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ON THE APPLICATION OF MULTI-BODY AND FINITE ELEMENT ANALYSIS FOR THE STRUCTURAL SYNTHESIS OF A MINI-BAJA VEHICLE

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Abstract. *This work deals with the use of CAE tools for the optimization of a mini-baja prototype chassis frame. Multi – body analysis is used in the assessment of the loads that act over the mini-baja structure. Once these loads are identified, they are fed into a finite element model that describes the mechanical behaviour of an initial version of the mini-baja frame. A numerical optimiser is then used to generate subsequent versions of the structure until an optimal design is reached. The optimization strategy combines sizing (tuning of intrinsic elemental properties) and shape (relocation of nodes) optimization techniques, along several different statements of the optimization problem in terms of objectives and constraints definition. The analysis and comparisons of the results obtained lead to the conclusions and the outline of future research work.*

Keywords *multi-body analysis, finite element analysis, shape and sizing optimization*

1. INTRODUCTION

The use of computer aided engineering (CAE) tools is a well established design development practice both in academia and in industry.

This spread is due to the availability of powerful computational resources at affordable cost, as well as the existence of numerical methods (Griffiths and Smith, 1991) that fulfil many of the needs in the field of design development.

The use of computers in engineering design development offers the advantage of reducing the time and the cost associated to experimental methods, besides making it easier to obtain relevant information about the system regarding different configurations.

Another positive aspect is related to the fact that mathematical models are obtained from physical laws, which leads to a clear and complete statement of the influence parameters and their action pattern over the system of interest. This reduces the possibility of analysis errors due to limitations on the experimental conditions.

2. NUMERICAL OPTIMIZATION AND AUTOMATIC SYNTHESIS

Numerical optimization techniques take advantage of computer automation capabilities through a set of mathematical methods. The standard mathematical formulation of the optimization problem is as follows (Vanderplaats, 1998):

$$\max,\min\left(F\left(\{X\}\right)\right) \tag{1}$$

that is, find the best possible (minimum or maximum) value of a function that represents a performance criterion, subject to

$$G_j\left(\{X\}\right) \leq 0 \tag{2}$$

standing for a set of threshold values to j aspects of system performance

$$H_k\left(\{X\}\right) = 0 \tag{3}$$

that is, a set of target values to k aspects of system performance, and

$$\{X\}^{LB} \leq \{X\} \leq \{X\}^{UB} \tag{4}$$

which are bounds to the values of the elements contained by the vector {X}. These elements are called design or decision variables (whose initial values are denoted as X^0), and all the functions (F, G and H) involved in the optimization problem depend upon these variables.

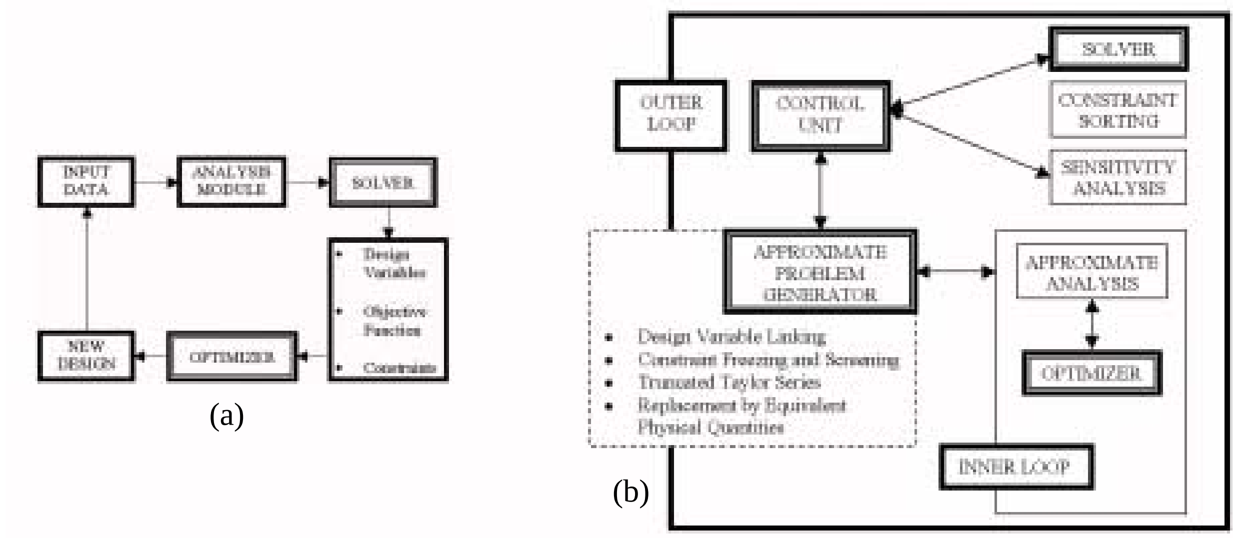

Figure 1. Comparison between direct coupling and synthesis approach for optimization

It is extremely important to notice that the definition of equations (1) to (3) can be done both in terms of analytical expressions (closed form equations) and numerical data (discrete values resulting from numerical software). Of course, a closed form representation is desirable, since it is much more convenient, but they are usually available only for simple design cases. Most of the times, numerical data calculated by means of analysis modules are the only available form to represent the functions involved in the statement of the optimization problem. This situation suggests the coupling between optimization and analysis modules, and the simplest form to do so could be as described by Fig. (1.a):

The direct coupling between analysis and optimization modules, as indicated in Fig. (1.a), results in the evaluation of the entire numerical model, by means of the analysis module, at all iterations performed by the optimizer along its search for an optimal solution. Since the CPU effort associated to a single analysis is multiplied by the (usually high) number of function evaluations during optimization, this scheme is often associated with a computatopnal overhead that disencourages its use.

Based on the fact that not all the parts of the model contribute all the time to the progress of the optimization procedure, a set of algorithms known as structural synthesis techniques (Schmidt, 1960) has been proposed to rationalize the use of computer resources when coupling optimizers to numerical analysis software. Figure (1.b) presents the framework of the synthesis approach.

It is important to highlight the role of the approximate problem generator in the framework presented in Fig. (1.b). This set of algorithms is the ultimate responsible for the feasibility of coupling numerical optimizers to analysis software, since its use results in signifficant drop of the need for computer resources in comparison to the optimization scheme presented in Fig. (1.a).

The following sub-sections present a brief description of the techniques implemented within the approximate problem generator.

2.1. Design variable linking

By establishing linear relations among design variables, one can reduce the amount of independent variables to be evaluated. Economy of CPU resources is noticeable due to the drop in the number of partial derivatives of the responses, to be caculated with respect to the independent design variables. Additonal advantages introduced by this technique are:

- The relations among design variables are directly controlled by the user. This helps to keep physical insight.
- The laws of dependence can be useful to enforce desirable design features, such as symmetry and parallelism, as shown in Fig. (2).

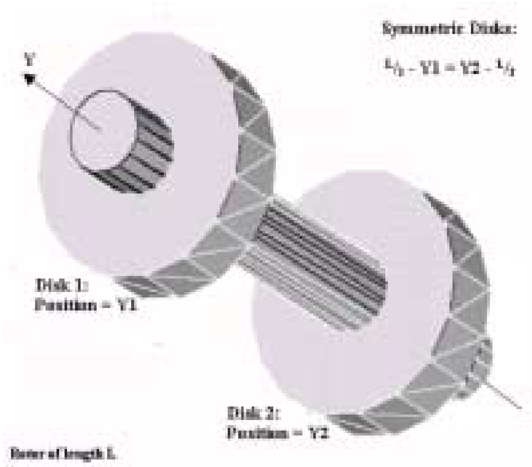

Figure 2. Illustration of design variable linking

2.2. Constraint freezing and screening

If a given subset of all the prescribed constraints has no risk of violation, it is useless to waste CPU time with their evaluation and the calculation of their derivatives with respect to the design variables. Hence, for the sake of feasibility, the constraints far from the violation threshold (TRS) can be neglected until their importance grows (i.e., risk of violation arises) at a different stage of the automated design process. This concept is illustrated graphically in Fig. (3.a).

For the purpose of further CPU economy, it should be considered that numerical models are discrete, which leads to the deployement of the constraints prescribed in the statement of the optimization problem. For instance, consider a group of thousands of shell finite elements employed to model an aeronautic airfoil. If one desires to minimize the structure's weight keeping track of fatigue stress levels, constraints must be prescribed over all the elements of the airplane's wing. Just a few (NSTR) elements, however, can be considered at each optimization iteration, due to their superior representativity in comparison with the others. The effect of NSTR on constraint screening is illustrated in Fig. (3.b).

It should be noted that the procedures of constraint freezing and constraint screening are overlapped (i.e., used in conjunction) in order to avoid the heavy calculations involved in the evaluation of unnecessary constraint functions: the number of such functions is thus reduced to the least possible.

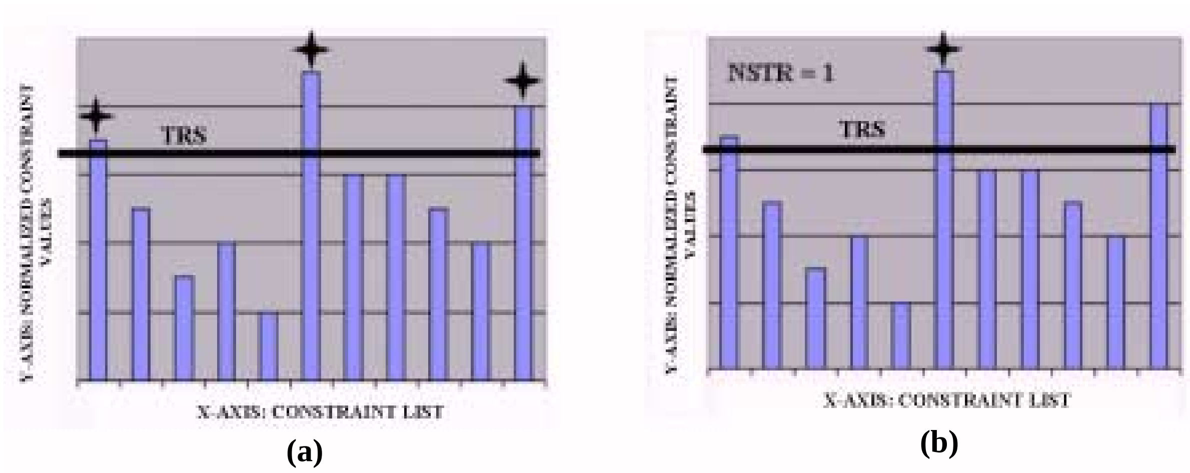

Figure 3. Illustration of constraint freezing and screening procedure

2.3. Truncated Taylor series

Up to this moment, the model reduction techniques presented acted on quantitative basis, that is, alternatives to reduce the number of independent design variables and constraints were indicated.

Although these solutions are efective, more CPU power can be saved if one analyses the qualitative aspects related to the functions evaluated during the optimization procedure. If several simple functions are eliminated by means of constraint deletion and screening but a few very complex, highly non-linear functions remain, still too much computer effort will be employed.

For this reason, it is very interesting to simplify the functions involved by means of linearization. This can be performed by expanding the functions in a Taylor Series to be truncated at the first term, as indicated in the Eq. (5).

$$
\begin{aligned}
f\left(x^0+\Delta x\right) &= f\left(x^0\right)+\left.\frac{df}{dx}\right|_{x^0}\bullet\Delta x+\left.\frac{d^2 f}{dx^2}\right|_{x^0}\bullet\frac{\Delta x^2}{2!}+\left.\frac{d3f}{dx^3}\right|_{x^0}\bullet\frac{\Delta x^3}{3!}+\ldots \\
\tilde{f}\left(x^0+\Delta x\right) &= f\left(x^0\right)+\left.\frac{df}{dx}\right|_{x^0}\bullet\Delta x
\end{aligned}
\qquad (5)
$$

2.4. Replacement by equivalent physical quantities

Still in the effort of simplifying the functions involved in the numerical non-linear optimization process, a special group of algorythms was developed, having in mind engineering situations often present in design optimization tasks.

2.4.1. Use of Internal forces inplace of stresses (Vanderplaats and Salajegheh, 1989)

A very common structural optimization task aims to obtain the minimum possible mass without violating stress constraints. Since the material (and consequently its desnsity) are very seldom considered as design variables, the optimizer has to impose changes to the geometric parameters and the area is chosen, most of the times, as the design variable.

In such a formulation, the objective function displays a linear, explicit relation with respect to the design variables. The same, however, does not hold true for the constraints, because stresses and areas relate with each other by means of a reciprocal mathematical function. This situation poses a special difficulty for the optimizer because the optimization problem is usually strongly driven by the constraints, which get involved with this nonlinearity problem in this particular kind of formulation.

Indeed, one would prefer, for the sake of computational economy, the objective function to became nonlinear, and the constraints linear with respect to the design variables. This switch can be done if the design variables became the reciprocal of the areas, which is equivalent to integrate the stresses with respect to the areas, obtaining the internal forces. For this reason, it is a usual procedure to replace stresses by internal forces in structural synthesis problems.

2.4.2. Rayleigh coefficient instead of eigenvalues (Canfield, 1990)

Another very usual optimization challenge in design engineering environment is the achievement of an ideal dynamic behaviour in terms of the structure natural frequencies.

The mathematical model used to deal with dynamic systems, however, leads to complicated matrix manipulations used to determine the eigenvalues.

Keeping the objective of saving computational effort, the natural frequencies can be estimated based on the Rayleigh coeficient, a dimensionless scalar that relates the kinetic and elastic energies of the vibrating structure:

$$\lambda = \omega^2 = \frac{\phi^T \cdot K \cdot \phi}{\phi^T \cdot M \cdot \phi} = \frac{U}{T} \tag{6}$$

where K, M and ϕ are the stiffness, mass and modal matrices, respectivelly. The scalars ω, λ, U and T stand for the natural frequency, its associated eigenvalue and the potential and kinetic energies.

3. TYPES OF OPTIMIZATION STRATEGIES FOR STRUCTURAL SYNTHESIS

3.1. Sizing Optimization

In a sizing optimization problem, as illustrated by Fig. (4), the design engineer deals with design variables that have some sort of linear or nonlinear relationship with intrinsic model properties, such as finite element cross section dimensions, thicknesses and the like. In the case of multi-body analysis, sizing optimization may be used to relate design variables to properties such as spring stifnessess and dumper constants.

One interesting feature regarding sizing optimization is its flexibility, since the design variable values can overwrite model properties with no need to formulate a brand new model for each tentative configuration.

Sizing Design Variables:

- h
- t_1
- t_2
- b_1
- b_2
- t_w

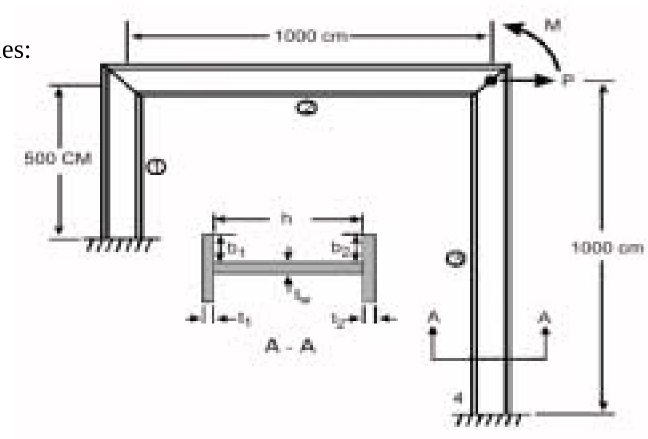

Figure 4. Illustration of a sizing optimization procedure

3.2. Shape Optimization

Shape optimization techniques aim at defining geometric configurations that meet a stated optimization criterion. In the case of finite element based structural optimization, the mesh corresponding to the initial design undergoes certain perturbations (as illustrated by Fig. (5)) so that the nodes are forced into new positions, in order to optimize the responses of interest. Through the design variable values, the optimizer controls the magnitude of the perturbations imposed over the positions of the nodes.

The general approach consists of generating the basis of a vector space. The displacements of all nodes during the shape optimization procedures are a linear combination of the basis.

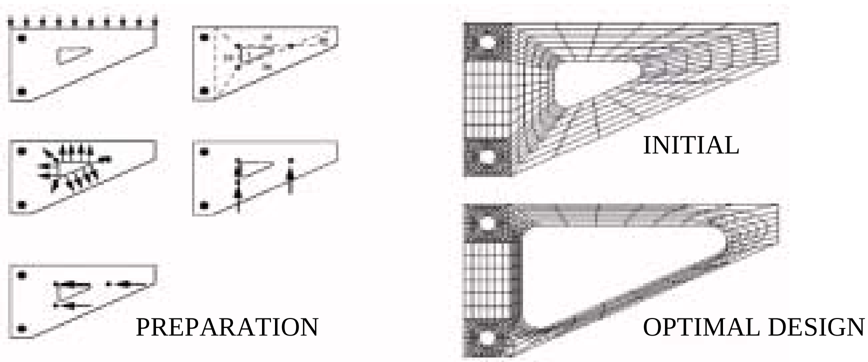

Figure 5. Illustration of a shape optimization procedure

4. ILLUSTRATIVE CASE STUDY

The diagram displayed in Fig. (6) presents the improvement strategy chosen for the design of the mini-baja chassis frame:

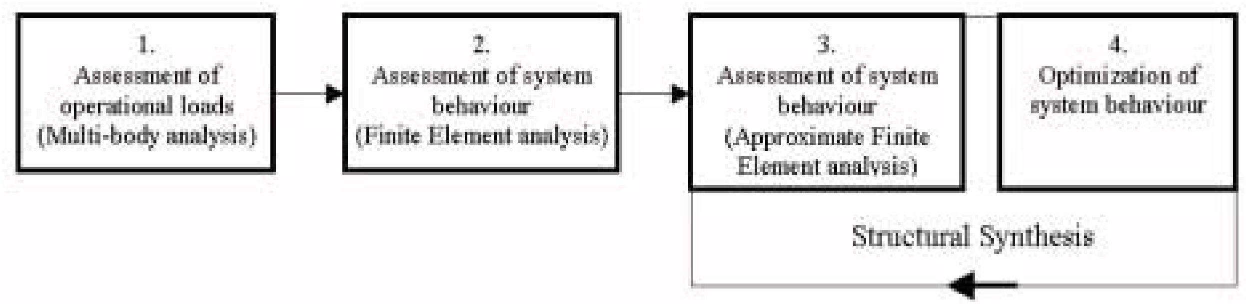

Figure 6. Strategy for the improvement of the mini-baja design

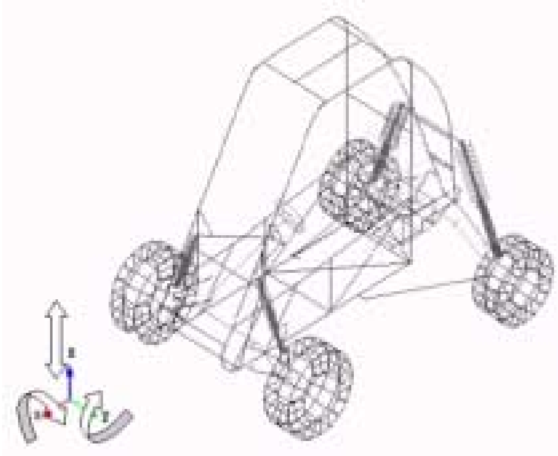

Figure 7. Multi-body model of the initial version of the mini-baja prototype

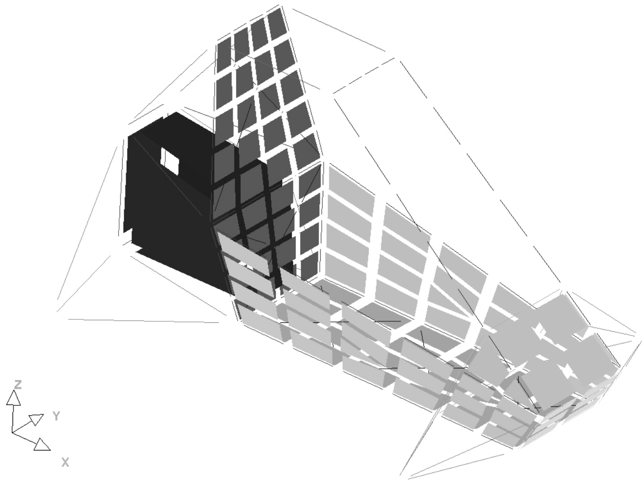

Figure 8. Finite element model of the initial version of the mini-baja prototype chassis

4.1. Assessment of operational loads

The characterization of the loads acting over the structure is implemented through the dynamical analysis of a mathematical model whose formulation in based on the multi – body technique, as shown in Fig. (7). The structure is regarded as a rigid component to which the suspension and the remaining parts subject to joint motion constraints are attached. The Mini – Baja prototype model is composed of 24 parts comprising a total of 11 degrees of freedom. Its configuration allows the analysis of the linear and angular motions along the longitudinal, transversal and vertical directions.

The mass and inertia parameters of the majority of the components are obtained through geometric solid models defined by CAD tools. The springs and tires stiffnesses are estimated by means of tension and compression experimental tests developed with controled displacements and speeds.

The loads acting over the structure originate at the contact between the vehicle's tires and the ground, due to displacements created by irregularities present along the track surface. The irregularities profile is defined as a series of harmonic functions whose amplitude is calculated based on the spectral density function defined for an off – road track, along a frequency band comprised between 0.10 Hz to 7.00 Hz. Besides the inherent irregularities, the track also contains individual obstacles that arise from the standard track profile and are mathematically represented as step functions.

4.2. Assessment of system behaviour (initial version)

Given the load characterization as in section 4.1, a finite element model of the mini-baja (Fig. (8)) prototype reveals its behaviour under a set of operational conditions. Among these analysis results, it should be highlighted that:

- The system mass should be reduced to improve performance;
- The resonance involving the engine excitation at maximum torque rotation (2600 rpm ⇒ 43.33 Hz) and maximum power rotation (4000 rpm ⇒ 66.67 Hz) and the natural frequencies of orders two, three and four should be avoided;
- The stresses and dynamic forces (due to track and powertrain excitations) have to be kept within acceptable boundaries to ensure the prototype durability.

4.3. Fine tuning of system parameters by automatic structural synthesis

The observations listed in the precedent section give rise to the definition of an optimization problem based upon the design variables and responses listed in Tab. (1) and Tab. (2), respectivelly.

Table 1. Design variables for the optimization problem regarding the mini-baja prototype

Labels	Description	Type	X^{LB}	X^{0}	X^{UB}
X1 – X8	Outer diameters of frame tubes	Sizing	19.4 mm	19.4 mm	29.12 mm
X9 – X12	Outer diameters of engine support tubes	Sizing	15.4 mm	19.4 mm	29.12 mm
X13 – X20	Wall thickness of frame tubes	Sizing	2.1 mm	3.0 mm	4.0 mm
X21 – X24	Wall thickness of engine support tubes	Sizing	1.0 mm	3.0 mm	4.0 mm
X25 – X26	Shell thicknesses	Sizing	1.0 mm	2.0 mm	4.0 mm
X27	Shell thicknesses	Sizing	0.4 mm	0.7 mm	2.0 mm
X28 – X33	Perturbation scale factors for the rear suspension anchorage points	Shape	-1.0	0.0	1.0
X34 – X57	Perturbation scale factors for the front suspension anchorage points	Shape	-1.0	0.0	1.0
X58 – X66	Perturbation scale factors for the curvatures of the frame tubes	Shape	-1.0	0.0	1.0
X67	Auxiliary design variable for artificial objective function formulation	-	1.0	5.0	1.0E+10

Table 2. Responses of interest for the optimization problem regarding the mini-baja prototype

Labels	Description
R100	Overall system mass
R21 – R27	Seven natural frequencies of vibrartion within the 45 – 75 Hz interest band
R4 – R15	Stresses acting over tube elements in the vehicle frame
R16 – R17	Stresses acting over shell elements in the vehicle frame
R18 – R19	Dynamic forces acting over suspension
R1000	Artificial design objective for simultaneous optimization of physical responses

Design variable X67 (Tab. (1)) and response R1000 (Tab. (2)) are related to each other in the statement of an optimization problem capable of simultaneously optimizing all physical responses of interest, considering all of them as having the same priority. This is obtained through an artificial objetive function as in Eq. (7). The optimizer is set to maximize the artificial objective function, which is a very trivial task given that the value of X67 may be maximized almost arbitrarily, until reaching 1.0E+10. Thus, the real optimization goal is to bring the values of the constraints imposed over the remaining responses (all of them effective physical quantities of interest) within the feasible region.

$$R1000\left(X67\right)=\left(X67\right)^{2} \tag{7}$$

Thus, a simultaneous sizing/shape optimization procedure is implemented within the software VRAND Genesis version 6.0 in order to improve the design aspects highlighted at section 4.2. Details concerning both components of the overall optimization strategy are now presented.

4.3.1. Sizing optimization

The primary goal of the sizing design variables in the case of the mini-baja prototype is mass reduction. For this reason, these variables are attached to dimensions of the tubes (outer diameters and wall thicknesses) and the shells (thicknesses) that ultimately define the amount of material present in the system.

4.3.2. Shape optimization

Shape optimization of the mini-baja structure is intended at the improvement of load distributions over the structure. The curvatures of the frame tubes are designed in order to impruve structural stiffness using a minimum amount of material.

Another group of shape design variables controls the positions of suspension anchorage points (Fig. (9)) so that track excitations are distributed in a favorable manner, that is, the resulting stresses assume the lowest possible values.

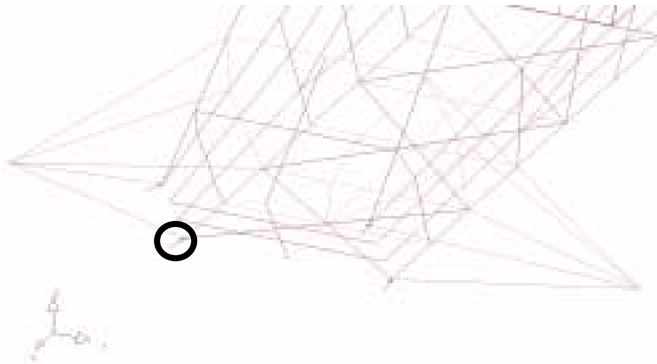

Figure 9. Illustration of shape optimization perturbation in the mini-baja frontal panel

4.3.3. Results

Among the results obtained, the most important one is the mass reduction achieved after the combined sizing/shape optimization procedure. The overall system mass droped from 82.35 kg to 54.07 kg, which represents a 34.34% reduction within the feasible region with respect to the constraints imposed over the stresses and the dynamic forces.

Another important result is related to the resonant frequencies initially valued at 54.65 Hz and 55.05 Hz. This situation is not good due to important powertrain excitation occuring at 54.00 Hz. After optimization, the values of the frequencies are raised to 55.70 Hz and 58.90 Hz, which is expected to minimize dynamic amplification effects responsible for vibroacoustic problems along the mini-baja shell panels.

The final tube curvatures is roughly the same of the initial version. Considering the changes in the responses of interest, this result indicates that these parameters are not influential with respect to stress distribution and overall system stiffness.

All in all, the performance of the optimized version can be considered as being superior to that of the initial version, considering all performane aspects. This improvement is obtained at a reasonable computer cost.

5. CONCLUSIONS AND OUTLINE OF FUTURE RESEARCH WORK

This paper has presented a design methodology based on computer aided engineering (CAE) tools, which allow the rapid assessment of the operating loads and the structural behaviour of mechanical systems. Such an efficient characterization is, indeed, a key diagnostic tool useful in defining system optimization strategies.

The design optimization relies heavily on the availability of algorithms designed to provide a relief on the use of computational resources, namelly, the automatic synthesis techniques. The implementation of these techniques, along with the flexibility of different optimization strategies (shape and sizing optimization), provide efficient design tools to cope with systems of high complexity.

Future research work may be developed in order to implement more aggressive shape optimization strategies that not only define modifications to existing structures but also help in creating brand new geometrical concepts. Such an optimization task is more likely to be successful if topology optimization is used prior to shape optimization for the conceptual design configuration. Since topology optimization usually results in heavy computations, synthesis techniques will again play a key role in assuring feasibility in terms of CPU time.

6. ACKNOWLEDGEMENTS

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