured at the target environment there will exist a time delay from the temperature measurement to the controller response, then

$$\frac{Q(s)}{\dot{a}(s)} = G_{21}(s) = K_a \frac{(T_{a3}\ s+1)}{(T_{a1}\ s+1)(T_{a2}\ s+1)} e^{-t s} \tag{6}$$

$$\frac{Q(s)}{\dot{q}(s)} = G_{22}(s) = K_q \frac{(T_{q3}\ s+1)}{(T_{q1}\ s+1)(T_{q2}\ s+1)} e^{-t s} \tag{7}$$

with

$G_{21}(s)$ = Transfer function from $\dot{a}(s)$ to $Q(s)$
$G_{22}(s)$ = Transfer function from $\dot{q}(s)$ to $Q(s)$
K_* = Static gains
T_* = Time constants
t = Time delay

3. WHY PREDICTIVE CONTROL?

Predictive control algorithms were originally proposed to deal with complex nonlinear plants. An important characteristic of MPC algorithms is that they can incorporate in their formulation plant constrains, actuators nonlinear characteristics and control-loop time delays. Due to their computationally costly algorithms, the application of these controllers became initially restricted to large industrial plants. Currently, with the appearance of new and powerful small computers, the implementation of complex algorithms became economically feasible even for small plants. This is the case of modern cooling systems, which are relatively small plants with sufficiently complex dynamics that requires the use of model based predictive control (MBPC) techniques.

Finally, the cost of modern micro-controllers has been continuously reducing and their computational capacity increasing to the point of having comparable costs of classical PID controllers. These facts have made economically feasible the implementation of powerful predictive control algorithms for relatively small plants like cooling machines.

4. THE MBPC - A BRIEF REVIEW

This section presents a very short overview on a particular case of MBPC, the Generalized Predictive Control (GPC) algorithm. Since the GPC basis is not in the scope of this paper, the reader in invited to address his/her interest for further discussion and details on GPC to Camacho et al (1999), Clarke et al (1989), Garcia (1986), Kinnaert (1989) and Morari (1994). Figure (2) shows a block diagram of the classical configuration of the predictive control scheme.

The GPC algorithm starts with a CARIMA (Controlled Auto Regressive Moving Average) model of the plant. Equation (8) gives the general form of the CARIMA model.

$$A(z)y(k) = z^{-d}B(z)u(k-1) + \frac{C(z)}{\Delta(z)}z(k) \tag{8}$$

with

$$A(z) = 1 + \sum_{i=1}^{na} a_i\, z^{-i} \tag{9}$$

$$B(z) = \sum_{i=1}^{nb} b_i\, z^{-i} \tag{10}$$

$$C(z) = \sum_{i=1}^{nc} c_i\, z^{-i} \tag{11}$$

and

$$\Delta(z) = 1 - z^{-1} = \frac{z-1}{z} \tag{12}$$

The output future behavior can be obtained by the predictor equation given by

$$\hat{y}(k+j) = \frac{E_j(z)B(z)\Delta(z)u(k+j-1) + F_j(z)y(k)}{C(z)} \tag{13}$$

for C(z) = 1

$$\hat{y}(k+j) = G_j(z)\Delta(z)u(k+j-1) + F_j(z)y(k) \tag{14}$$

Equation (14) represents the future output sequence that can be obtained through the recursive solution of the Diophantine Equation given by

$$C(z) = E_j(z)A(z)\Delta(z) + z^{-j}F_j(z) \tag{15}$$

with

$$E_j(z) = \sum_{i=0}^{j-1} e_{j,i}\, z^{-i} \tag{16}$$

$$F_j(z) = \sum_{i=0}^{n} f_{j,i}\, z^{-i} \tag{17}$$

The control sequence $Du(k)$ is found such that it minimizes a previously defined cost function J. Equation (18) gives the general form of the cost function J.

$$J = E\left\{ \sum_{j=N_1}^{N_2} \left[r(k+j) - \hat{y}(k+j)\right]^2 + \sum_{j=1}^{Nu} l_j \left[\Delta u(k+j-1)\right]^2 \right\} \tag{18}$$

with

N_1	=	Minimum Cost Horizon
N_2	=	Maximum Cost Horizon
N_u	=	Control Horizon
l_j	=	Control Weighting Sequence

Figure (3) shows the proposed predictive control scheme. It consists on two SISO GPCs, the first one to regulate the superheating and the second one to control the freezing power. The main feature of this structure is that it simplifies the on-line implementation of the control algorithm minimizing hardware investment.

5. SIMULATION RESULTS

The cooling machine model as proposed by Machado (1996) is adopted here for analysis and simulation purposes. In this case, the four SISO transfer functions of Eq. (5) are given by

$$G_{11}(s) = -5.62\,\frac{1}{(45\ s+1)} \qquad G_{12}(s) = 2.49\,\frac{(70\ s+1)}{(59.52\ s+1)}$$

$$G_{21}(s) = 33.89\,\frac{(-36.37\ s+1)\,e^{-t\,s}}{(25.65\ s+1)(67.79\ s+1)} \qquad G_{22}(s) = 22.20\,\frac{(630\ s+1)\,e^{-t\,s}}{(80\ s+1)(90\ s+1)}$$

It can be seen that the open loop plant is nominal stable since it does not have any poles at the right half-plane. However, the gain and phase margins of $G_{22}(s)$ are not sufficiently large to guarantee the closed loop stability under parametric uncertainty.

Figure (4) shows the open loop response for a freezing power step input. It should be notice that the effects of the time delay (τ = 1 sec.) in the plant response are apparently negligible with respect to the long settling time of the system.

Figure (5) presents the $G_{22}(s)$ system frequency response (using the Pade approximation for the time delay). It can be observed that, in this case the $G_{22}(s)$ system is already closed loop unstable for τ = 1 sec.

Table (1) shows the full plant conditioning number. It can be noticed that the plant is open loop ill conditioned.

Table 1. Plant Conditioning Numbers

Matrix:	Controllability	Observability
Conditioning Number:	8.1798×10^4	5.1169×10^8

Figures (6) and (7) show the frequency dependence of the GPC algorithm. The tests were performed using sinusoidal thermal loads.

Figures (8) to (11) present the proposed control scheme performance. Figure (8) shows the closed loop system response for a step input applied to the freezing power input. Figure (9) displays the closed loop system response for a super heating step input. Finally, Figures (10) and (11) present the generated control signals for the step inputs.

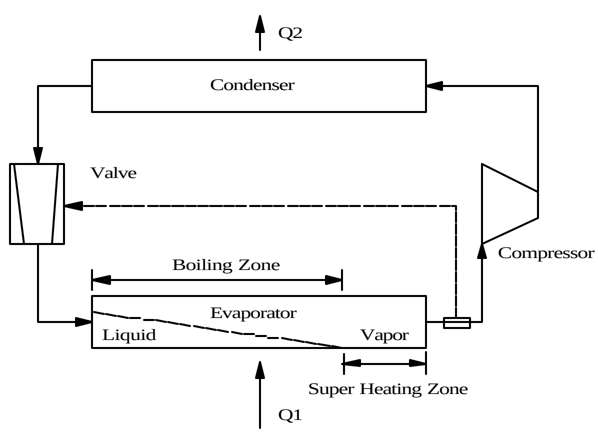

Figure 1 - Cooling Unit Diagram.

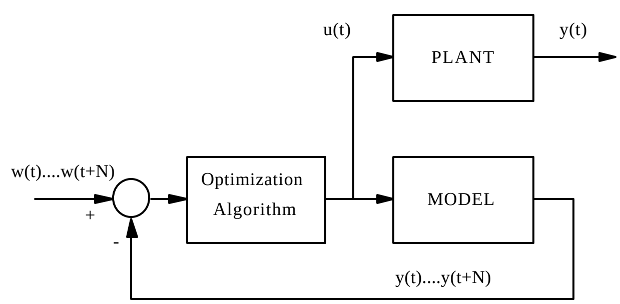

Figure 2 - Model Predictive Control.

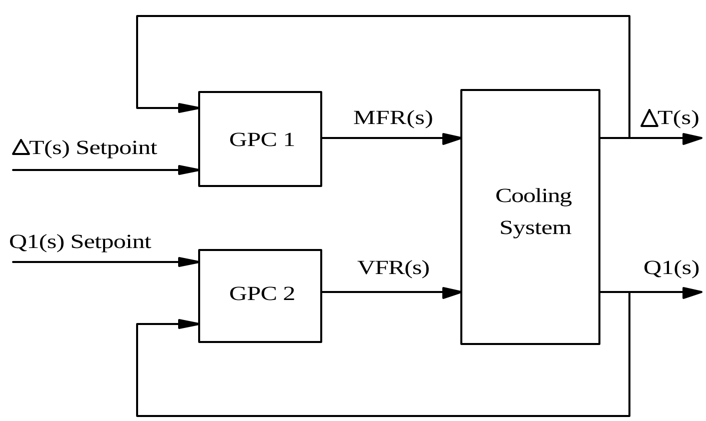

Figure 3 – The Proposed Control Scheme.

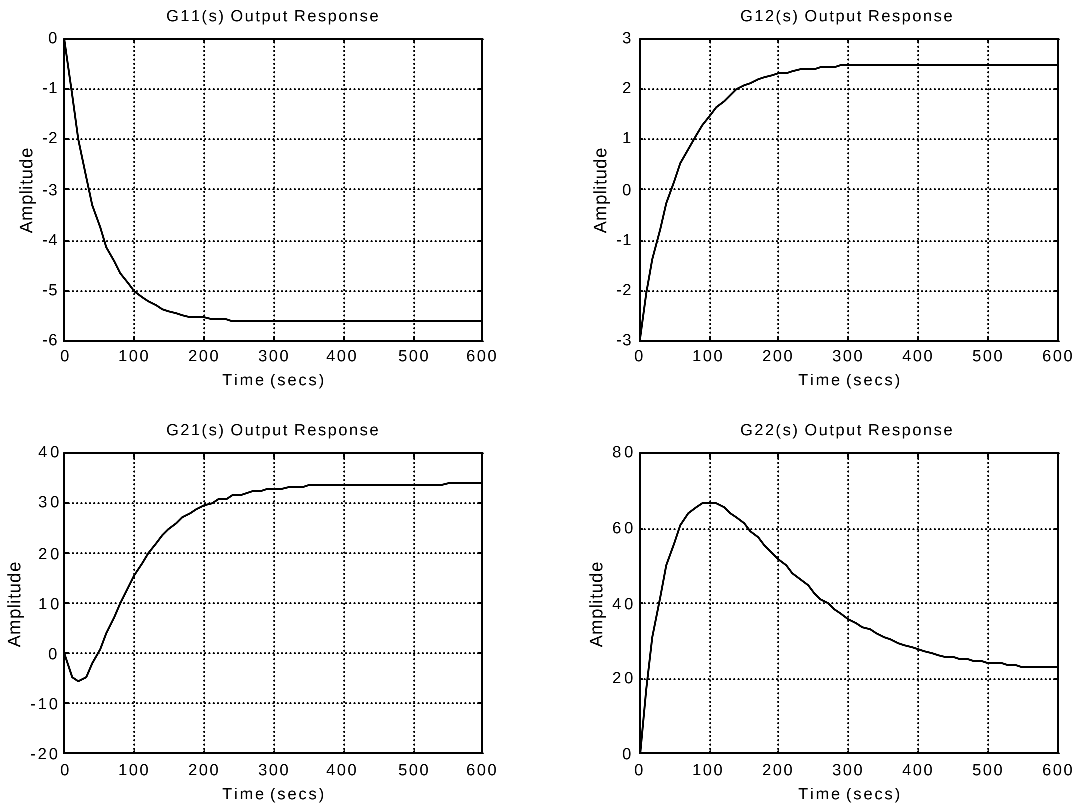

Figure 4 - G(s) Open Loop Step Response (τ=1s).

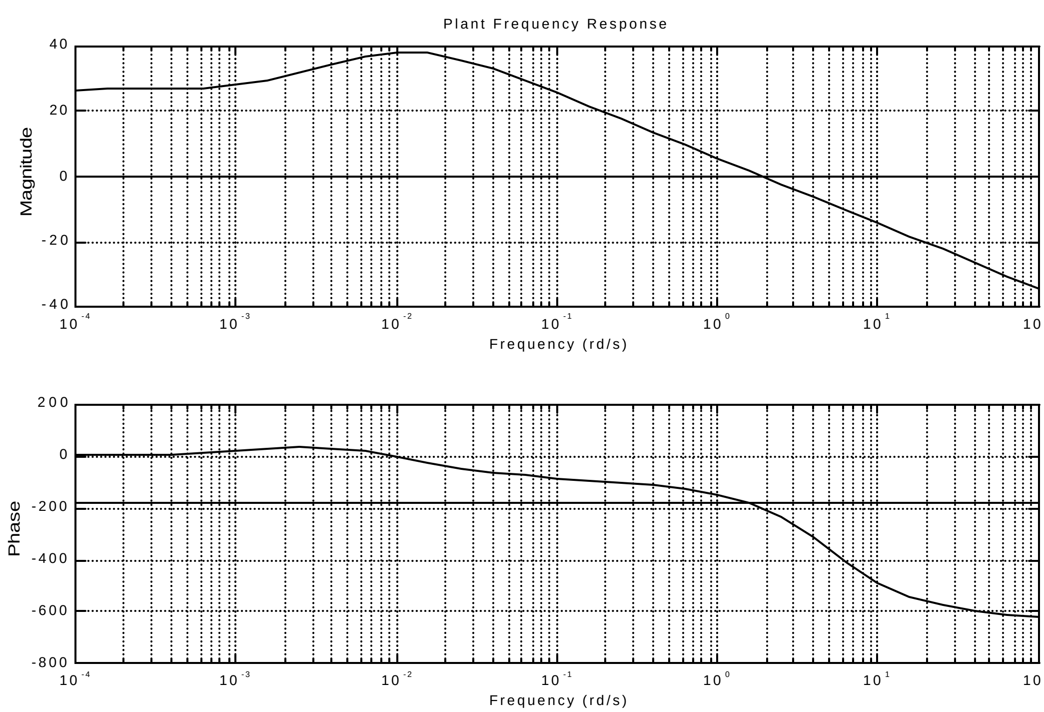

Figure 5 - $G_{22}(s)$ Frequency Response (τ=1s).

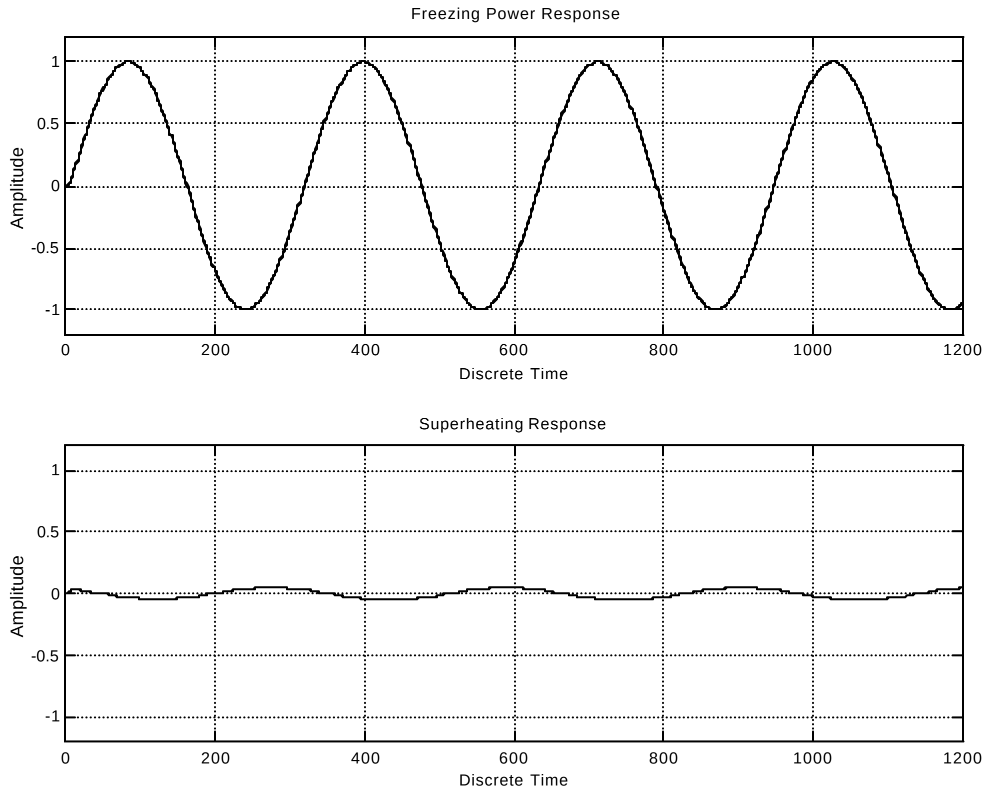

Figure 6 – GPC Frequency Response Dependence.

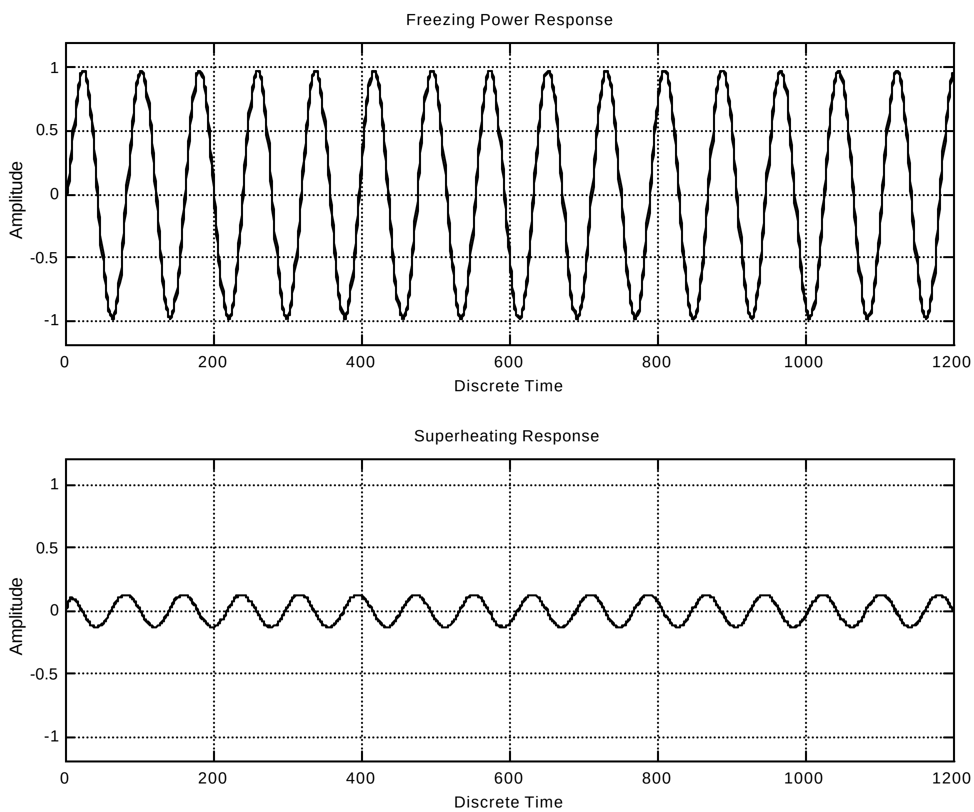

Figure 7 – GPC Frequency Response Dependence

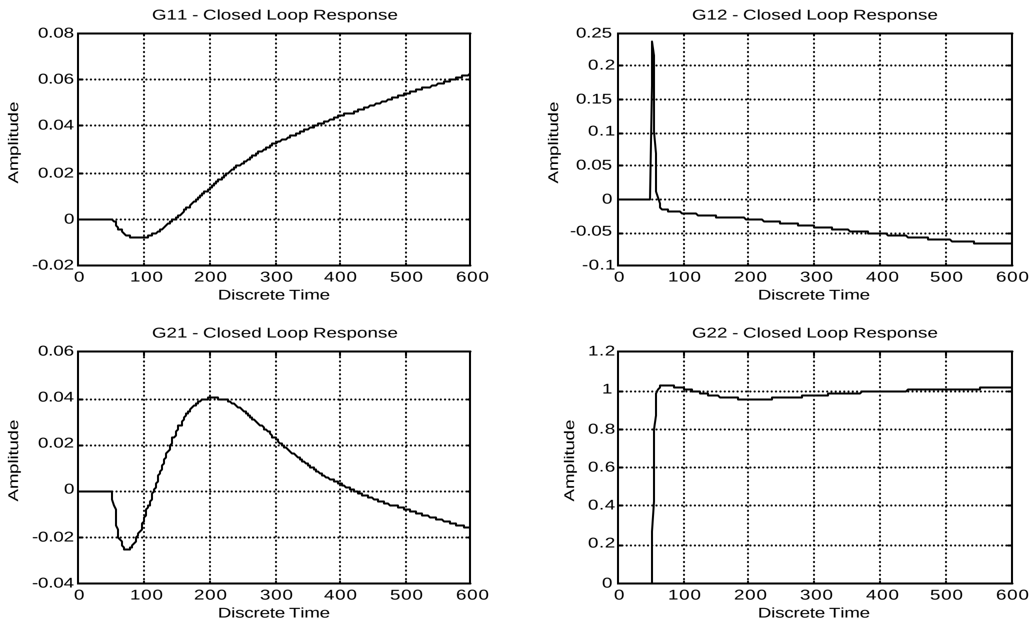

Figure 8 – Plant Response for a Freezing Power Step Input.

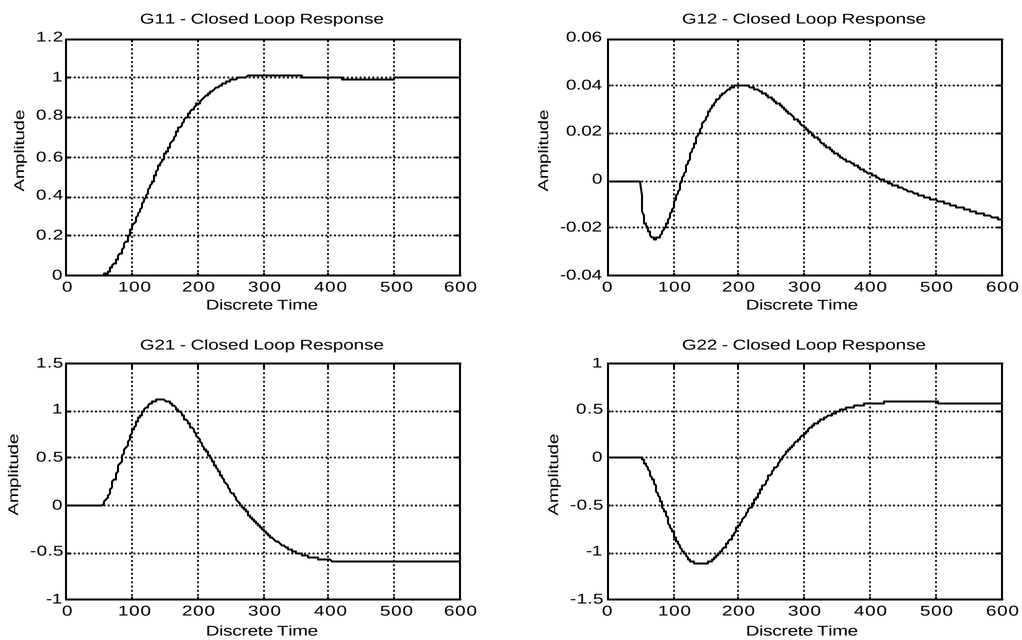

Figure 9 - Plant Response for a Super Heating Step Input.

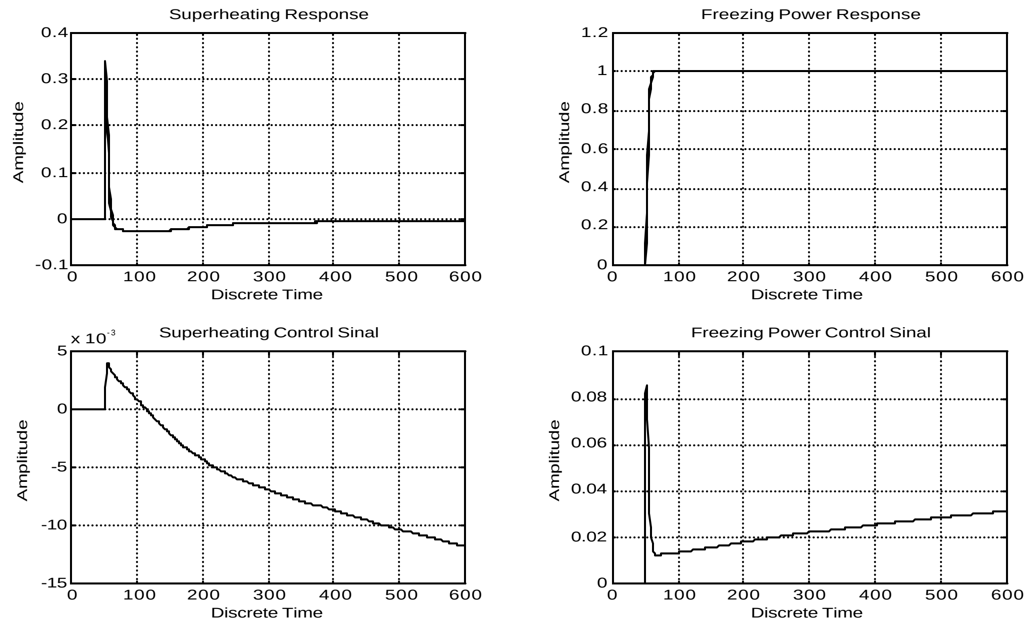

Figure 10 – Control Signals for a Freezing Power Step Input.

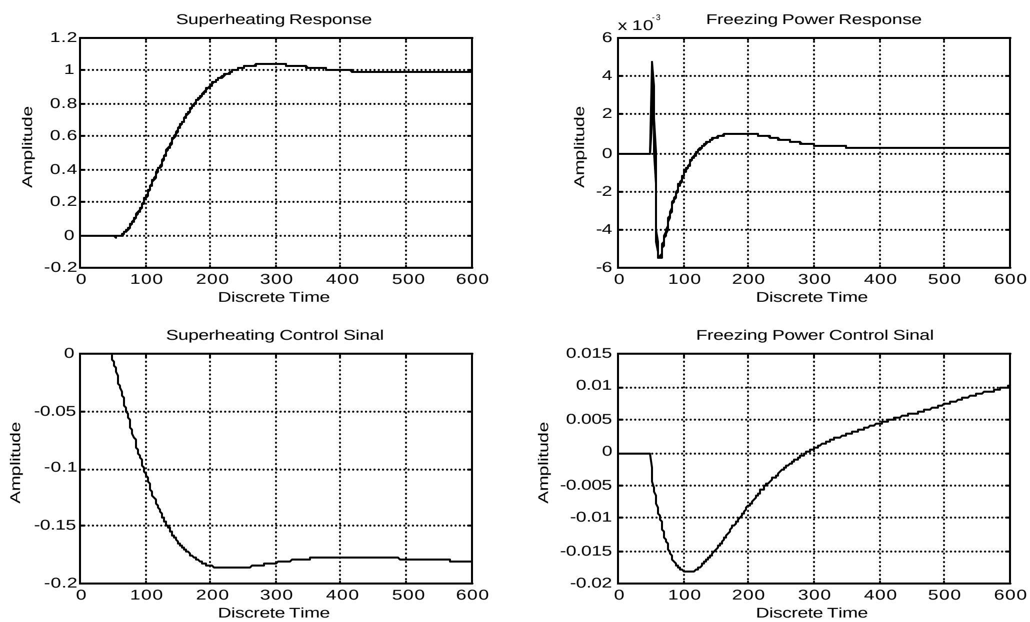

Figure 11 – Control Signals for a Super Heating Step Input.

6. FINAL COMMENTS AND CONCLUSIONS

This work assessed the performance of the GPC algorithm applied to the control of a cooling machine based on vapor compression.

For analysis and simulation purposes a 2x2 MIMO model, with strong cross coupling between its inputs and outputs, represented the plant. In this case, the mass flow rate and the volume flow rate were considered as the inputs and the freezing power and the super heating as the outputs.

A control scheme was defined and implemented as two SISO GPC algorithms in a parallel configuration. This configuration was chosen to facilitate its real-time implementation in small micro-controllers. Also, the adopted configuration allowed the GPC parameters to be independently tuning for the freezing power and super heating, improving in this way the overall controller performance.

The simulation results showed a high performance of the GPC algorithm working with a MIMO and potentially unstable plant. The system cross coupling between its inputs and outputs was easily compensated by the proposed control structure.

Finally, based on the results it can be concluded that predictive control algorithms are natural candidates for the control of cooling machines in the near future.

7. ACKNOWLEDGEMENTS

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