II CONGRESSO NACIONAL DE ENGENHARIA MECÂNICA
II NATIONAL CONGRESS OF MECHANICAL ENGINEERING
12 a 16 de Agosto de 2002 - João Pessoa – PB

A COMPARISON BETWEEN GRADIENT BASED AND INTELLIGENT OPTIMIZATION TECHNIQUES FOR THE DESIGN OF AN AUTOMITIVE COMPONENT

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Abstract. *This paper presents a case study involving a heavy truck side bumper and the optimal design concerning its vibrating aspects. The dynamical behaviour is modeled by means of response surfaces constructed from the results of computer experiments. These statistical meta models, presented as closed form polynomials are then used within different optimization approaches: gradient optimization techniques, such as the modified feasible directions method, in comparison with intelligent optimization methods, such as genetic algorithms, simulated annealing, and tabu search. Besides comparing the optimization results obtained from the two basic approaches mentioned above, the robustness of the intelligent algorithms is verified by varying the random sets of initial designs. It is important to mention that intelligent techniques are difficult to be used in many design applications because of the high computational cost to compute the fitness function. This computational overhead can be significantly reduced when using meta models to represent the fitness function. In this paper this is achieved by creating response surfaces. Numerical results support these assumptions*.

Keywords *Gradiente Based Optimization, Heuristic Optimization, Meta Models*

1. INTRODUCTION

Numerical optimization techniques have been widely used in general design of mechanical systems. More specifically, in the field of mechanical systems dynamics, many applications have been developed by involving automatic design modifications: resonance and critical speed avoidance, vibration level reduction, model updating and the like.

Numerical optimization techniques take advantage of computer automation capabilities through a set of mathematical methods. The standard mathematical formulation of the optimization problem is as follows (Vanderplaats, 1998):

$$\max, \min\left(F\left(\{X\}\right)\right) \tag{1}$$

that is, find the best possible (minimum or maximum) value of a function that represents a performance criterion, subject to

$$G_j\left(\{X\}\right) \leq 0 \tag{2}$$

standing for a set of threshold values to j aspects of system performance

$$H_k\left(\{X\}\right) = 0 \tag{3}$$

that is, a set of target values to k aspects of system performance, and

$$\{X\}^{LB} \leq \{X\} \leq \{X\}^{UB} \tag{4}$$

which are bounds to the values of the elements contained by the vector {X}. These elements are called design or decision variables (whose initial values are denoted as X^0), and all the functions (F, G and H) involved in the optimization problem depend upon these variables.

2. GRADIENT BASED OPTIMIZATION METHODS

These are the most traditional and widely used design optimization methods, due to their reliability and efficiency in a wide range of engineering applications. Three fundamental steps are usually necessary to implement them:

- *Definition of the search direction* – This procedure is the optimization algorithm itself. Gradients of the objective function (in the sequential mehtods) and both objective and constraint functions (in the direct methods) are manipulated in order to establish search directions along the design space.

- *Definition of the step in the search direction* – Once a search direction {S} is defined in the previous step, the general optimization problem is restricted to a one – dimensional search. The quantity α in Eq. (5) is the size of the optimizer's move along the search direction in order to update the design configuration from $\{X\}^i$ to $\{X\}^{i+1}$.

$$\{X\}^{i+1} = \{X\}^i + \alpha \cdot \{S\} \tag{5}$$

- *Convergence verification* – Convergence is achieved for the design variable set $\{X^*\}$ upon the satisfaction of the Kuhn – Tucker conditions, expressed by Eqs. (6) to (8):

$$G_j\left(\{X^*\}\right) \leq 0 \tag{6}$$

$$\lambda_j \cdot G_j\left(\{X^*\}\right) = 0 \tag{7}$$

$$\left[\nabla F\left(\{X^*\}\right) + \sum_j \lambda_j \cdot G_j\left(\{X^*\}\right)\right] = 0 \tag{8}$$

where λ_j are the Lagrange multipliers.

3. HEURISIC OPTIMIZATION METHODS

Also known as “random” and “intelligent” optimization strategies, this group of optimization methods vary the design parameters according to probabilistic rules. It is common to resort to random decisions in optimization whenever deterministic rules fail to achieve the expected success. On the other hand, however, heuristic techniques tend to be more costly, sometimes to the point that certain applications are not feasible unless alternative formulations, designed to spare computational resources, are introduced. Such formulations comprise the response surface meta – modeling method, that is used in this paper to represent system responses to be optimized by heuristic methods, as well as the deterministic (gradient based) ones.

3.1. Genetic algorithms

Genetic Algorithms are random search techniques based on Darwin’s “survival of the fittest” theories, as presented by Goldberg (1989). Genetic algorithms were originated with a binary representation of the parameters and have been used to solve a variety of discrete optimization problems. A basic feature of the method is that an initial population evolves over generations to produce new and hopefully better designs. The elements (or designs) of the initial population are randomly or heuristically generated.

A basic genetic algorithm uses four main operators, namely *evaluation*, *selection*, *crossover* and *mutation* (Michalewicz, 1996), which are briefly described in the following:

- *Evaluation* – the genetic algorithms require information about the fitness of each population member (fitness corresponds to the objective function in the classical optimization techniques). The fitness measures the adaptation grade of the individual. An individual is understood as a set of design variables. No gradient or auxiliary information is required, only the fitness function is needed.
- *Selection* - the operation of choosing members of the current generation to produce the prodigy of the next one. Better designs, viewed from the fitness function, are more likely to be chosen as parents.
- *Crossover* – the process in which the design information is transferred from the parents to the prodigy. The results are new individuals created from existing ones, enabling new parts of the solution space to be explored. This way, two new individuals are produced from two existing ones.
- *Mutation* – a low probability random operation used to perturb the design represented by the prodigy. It alters one individual to produce a single new solution that is copied to the next generation of the population to maintain population diversity.

3.2. Simulated annealing (Saramago, Assis and Steffen, Jr, 1999)

Annealing is a term from metallurgy used to describe a process in which a metal is heated to a high temperature, inducing strong perturbations to its atoms positions. Providing that the temperature drop is slow enough, the metal will eventually stabilize into an orderly structure. Otherwise, an unstable atom structure arises.

Simulated annealing can be performed in design optimization by randomly perturbing the decision variables and keeping track of the best resulting objective value. After many tries, the most successful design is set to be the center about which a new set of perturbations will take place.

In an analogy to the metallurgical annealing process, let each atomic state (design variable configurations) result in an energy level (objective function value) E. In each step of the algorithm, the atoms positions are given small random displacements due to the effect of a prescribed temperature T (standard deviation of the random number generator). As an effect, the energy level undergoes a change ΔE (variation of the objective function value). If $\Delta E \leq 0$, the objective stays the same or is minimized, thus the displacement is accepted and the resulting configuration is adopted as the starting point of the next step. If $\Delta E > 0$, on the other hand, the probability that the new configuration is accepted is given by Eq. (9):

$$P(\Delta E) = e^{\left(-E/k_b T\right)} \tag{9}$$

where k_b is the Boltzman constant, set equal to 1.

Since the probability distribution in Eq. (9) is chosen, the system evolves into a Boltzman distribution. The random numbers r are obtained according to an uniform probability density function in the interval (0,1). If $r < P(\Delta E)$ the new configuration is retained. Otherwise, the original configuration is used to start the next step.

The temperature T is simply a control parameter in the same units as the objective function. The initial value of T is related to the standard deviation of the random number generator, whilst its final value indicates the order of magnitude of the desired accuracy in the location of the optimum point. Thus, the annealing schedule starts at a high temperature which is discretely lowered (using a factor $0 < r_t < 1$) until the system is "frozen", hopefully at the optimum, even if the design space is multimodal.

3.3. Taboo search (Pham and Karaboga, 2000)

This method was originally developed to solve combinatorial optimization problems. It is a kind of iterative search and is characterized by the use of a flexible memory, being regarded for its optimization capabilities even when multimodal design spaces are concerned.

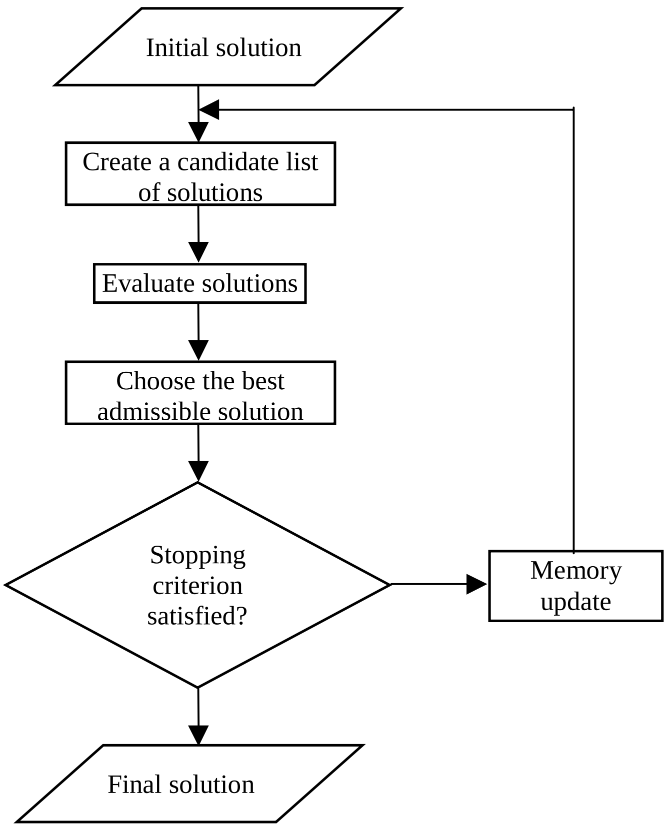

Figure 1. Basic taboo search flowchart

The basic taboo search algorithm is performed according to four fundamental elements:

- *Forbiding strategy* – It is employed to avoid cycling problems by forbidding certain types of design variable changes, that is, classifying them as “taboo”. Ideally, a taboo list with all the points of the design space which were previously visited by the optimizer may be created. Due to memory and CPU power limitations, however, the taboo list is reduced to the positions visited along the last *Ts* iteractions, and it is assumed that the cycling problem is prevented if these design space points are checked. If the taboo list size *Ts* is small, the probability of cycling rises. On the other hand, too long *Ts* values may lead the optimizer to leave certain design space regions before a sufficiently detailed search is performed. Once the taboo list is full, replacement is performed on a *first-in-first-out* (FIFO) basis.
- *Aspiration criteria* – Are used to set performance specifications that guide the search procedure. If a solution is of sufficient quality (that is, close enough to the aspiration criterion), it is made free from the taboo list. Asiration criteria can be either time independent (constant along the optimization problem) or, in a more sophisticated case, time dependent (the quality of visited solutions is used to set new aspiration criteria).
- *Freeing strategy* – It is used to decide what leaves the taboo list in order to be reconsidered in further search steps. The freeing strategy is concerned if a given solution has passed the aspiration criterion test.
- *Overall strategy* – It manages the interplay between the three precedent elements, according to the flowchart presented in Fig. (1).

4. META MODELING AND RESPONSE SURFACE APPROXIMATIONS

Meta models are statistical surrogates used to represent cause – and – effect relationships between design parameters and responses of interest within a given design space. They are constructed according to the four fundamental steps whose brief descriptions follow:

a) Experimental design – a design space, including a range of design possibilities, is sampled in order to reveal its contents and tendencies. Each sample is a combination of design variable values;
b) Choice of a model – the nature of the “meta-model” itself is determined, taking into account that the relations contained in the data gathered in the previous step have to be symbolically represented, with the highest possible accuracy;
c) Model fitting – the model whose shape is defined in “b” is fitted to the data collected in “a”. Differences in fitting schemes may affect the efficacy of “meta-modeling” techniques in the solution of a given problem. In the case of the Response Surface Method (R.S.M.), the least squares formulation is adopted, as shown in Eqs. (10) and (11):

$$\{Y\} = [E]\cdot\{B\} + \{\delta\} \tag{10}$$

where {Y} is the vector of responses (dependent variables) obtained for each line of the matrix [E] which corresponds to the experimental design stage of meta-modeling. The vector {δ}contains free, random error terms. The vector of model parameters {B} can be estimated as follows:

$$\{B\} = \left([E]'\cdot[E]\right)^{-1}\cdot[E]'\{Y\} \tag{11}$$

where the term $\left([E]'\cdot[E]\right)^{-1}$ comes directly from the experimental matrix and is called the variance-covariance matrix, a very important element in evaluating the quality of the meta-model, as referred to in item “d”.

d) Verification of model accuracy – the three precedent steps are sufficient to build a first tentative model, whose overall quality and usefulness have to be evaluated by adequate sets of metrics. Each combination of design space sampling, model choice and fitting procedure leads to the use of specific verification procedures.

5. COMPROMISE OPTIMIZATION FORMULATION

Vanderplaats (1998) presents a compromise optimization formulation as in Eq.(12):

$$F(X)=\left\{\sum_{k=1}^{K}\left[\frac{W_k\{F_k(X)-F_k^*(X)\}}{F_{pk}(X)-F_k^*(X)}\right]^2\right\}^{\frac{1}{2}} \tag{12}$$

where:

- $F(X)$ is a compromise objective function
- F_k is the *k-th* response of interest, in a total of K
- F^*_k is the target value for the *k-th* response
- F_{pk} is the worst value accepted for the *k-th* response
- W_k is the weighting factor applied for the *k-th* response of interest

This formulation is well regarded because it considers engineering specifications through F^*_k and F_{pk}, which helps in keeping a practical insight over the optimization problem.

It should be noted that the optimization problem defined through Eq. (12) is unconstrained because the K responses encompass both objective and constraint functions. This is useful when heuristic optimization techniques are used because most of the times it is not trivial to implement the handling of explicit constraints when such methods are used.

6. ILLUSTRATIVE CASE STUDY

(Butkewitsch and Steffen, Jr, 2001) describe the meta – modeling procedure used to build response surface surrogates for the three first natural vibrating frequencies of the heavy truck side bumper whose finite element model is depicted in Fig. (2).

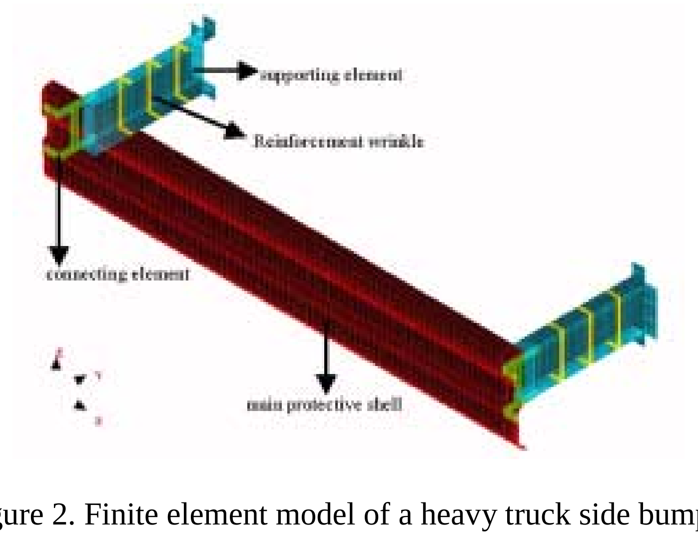

Figure 2. Finite element model of a heavy truck side bumper

Figure 2 highlights constructive characeristics that can be used as design variables:

- V1: thickness of the main protective shell;
- V2: thickness of supporting element shell;
- V3: thickness of connecting element shell;
- V4: position of the first reinforcement wrinkle;
- V5: position of the second reinforcement wrinkle;
- V6: position of the third reinforcement wrinkle;
- V7: width of the connecting element;
- V8: length of the connecting element;
- V9: height of the curvature in the main protective shell;

The resulting response surface models for the first three natural vibrating frequencies, written in terms of design variables *V1* to *V9*, are given by Eqs. (13) to (15).

$$F1(V1,V7,V8,V9) = 16.00857 - 0.92484 \cdot V1 - 0.28596 \cdot V7 - 0.62578 \cdot V8 - 0.45209 \cdot V9 - 0.40292 \cdot V8^2 \quad (13)$$

$$F2(V1,V8,V9) = 28.18584 - 0.81971 \cdot V1 + 0.35551 \cdot V8 + 2.90590 \cdot V9 - 1.3639 \cdot V9^2 \quad (14)$$

$$F3(V1,V2,V7,V8,V9) = 36.52211 - 0.81285 \cdot V1 + 0.29906 \cdot V2 + 0.22171 \cdot V7 + 0.35785 \cdot V8 + 3.86218 \cdot V9 - 0.20076 \cdot V7^2 - 0.14615 \cdot V8^2 - 1.02648 \cdot V9^2 \quad (15)$$

It should be noted that design variables *V3*, *V4* and *V5* do not participate of the response surface equations because they are not statistically signifficant (Box and Draper, 1987).

Equations (13) to (15) are then used to build a compromise optimization function that reflects the design optimization problem summarized in Tab. (1):

Table 1. Target and worst admissible values for natural frequency optimization

	F1 (Hz)	**F2 (Hz)**	**F3 (Hz)**
Target Value (F^*_k)	18.00	30.00	40.00
Worst Admissible Value (F_{pk})	16.00	28.00	36.50

It should be noted that the worst admissible values correspond to the initial design X^0 (where natural frequencies coincide with typical excitation frequencies of the system). At this configuration, all design variables are set to zero, according to the reponse surface coding adopted in this work. Thus, the response values are equal to the grand avrerage or free terms of the response surface equations (Eqs. (13) to (15)).

The results obtained by means of gradient based, genetic algorithms, simulated annealing and taboo search are shown in Tab. (2).

Table 2. Response values obtained through gradient based and heuristic optimization methods

	Gradient Based	Genetic Algorithm	Simulated Annealing	Taboo Search
F1 (Hz)	17.11	16.72	16.77	16.27
F2 (Hz)	30.18	30.52	30.48	29.12
F3 (Hz)	39.25	40.19	39.97	37.92

It is important to highlight that the results obtained by the heuristic techniques may vary each time the optimization procedure is repeated, due to the random nature of these methods. For this reason, their utilization is repeated 1000 times and the robustness of the optimum results can be checked by means (μ) of the mean and standard deviation (σ) values displayed in Tab. (3). The subscipts GA, AS and TS stand for the initials of each method's names: Genetic Algorithms, Simulated Annealing and Taboo Search, respectivelly.

Table 3. Means and standard deviations of heuristic optimum solutions

	Genetic Algorithm		Simulated Annealing		Taboo Search	
	μ_{GA}	σ_{GA}	μ_{SA}	σ_{SA}	μ_{TS}	σ_{TS}
F1 (Hz)	16.72	3.25e-005	16.62	0.14	15.90	0.70
F2 (Hz)	30.52	1.07e-005	30.50	0.06	27.61	1.83
F3 (Hz)	40.19	4.97e-005	40.09	0.19	35.92	2.36

It is also possible to have a graphical insight of the data in Tab. (3) by means of the ten category histograms depicted in Figs. (3) to (5).

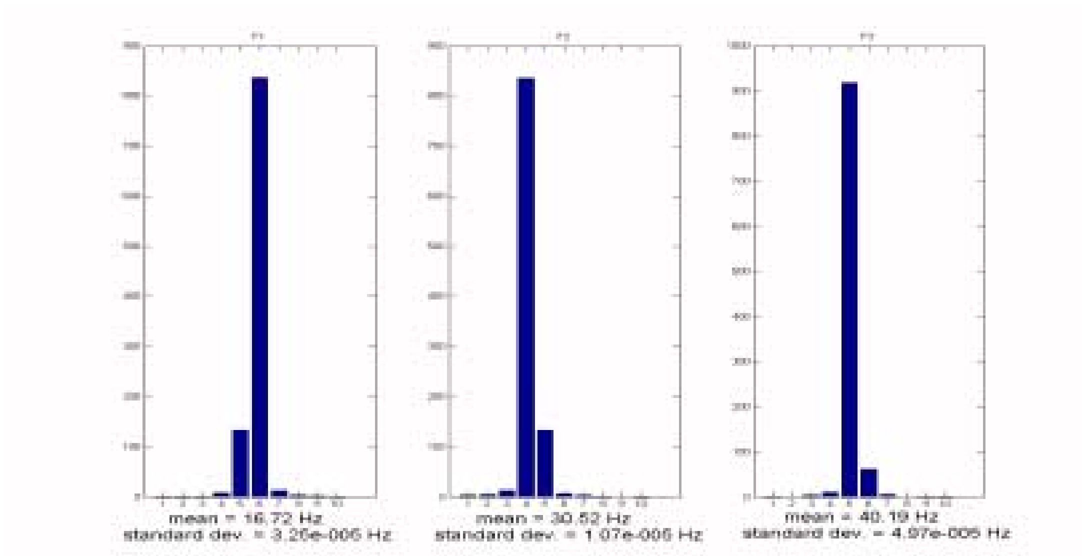

Figure 3. Histogram distribution of the optimum response values obtained with the Genetic Algorithm method

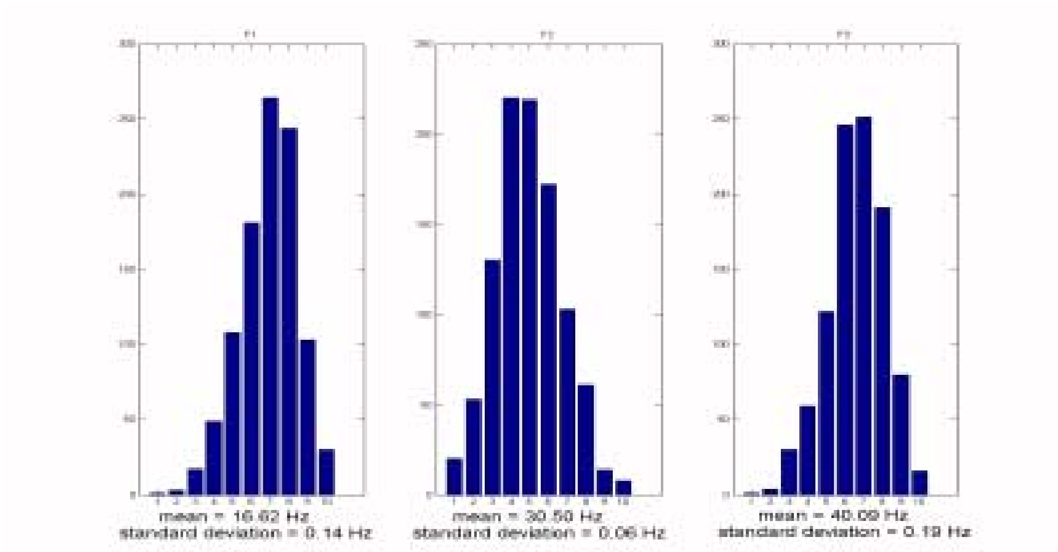

Figure 4. Histogram distribution of the optimum response values obtained with the Simulated Annealing method

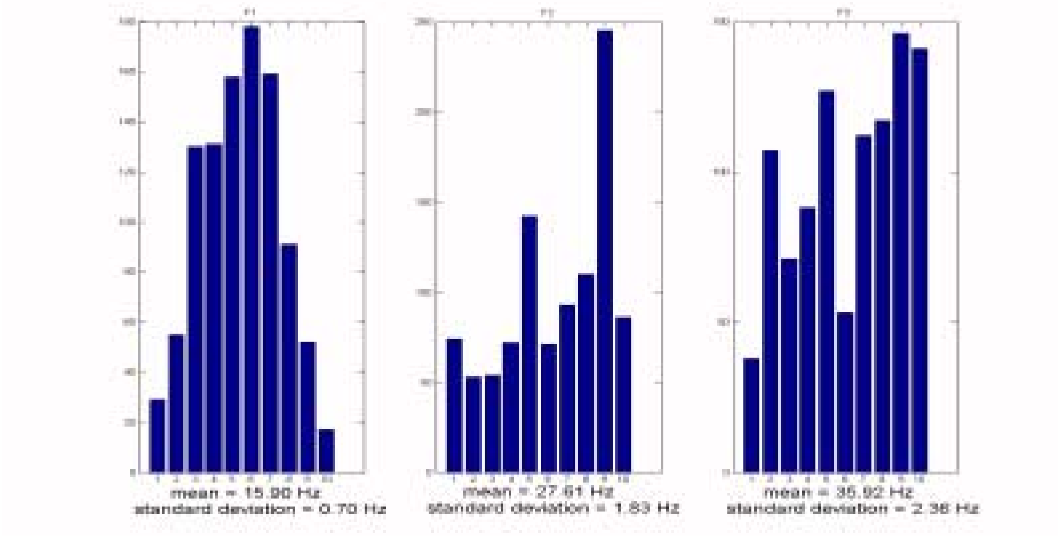

Figure 5. Histogram distribution of the optimum response values obtained with the Taboo Search method

7. ANALYSIS, CONCLUSIONS AND OUTLINE OF FUTURE RESEARCH

This work shows a comparison between gradient based and inteligent optimization techniques. The theoretical framework of these methods is outlined and an optimization problem associated to the vibratory behaviour of an automotive component is solved in order to illustrate the theory with numerical data from an engineering application.

The results listed in Tabs. (2) and (3) show that gradient based, genetic algorithms and simulated annealing are methods which perform very similarly for the problem presented in section 6. This may be attributed to the fact that response surface models generate well conditioned design spaces, reducing the likelihood of gradient based methods being trapped into local optima. On the other hand, numerical evaluation of the same response surface surrogates is usually unexpensive, which reduces the C.P.U. overhead associated with the use of heuristic optimization methods.

These two aspects mean that when responses of interest in engineering problems are represented by means of statistical meta – models, the range of applicable optimization methods may be increased, helping to find superior solutions and to assure their optimality, when similar optimum values are determined by means of several different methods.

Table 3 and Figs. (3) to (5) show interesting facts about the robustness of the three heuristic optimization methods used in the solution the side bumper natural frequencies repositioning.

Genetic algorithms are capable of obtaining results with minimum variation, showing low sensitivity to the randomness of the several parameters that participate of the optimization procedure.

The variation of the optimum solutions obtained with the simulating annealing method is somewhat higher to that of the genetic algorithm approach. However, its distribution adheres quite well to a normal probability density function, showing that result variation is prone to behave in a very predictable manner.

Tables (2) and (3), besides Fig. (5) lead to different conclusions about the taboo search techique. In the average, this method obtained very little progress in optimizing the objective function, and its results present the highest dispersion figures among the heuristic methods tested in this work. The literature available, very sparse in comparison to that of the other two methods, is not capable of establishing safe guidelines about the configuration of a set of parameters to which the method is very sensitive. The most important one is *Ts* the size of the taboo list. Several different values were tried and led to very variable, yet not satisfactory results.

Thus, the application of taboo search to complex engineering problems deserves deeper research, because in this situations the method's performance may be different of that obtained with simple test functions.

Future research work should also be devoted to the study of neural networks as optimization tools, since they operate under the principle that a performance criterion has to be optimized. Hopfield networks and Kohonen self – organising maps are regarded as suitable architectures for design optimization applications.

Another interesting alternative comprises the use of more sophisticated meta – models to represent the inherent non – linearities of complex design spaces. In such situations, the risk of multimodalities rises, and the performance differences between gradient based and heuristic methods may lead to new findings.

8. ACKOWLEDGEMENTS

The authors acknowledge the financial support granted by FAPEMIG through TEC 2243/97.

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