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IMPROVING THE ACCURACY OF STRAIN MEASUREMENTS USING KALMAN FILTERS

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Abstract. *This paper shows the development of a procedure for white noise reduction to be used in quasi-static strain measurement. It is used a mathematical procedure called Kalman filter. Simulations show that this method is really a good choice to improve the accuracy of the measurement since the signal-noise ratio is increased. This method can be implemented in on-line measurement systems, as in a computer with a Digital Acquisition Card.*

Keywords: *Kalman filter, strain measurement, simulation, noise reduction*

1. INTRODUCTION

In any measurement system, the effect introduced by the noise is to corrupt the signal, affecting its accuracy (Brindle et al., 1992 and 1993). This source of error is always present in any measurement, since it is produced by the instrumentation system. For example, electronic amplifiers have a characteristic signal-noise ratio that must be the largest possible in order to reduce the noise present in the measurement.

In order to reduce the presence of noise, a filtering method must be used. The most popular filtering method is the frequency selective filtering. This type of filtering is present in many instrumentation systems. This filter can be analog or digital, and there are many types of filter to be chosen depending on the desired result.

If a signal is corrupted by noise, it is desirable to make the "best" possible separation of them. "Best" in this case means minimum mean-square error. It leads to a question that is, what linear operation on an additive combination of signal and noise will lead to this "best" separation. R.E. Kalman has presented this method of filtering in 1960, when he published his famous paper describing a recursive solution to the discrete-data linear filtering problem. This contribution has been especially significant in applied work, because his solution is readily implemented with modern digital methods.

This paper shows the implementation of a Kalman filter in a strain measurement system to remove its white noise in order to improve its accuracy. A HP 3852A Data Acquisition Unit using a 5-½ digit integrating voltmeter model HP 44701A and strain gages accessories model HP 4419A composes the system. The signal is transferred to a notebook via a GPIB interface. The notebook is equipped with a GPIB PCMCIA Card from National Instruments and the software Labview is used to control the data transfer and to filter the signal using the Kalman filter.

2. THE KALMAN FILTER

The Kalman filter is a multiple-input, multiple-output filter that can optimally estimate, in real time, the state of system based on its noisy outputs. These states are all the variables needed to completely describe the system behavior as a function of time (such as position, velocity, voltage levels, and so forth). In fact, one can think of the multiple noisy outputs as a multidimensional signal plus noise, with the system states being the desired unknown signals. The Kalman filter then filters the noisy measurements to estimate the desired signals. The estimates are statistically optimal in the sense that they minimize the mean-square estimation error.

The Kalman filter is a set of mathematical equations that provides an efficient computational (recursive) solution of the least-square method. The filter is very powerful in several aspects; it supports estimations of the past, present, and even future states, and it can do so even when the precise nature of the modeled system is unknown. The Kalman filter tries to estimate the state $\boldsymbol{x} \in \hat{A}^n$ of a discrete time controlled process that is governed by the linear stochastic difference equation

$$x_k = A \cdot x_{k-1} + B \cdot u_k + w_{k-1} \quad (1)$$

with a measurement $\boldsymbol{z} \in \hat{A}^n$ that is

$$z_k = H \cdot x_k + v_k \quad (2)$$

where

x_k is the (n x 1) process state vector at time t_k.
A is the (n x n) state transition matrix that relates x_k to x_{k+1} in the absence of a forcing function.
w_k is the (n x 1) process noise vector that models the uncertainty of the process.
z_k is the (m x 1) process measurement vector at time t_k.
H_k is the (m x n) measurement matrix that relates state to measurement.
v_k is the (m x 1) measurement noise that models the noise in the measurement.

The random variables w_k e v_k are assumed to be independent (of each other), and with normal probability distributions.

$$p(w) \sim N(0, Q)$$
$$p(v) \sim N(0, R)$$

In practice, the process noise covariance Q and measurement noise covariance R matrices might change with each time step or measurement, however they will be assumed constant.

The discrete Kalman filter algorithm that will be used is shown in Equations (3-7).

$$\hat{x}_k^- = A \cdot \hat{x}_{k-1} + B \cdot u_k \quad (3)$$
$$P_k^- = A \cdot P_{k-1} \cdot A^T + Q \quad (4)$$
$$K_k = P_k^- \cdot H^T \cdot \left(H \cdot P_k^- \cdot H^T + R\right)^{-1} \quad (5)$$
$$\hat{x}_k = \hat{x}_k^- + K_k \cdot \left(z_k - H \cdot \hat{x}_k^-\right) \quad (6)$$
$$P_k = \left(I - K_k \cdot H\right) \cdot P_k^- \quad (7)$$

3. MODEL OF THE PROCESS

It will be supposed that a system is subjected to strain; it is desired to obtain this strain from a measurement that includes noise. The measurement noise is supposed to be a white noise. Therefore, it is necessary to model the process as described in Eq.(1) .

It will be supposed that the third derivative of the strain is constant between two samples ($\dddot{e}(t)=\dddot{e}_k, t_k < t < t_{k+1}$), but with random amplitude with zero mean, with variance Γ. The reason for this choice is to have a third order Kalman filter that is able to follow a second order signal.

The strain, its first and second derivatives between two samples have the following relations:

$$\begin{aligned} e_{k+1} &= e_k + \dot{e}_{k-1} \cdot \Delta t + \ddot{e}_{k-1} \frac{\Delta t^2}{2} + \dddot{e}_{k-1} \frac{\Delta t^3}{6} \\ \dot{e}_{k+1} &= \dot{e}_{k-1} + \ddot{e}_{k-1} \cdot \Delta t + \dddot{e}_{k-1} \frac{\Delta t^2}{2} \\ \ddot{e}_{k+1} &= \ddot{e}_{k-1} + \dddot{e}_{k-1} \cdot \Delta t \end{aligned} \tag{8}$$

Then, the corresponding state equation is:

$$\begin{bmatrix} e_{k+1} \\ \dot{e}_{k+1} \\ \ddot{e}_{k+1} \end{bmatrix} = \begin{bmatrix} 1 & \Delta t & \Delta t^2/2 \\ 0 & 1 & \Delta t \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} e_k \\ \dot{e}_{k-1} \\ \ddot{e}_{k-1} \end{bmatrix} + \begin{bmatrix} \Delta t^3/6 \\ \Delta t^2/2 \\ \Delta t \end{bmatrix} \cdot \dddot{e}_k \tag{9}$$

and the observation equation is

$$z_n = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} e_n \\ \dot{e}_k \\ \ddot{e}_k \end{bmatrix} + w_k \tag{10}$$

where $\dddot{e}_n$ is a white noise with zero mean and variance *G*, w_n is the measurement noise that is supposed to be white with zero mean and variance *R*.

Therefore the equations of the strain correspond to the classic model of the Kalman Filter with:

$$A = \begin{bmatrix} 1 & \Delta t & \Delta t^2/2 \\ 0 & 1 & \Delta t \\ 0 & 0 & 1 \end{bmatrix} \tag{11}$$

$$B = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \tag{12}$$

$$u = 0 \tag{13}$$

$$Q = \begin{bmatrix} \Delta t^6/36 & \Delta t^5/12 & \Delta t^4/6 \\ \Delta t^5/12 & \Delta t^4/4 & \Delta t^3/2 \\ \Delta t^4/6 & \Delta t^3/2 & \Delta t^2 \end{bmatrix} \cdot \Gamma \tag{14}$$

$$H = [1 \quad 0 \quad 0] \tag{15}$$

4. SIMULATION OF A MEASUREMENT

The validity of this model to filter signals with low variance was simulated using the signal shown in Fig. 1 in MATLAB. It was simulated a strain that is sampled at a rate of 10Hz. The signal is zero at the beginning of the measurement, then increases in a constant rate from zero to 1, stays constant and then decreases to zero. It is obtained a noisy measurement that is shown in Fig. 1.

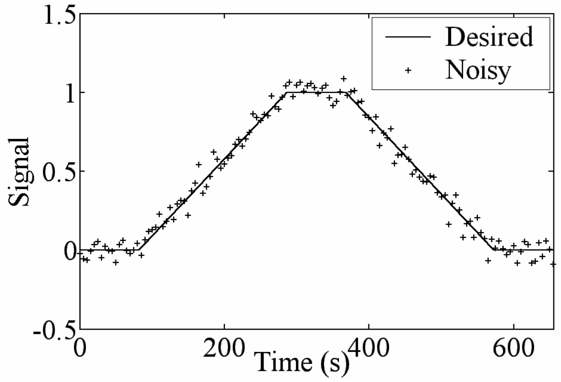

Figure 1: Simulation of a signal and its noisy measurement.

In order to apply the filter algorithm, it is necessary to estimate the measurement noise and the process covariance *R* and *Q*.

The measurement noise covariance *R* is usually measured prior to operation of the filter. Measuring it is generally possible in an off-line sample measurement.

The process noise covariance *Q* is more difficult to determine. In general, it is difficult to anticipate the process under observation. Sometimes a relatively simple process model can produce acceptable results if one “injects” enough uncertainty into the process via the selection of *Q*.

The value of *R* was easily obtained from the measurement of the variance generated by the MATLAB. It was used a value of $\Gamma=10^{-9}$.

The resulting of the filtering is shown in Fig. 2.

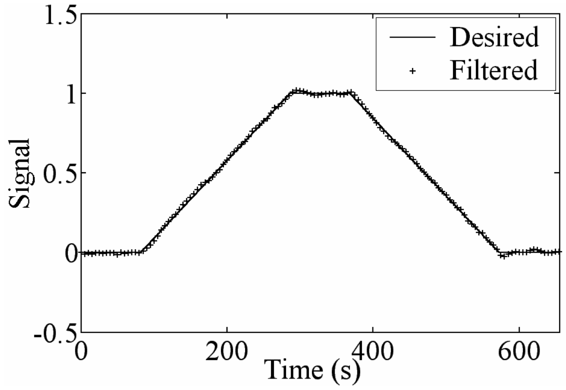

Figure 2: Simulation of filtering of the signal shown in Figure 1 using the Kalman filter.

Figure 3 shows the comparison between the noise in the measured signal and the noise present in the signal after using the Kalman filter. It can be verified that the Kalman filter was able to remove the noise without introducing the common distortions present in classic frequency selective filters.

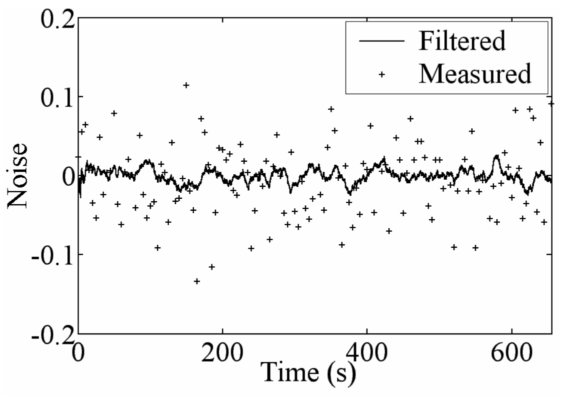

Figure 3: Comparison between the signal present in the measured and the filtered signal.

5. COMPARING WITH ANOTHER FILTER METHODS

The signal was then filtered using a Butterworth low-pass filter fourth order with cut-off frequency of 0.3Hz. The resulting signal is shown in Fig. 4. In order to remove the noise, a frequency selective filter like this must have a very low cut-off frequency. It introduces a delay in the signal that distorts it.

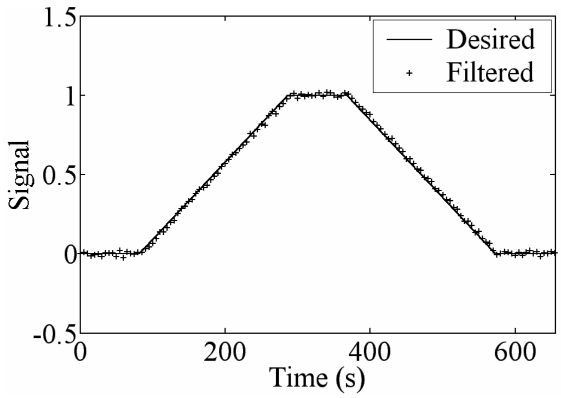

Figure 4: Resulting of the filtering of the signal using a Butterworth low-pass filter of fourth order with a cut-off frequency of 0.3Hz.

Figure 5 shows the error of the obtained signal when the Kalman filter and the Butterworth filter are used.

The larger error present in the signal where the Butterworth filter was applied is mainly caused by the delay introduced by this filter. It shows that this filter removes the noise, but the accuracy is not improved.

Even though it was used an IIR Butterworth filter, this result can be extended to other filters, since the problem verified is the delay that will be present in any IIR or FIR filters.

The Kalman, the IIR and the FIR filters use past values of the signal to calculate its present value, but the Kalman filter uses the knowledge of the behavior of the system to better estimate the present value.

Another known method of filtering is the software averaging. Software averaging is a simple and effective technique of digitally filtering; for every data point it needed, the system acquires many voltage readings. For example, a common approach is to acquire 100 points and average those points for each measurement that is needed. For slower applications in which oversampling can be used, averaging is a very effective noise filtering technique.

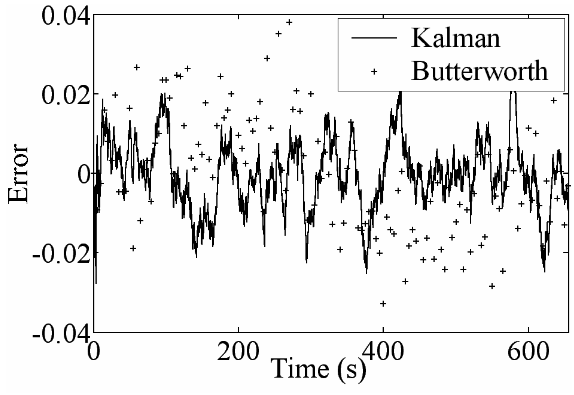

Figure 5: Comparison between the errors of the signal filtered using the Kalman filter and the one using the Butterworth filter.

Figure 6 shows the application of software averaging to the signal shown in Fig. 5. It presents the same delay verified in the Butterworth filter. This delay is not surprising simply because this method is equivalent to a FIR filter.

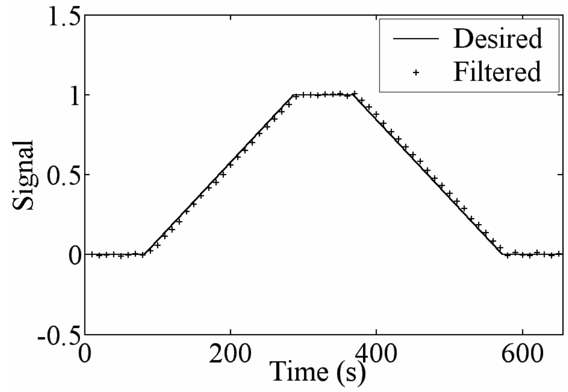

Figure 6: Resulting of the software averaging of 100 points.

In order to make the error introduced by the delay very small, it is necessary to increase the sample rate used in the measurement system. If it is not possible to do it, the software average will not be a good choice and these simulations show that the Kalman filter can do this task better than the classic filters.

6. FILTERING A REAL SIGNAL

Figure 7 shows the signal of a strain measured in a cantilever beam. It can be verified the presence of noise that diminish the accuracy of the measurement. The value of R was obtained in an off-line measurement and the value of Γ was estimated as 10^{-6}.

Since the value of Γ is an estimate of the uncertainty of the model, it is desired that it will be the lowest possible. A zero value can be used only if the correct model is used.

If the model is not the exact one, a large value results in very poor filtering. If a value too small is used, the filtered signal doesn't follow the desired signal. Using trial and error method, the best value found for Γ was 10^{-6}.

Evidently, this is not the best method to estimate this value, but, since this work is the its beginning, such methods are not being used yet.

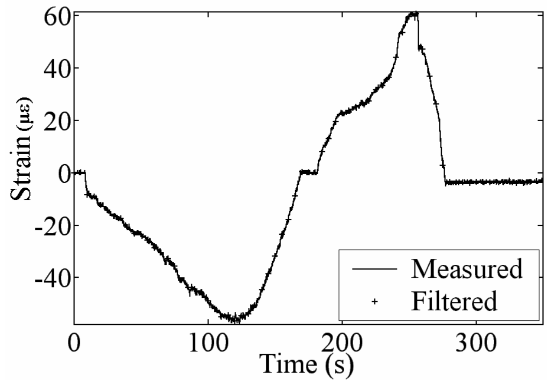

Figure 7: Comparison between a strain and its filtered signals using a Kalman filter.

Figure 8 shows the zoom of the signal in the interval from 0 to 30 seconds. The fact that the signal be optimally filtered really improves the accuracy of the measurement, since the random variation introduced by the noise is decreased. It can be verified by the fact that the filtered signal tends to converge to the actual signal (without measurement noise).

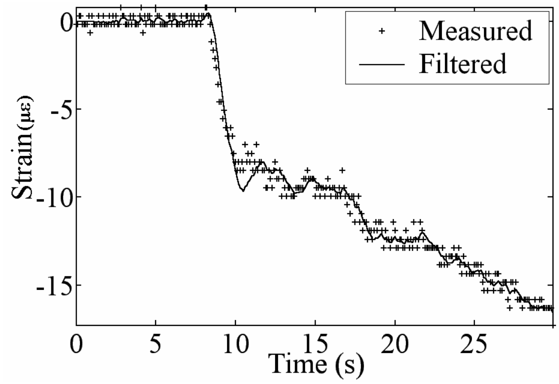

Figure 8: Zoom of the Figure 7 between 0 and 30 seconds.

7. CONCLUSIONS

The Kalman filter is a very powerful tool to improve the accuracy of any measurement system. In this paper, this method was applied to measurements made with a strain gage and showed to be very promising.

This method can be implemented in an on-line measurement system and the proposal of future works is to implement it using the software Labview in order to use it in a strain measurement system that uses 4 wires systems, since the problem of this method is the accuracy of the available systems to read directly resistence.

The disadvantage of this method is that it is not simple to be used, since the result is very sensitive to the model used. A wrong model will lead to very wrong results.

8. REFERENCES

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