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ON THE APPLICATION OF MULTI-BODY AND FINITE ELEMENT ANALYSIS FOR THE STRUCTURAL SYNTHESIS OF A MINI-BAJA VEHICLE

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Abstract. *This work presents a comparison between discrete and continuous optimization techniques applied to improve the dynamics and the mechanical strength of a mini-baja prototype. The behaviour of the vehicle under study is represented by polynomial models known as response surfaces, which are obtained by the statistical manipulation of the results of a series of finite element analysis of the mini-baja structure. These finite element analysis are performed under different operational conditions in order to verify the robustness of each particular design configuration. Response surfaces are also built aiming at approximate representations of the robustness metrics. Next, the physical behaviour, as well as the robustness of the vehicle under study, both of them modelled by means of response surfaces, are subject to numerical optimization. In a first optimization attempt, the design variables are regarded as continuous entities, ranging freely from the lower to the upper side constraint. Then, their variation is restricted to a set of prescribed discrete values. The results of both approaches are compared, giving rise to some conclusions and perspectives for future research work.*

Keywords *Discrete Optimization, Continuous Optimization, Response Surfaces, Robust Design*

1. INTRODUCTION

The Response Surface Method (R.S.M.) has been chosen for the application in this paper due to its advantages concerning reduction of computational cost and improvement of the numerical conditioning in the performance prediction and optimization of engineering systems (Box and Draper, 1987). It is important to notice, however, that better numerical conditioning only results when the point set used to build the response surfaces is well balanced, according to the statistical fundamentals of experimental design (D.O.E.). This statistical framework, on the other hand, allows for the realization of interesting studies related to the optimality of engineering systems when uncertainty issues are considered on their operating conditions (Taguchi et al., 1999).

Moreover, response surface models provide the designer with a global view of the design space, including useful insight into the relative significance and correlation of the individual design parameters influencing the responses of interest.

Finally, response surfaces are constructed based on response values only, and thus avoid having to compute design sensitivities, whose computation is not always a trivial task.

2. OVERVIEW OF META – MODELING TECHNIQUES

As all meta – models, response surfaces are obtained according to the phylosophy described in Fig. (1)

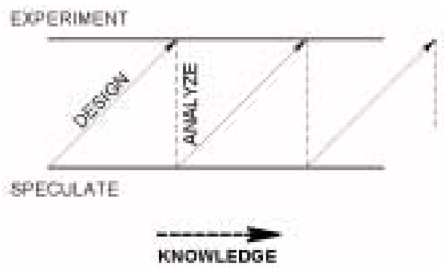

Figure 1. Illustration of the statistical meta – modeling rationale

which results in a procedure based on the four following steps:

a) Experimental design – a design space, including a range of design possibilities, is sampled in order to reveal its contents and tendencies;
b) Choice of a model – the nature of the “meta-model” itself is determined, tacking into account that the relations contained in the data gathered in the previous step have to be symbolically represented, with the highest possible accuracy;
c) Model fitting – the model whose shape is defined in “b” is fitted to the data collected in “a”. Differences in fitting schemes may affect the efficacy of “meta-modeling” techniques in the solution of a given problem. In the case of the R.S.M., the least squares formulation is adopted, as shown in Eqs. (1) and (2):

$$\{Y\} = [E] \cdot \{B\} + \{\delta\} \tag{1}$$

where {Y} is the vector of responses (dependent variables) obtained for each line of the matrix [E] which corresponds to the experimental design stage of meta-modeling. The vector {δ}contains free, random error terms. The vector of model parameters {B} can be estimated as follows:

$$\{B\} = \left([E]' \cdot [E] \right)^{-1} \cdot [E]' \{Y\} \tag{2}$$

where the term $\left([E]' \cdot [E] \right)^{-1}$ comes directly from the experimental matrix and is called the variance-covariance matrix, a very important element in evaluating the quality of the meta-model, as referred to in item “d”.

d) Verification of model accuracy – the three precedent steps are sufficient to build a first tentative model, whose overall quality and usefulness have to be evaluated by adequate sets of metrics. Each combination of design space sampling, model choice and fitting procedure leads to the use of specific verification procedures.

Since the response surface meta-models are available, they can be used in a variety of optimization procedures aiming at design improvement. Some of them are briefly addressed in sections 3 to 5, and used in the illustrative case study of section 5.

3. OVERVIEW OF ROBUST ENGINEERING

The robust design phylosophy is based on two fundamental principles (Taguchi et al. , 1999):

- When performance deviates from a given target value, a loss is caused, even if specifications are still met;
- Variation in operating conditions and intrinsic deviations from system ideal configurations result in performance variability. Continuously pursuing variability reduction is key to achieve high quality and reduce cost.

From the operational viewpoint, one of the possible approaches is to calculate robustness metrics and tune system parameters (design variables) so that these metrics are optimized. The Cdk metric, as shown in Fig. (2) and Eqs. (3) to (5) accounts for system performance variation against engineering specifications. Higher values of the Cdk metric indicate more robust designs.

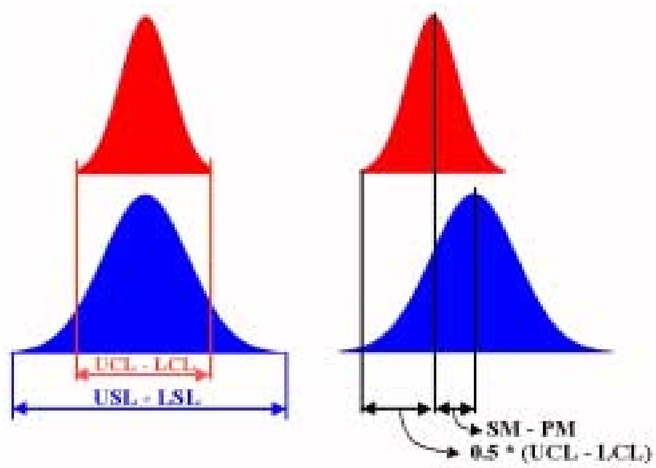

Figure 2. Graphical presentation of the robustness concept

$$C_d = \frac{USL - LSL}{UCL - LCL} \tag{3}$$

$$C_{dk} = C_d \cdot (1 - k) \tag{4}$$

$$k = \frac{|\, STV - ML \,|}{\frac{1}{2} \cdot (UCL - LCL)} \tag{5}$$

The parameters shown in Fig. (2) and Eqs. (3) to (5) are:

- UCL/LCL: upper and lower design control limits. The difference between UCL and LCL is the total design performance dispersion under the normal p.d.f (6σ);
- USL/LSL: upper and lower design specification limits;
- SM: Specified mean value (target performance);
- PM: Effective mean performance.

4. DISCRETE OPTIMIZATION FUNDAMENTALS (Vanderplaats, 1998)

Sometimes it is desirable to choose the values of the design variables to be taken from a set of discrete values. It should be noted, however, that the conventional and well established optimization approach is not directly applicable to such problems, due to the inability of non – linear programming methods to handle discrete design variable values. Quite often, one suggests to overcome this obstacle by solving an equivalent continuous problem and then just round the design variable values to the closest discrete values. Despite its simplicity, this approach may lead to nonoptimal or even infeasible design configurations.

More efficient approaches have evolved from the linear to the non – linear programming domain, such as the “Branch and Bound Methods” (B.B.M.), which are used in this paper. In he B.B.M., a continuous variable optimization is first performed, providing a starting point as well as a lower bound on the discrete solution. Then, one of the design variables is increased to its next discrete value and the optimization is performed with respect to the remaining variables. If the optimum is worse than before, the variable is set to its next lower value and the process is repeated. If an improvement is obtained, the search is continued in this direction until no improvement can be found. Then, this variable is allowed to change, subject to the pertaining bounds, and the process is repeated with the next discrete variable, until all of them are examined.

Although this method is theoretically correct for convex problems, numerical problems may be associated with the accuracy of non – linear programming solution. Besides, the relativelly large number of non – linear programming problems cannot be readily updated, leading to significant increase in computational cost, which should be counterbalanced by the introduction of meta – models to represent the responses of interest.

5. COMPROMISE OPTIMIZATION FOR MULTI – OBJECTIVE PROBLEMS

Multicriterion optimization problems arise in different engineering fields and considerable attention is being devoted to develop methods aiming at their solution (Osyczka, 1984; Eschenauer et al., 1990). The main difficulty to be considered is that the solution for multi – objective optimization problems is non – unique.

The case study presented in this paper (section 6) encompasses seven responses of interest, defining a situation prone to be formulated as a multicriterion optimization problem. Indeed, a compromise programming formulation (Vanderplaats, 1998) is adopted, mainly because it accounts for design specifications, similarly to the robust engineering approach presented in section 3. This aspect can be visualized through Eq. (6)

$$F(X)=\left\{\sum_{k=1}^{K}\left[\frac{W_k\left\{F_k(X)-F_k^*(X)\right\}}{F_{pk}(X)-F_k^*(X)}\right]^2\right\}^{\frac{1}{2}} \tag{6}$$

where:

- F(X) is a compromise objective function
- F_k is the k-th response of interest, in a total of K
- F^*_k is the target value for the k-th response
- F_{pk} is the worst value accepted for the k-th response

- W_k is the weighting factor applied for the k-th response of interest

It should be noted that the optimization problem defined through Eq. (6) is unconstrained because the K responses encompass both objective and constraint functions.

This formulation is well regarded because it considers engineering specifications through F^*_k and F_{pk}, which helps in keeping a practical insight over the optimization problem.

6. CASE STUDY PRESENTATION

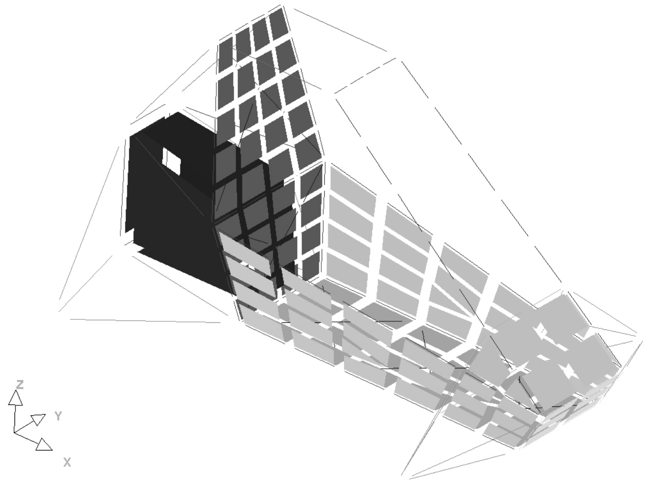

Figure 3 – Finite element model of the Mini – Baja prototype

The techniques presented in the previous sections are now demonstrated through the design optimization of a Mini – Baja prototype, whose finite element model is shown in Fig. (3). This optimization problem is stated by means of the design variables and responses described in Tab. (1) and Tab. (2) respectivelly.

Table 1. Design variables for the optimization problem regarding the mini-baja prototype

Variable	Lower Side Constraint (Code: -1.000)	Upper Side Constraint (Code: 1.000)
Vehicle Length (V1)	1.8831 [m]	2.5477 [m]
Roof Width (V2)	0.4000 [m]	0.5200 [m]
Baseline Width (V3)	0.5950 [m]	0.8050 [m]
Structure Tubes Inner Diameter (V4)	0.0209 [m]	0.0350 [m]
Structure Tubes Wall Thickness (V5)	0.0021 [m]	0.0035 [m]

Table 2. Responses of interest for the optimization problem regarding the mini-baja prototype

R1	Mass of the Mini-baja prototype [kg]
R2	Strain energy under standard operating conditions [J]
R3	Second natural frequency of vibration [Hz]
R4	Third natural frequency of vibration [Hz]
R5	Fourth Natural frequency of vibration [Hz]
R6	Maximum force acting over the structure [N]
R7	Maximum torque acting over the structure [N.m]

The meta – modeling of the system responses with respect to the design variables requires the realization of an experimental design such as the one displayed in Tab. (3).

Table 3. $2^{5-1}_{R=V}$ experimental design for response surface generation (five design factors)

Runs	Coded design variable values				
	V1	V2	V3	V4	V5
1	1.000	1.000	1.000	1.000	1.000
2	1.000	1.000	1.000	-1.000	-1.000
3	1.000	1.000	-1.000	1.000	-1.000
4	1.000	1.000	-1.000	-1.000	1.000
5	1.000	-1.000	1.000	1.000	-1.000
6	1.000	-1.000	1.000	-1.000	1.000
7	1.000	-1.000	-1.000	1.000	1.000
8	1.000	-1.000	-1.000	-1.000	-1.000
9	-1.000	1.000	1.000	1.000	-1.000
10	-1.000	1.000	1.000	-1.000	1.000
11	-1.000	1.000	-1.000	1.000	1.000
12	-1.000	1.000	-1.000	-1.000	-1.000
13	-1.000	-1.000	1.000	1.000	1.000
14	-1.000	-1.000	1.000	-1.000	-1.000
15	-1.000	-1.000	-1.000	1.000	-1.000
16	-1.000	-1.000	-1.000	-1.000	1.000

Table 4. Response values for $2^{5-1}_{R=V}$ experimental design

Runs	R1	R2	R3	R4	R5	R6	R7
1	97.704	1.550	47.139	68.540	76.025	878.300	1697.900
2	34.418	12.435	29.764	41.192	45.980	897.000	1781.700
3	54.529	2.617	41.042	68.114	73.083	974.800	1434.200
4	60.856	5.150	28.377	45.664	49.172	1004.400	1485.200
5	54.988	2.627	49.988	67.042	73.998	931.400	1592.400
6	61.369	5.560	34.991	44.780	49.788	962.300	1705.000
7	94.960	1.380	47.744	71.725	76.630	985.200	1395.500
8	33.451	9.957	30.172	43.323	46.314	976.500	1419.500
9	48.811	1.213	55.845	85.868	88.659	708.500	842.100
10	54.475	2.399	38.566	57.914	59.499	723.200	818.400
11	84.113	.646	53.840	90.181	95.203	786.100	517.900
12	29.630	4.656	32.864	54.672	57.683	815.200	497.200
13	84.926	.646	66.117	88.914	93.131	704.600	980.500
14	29.917	4.788	41.732	53.731	56.372	731.900	1016.700
15	47.218	1.221	58.034	86.126	92.718	793.600	514.100
16	52.697	2.375	40.062	57.934	62.279	820.200	501.700

The decision variable combinations, which define the design space samples, are arranged according to a standard two – level fractiobal factorial design (Montomery, 1996). The least squares interpolation procedure gives rise to the following linear response surface models, in order to represent the physycal quantities listed in Tab. (2):

$$R1 = 57.7539 + 3.7806 \cdot \mathrm{V1} + 13.1522 \cdot \mathrm{V4} + 16.1336 \cdot \mathrm{V5} \quad (7)$$

$$R2 = 3.7012 + 1.4582 \cdot \text{V1-2.2138} \cdot \text{V4-1.2380} \cdot \text{V5} \quad (8)$$

$$R3 = 43.5172\text{-}4.8652 \cdot \text{V1-2.5878} \cdot \text{V2} + 2.0005 \cdot \text{V3} + 8.9513 \cdot \text{V4} + 1.0873 \cdot \text{V5} \quad (9)$$

$$R4 = 64.1074\text{-}7.8099 \cdot \text{V1} + 14.2063 \cdot \text{V4} + 1.5991 \cdot \text{V5} \quad (10)$$

$$R5 = 68.5334\text{-}7.1596 \cdot \text{V1} + 15.1475 \cdot \text{V4} + 1.6827 \cdot \text{V5} \quad (11)$$

$$R6 = 855.8250 + 95.4125 \cdot \text{V1-38.6750} \cdot \text{V3} \quad (12)$$

$$R7 = 1137.5000 + 426.4250 \cdot \text{V1} + 166.8380 \cdot \text{V3} \quad (13)$$

The quality of these meta – models is measured by means of the adjusted multiple correlation coefficient (R_a^2), that measures the proportion of the data variability which is captured by the statistical model. The values of R_a^2 for responses R1 to R7 are respectivelly equal to 95.63%, 71.75%, 97.57%, 98.33%, 98.40%, 96.90% and 96.97%.

It is important to highlight the good levels of explained variance obtained with these response surfaces, expressed by means of the R_a^2 values, which are close to 100.00%. This situation is expressive considering that all models are linear, resulting from unexpensive experimental designs that demand few finite element runs in comparison to more sophisticated ones, aimed at constructing higher order models.

An exception holds for the strain energy (response R2), whose response surface is able to explain only 71.75% of the data variance. Since this value is just reasonable, caution is to be taken when interpreting estimates resulting from this particular response surface model.

Equations (7) to (13) are then combined in the following optimization problem statement:

Minimize the Strain Energy (Eq. (8)), subject to:

- Mass (Eq. (7)) ≤ 30 kg
- Second Natural Frequency of Vibration (F2 ⇔ Eq. (9)) ≤ 40 Hz
- Third Natural Frequency of Vibration (F3 ⇔ Eq. (10)): 48 Hz ≤ F3 ≤ 52 Hz
- Fourth Natural Frequency of Vibration (F4 ⇔ Eq. (11)): 55 Hz ≤ F3 ≤ 60 Hz
- Maximum Resultant Force (Eq. (12)) ≤ 800 N
- Maximum Resultant Torque (Eq. (13)) ≤ 1000 N.m

The real aim of this optimization problem statement is to reduce mass. Since it cannot be achieved at the expense of structural integrity, the optimizer is programmed to minimize the strain energy as well. It should be noted, however, that the strain energy, which is a global metric, may be minimized while local loads are increased, which is not desirable. For this reason, (local) maximum forces and torques are also constrained to safety values.

Constraints imposed over natural frequencies of vibrations address the issue of resonance avoidance. Second, third and fourth natural vibrating frequencies may not coincide with vibration induced by the vehicle’s powertrain at the maximum engine torque, power and rotation regimes.

7. RESULTS PRESENTATION AND ANALYSIS

7.1. Results of continuous optimization

In this first approach, the design variables are allowed to vary freely from their lower to their upper side constraint values. The design improvement thus obtained is shown by the comparison in Tab. (5). These results are obtained by means of changes in the design variables, as shwn in Tab. (6).

Table 5. Response values before and after continuous optimization procedure

Response	Initial Design Value	Optimal Design Value
Mass (R1, Eq. (7))	57.7539	30.0235
Strain Energy (R2, Eq. (8))	3.7012	4.7966
2nd Natural Frequency (R3, Eq. (9))	43.5172	45.1856
3rd Natural Frequency (R4, Eq. (10))	64.1074	61.8756
4th Natural Frequency (R5, Eq. (11))	68.5334	65.0083
Maximum Force (R6, Eq. (12))	855.8250	736.9431
Maximum Torque (R7, Eq. (13))	1137.5000	812.3185

Table 6. Design variable values before and after continuous optimization procedures

Design Value	Initial Design Value	Optimal Design Value
V1	2.2154	1.8831
V2	0.4600	0.4137
V3	0.7000	0.7637
V4	0.0280	0.0238
V5	0.0028	0.0021

7.2. Results of discrete optimization

The continuous optimization procedure shown in section 7.1 is meaningful for design variables V1, V2, and V3. The two remaining variables, however, have to comply to commercially available tube dimensions. It means that the optimizer is only allowed to change them according to a pre – defined discrete set of values, as shown in Tab. (7).

Table 7. Discrete value sets for design variables V4 and V5

Physical Values		D.O.E. Coded values	
Inner Diameter (V4)	Wall Thickness (V5)	Inner Diameter (V4)	Wall Thickness (V5)
21.200	2.100	-0.96176	-1.00000
20.930	2.870	-1.00000	0.057692
26.645	3.378	-0.19062	0.755495
35.052	3.556	1.00000	1.00000

Taking these additional constraints into account, the optimization procedure leads to changes in system performance and configuration, which are shown in Tab. (8) and Tab. (9) respectivelly.

Table 8. Response values before and after discrete optimization procedure

Response	Initial Design Value	Optimal Design Value
Mass (R1, Eq. (7))	57.7539	27.3182
Strain Energy (R2, Eq. (8))	3.7012	6.4308
2nd Natural Frequency (R3, Eq. (9))	43.5172	36.5931
3rd Natural Frequency (R4, Eq. (10))	64.1074	52.2597
4th Natural Frequency (R5, Eq. (11))	68.5334	55.4126
Maximum Force (R6, Eq. (12))	855.8250	801.6395
Maximum Torque (R7, Eq. (13))	1137.5000	1004.8662

Table 9. Design variable values before and after discrete optimization procedure

Design Value	Initial Design Value	Optimal Design Value
V1	2.2154	2.0701
V2	0.4600	0.4600
V3	0.7000	0.7339
V4	0.0280	0.0212
V5	0.0028	0.0021

Comparing response results of Tabs. (5) and (8), for the continuous and discrete optimization procedures, respectivelly, it is possible to observe that the more constrained design space resulting from the discrete formulation does not necessarily means that the its optimum is worse than the one from the continuous case. In the case of the mini – baja prototype, the optimizer managed to find a suitable search direction along the design space leading to an even better result with respect to the mass, which is the most important design parameter.

It should be noted, however, that this result is obtained for discrete but unlinked values for V4 and V5, which does not correspond to the reality of commercially available structural tubes. In an attempt to express V5 as a function of V4, which is set to vary discretelly, the optimizer faces numerical conditioning problems with the implicit derivatives that arise when calculating the sensitivities of the responses with respect to V5. This lack of numerical conditioning forces premature convergence to a sub – optimum.

7.3. Robust design results

Design robustness with respect to variable operating conditions is determined for strain energy, maximum force and maximum torque (R2, R6 and R7, respectivelly).

The metric Cd (Eq. (3)) is used to quantify the robustness of candidate designs. Since there is a Cd value associated to each run of Tab. (3), a response surface can be interpolated for this metric.

These response surfaces are then added, in the form of additional constraints, to the optimization problem developed in section 7.2.

By means of Tab. (10), it is possible to notice that the robustness constraint penalizes the mass reduction achieved in section 7.2. In order to cope with stringent operating conditions, the optimizer makes a stiffer structure, which rises both the mass and the natural frequencies of interest of the mini – baja prototype.

Table 10. Response values before and after robust design procedure

Response	Initial Design Value	Optimal Design Value
Mass (R1, Eq. (7))	57.7539	48.8114
Strain Energy (R2, Eq. (8))	3.7012	1.2125
2nd Natural Frequency (R3, Eq. (9))	43.5172	55.8445
3rd Natural Frequency (R4, Eq. (10))	64.1074	85.8675
4th Natural Frequency (R5, Eq. (11))	68.5334	88.6585
Maximum Force (R6, Eq. (12))	855.8250	708.5000
Maximum Torque (R7, Eq. (13))	1137.5000	842.1000

The new design variable values thus determined are shown in Tab. (11). When comparing the results in Tabs. (9) and (11), it is evident that the additional design constraints posed by the robustness requirements force the optimizer to search for configurations that are signifficantly different from the initial design. When robustness is not considered, on the other hand, the difference between the initial and the optimum design variable sets is less signifficant.

Table 11. Design variable values before and after robust design procedure

Design Value	Initial Design Value	Optimal Design Value
V1	2.2154	1.8831
V2	0.4600	0.5200
V3	0.7000	0.8050
V4	0.0280	0.0350
V5	0.0028	0.0021

8. CONCLUSIONS AND SUGGESTIONS FOR FUTURE RESEARCH WORK

The response surface based meta – modeling technique used in this work managed to represent the behaviour of a complex system in order to optimize its performance at the expense of reasonable amounts of computational resources, in both continuous and discrete formulations. Also, complex and abstract design aspects such as the robustness metrics find quite satisfactory representation in the response surface models. The absence of convergence problems regarding the numerical optimization procedure should also be noted for the continuous case, whilst the implicit design variable dependence in the discrete case resulted in some numerical difficulties.

The statistical methodology underlying the R.S.M. provides the means necessary to implement robust engineering studies, aiming at the maintenance of the system performance even if its operational conditions change.

The results obtained had shown, however, that optimality and robustness may be conflicting characeristics. For this reason, interesting research can be conducted in order to find adequate formulations so that the robustness issues are embedded into the optimization problem. This may help to improve the compatibility between optimality and robustness, leading to designs that may maintain high level average performance under a broad range of operating conditions.

This may be achieved by reformulating the response surface meta – models that describe the system behaviour, so that performance criteria are also expressed in terms of the operating conditions. This will ease and reduce the computational effort necessary to perform extensive simulation studies, whose resulting statistical metrics may then be embedded as objective and/or constraint functions within an optimization procedure.

9. ACKNOWLEDGEMENTS

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