



ON A CONTROL METHOD APPLIED TO A NON-IDEAL PORTAL FRAME FOUNDATION OF AN UNBALANCED ROTATING MACHINE

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Abstract. *In this paper, the numerical simulations of the non-linear control method using internal resonance and saturation phenomenon to suppress the steady-state vibrations of a non-ideal system are presented. An engineering application is presented, namely: a portal frame foundation of an unbalanced rotating machine (limited power supply). The control scheme is implemented by introducing controllers, which are coupled dynamically with the portal frame through a nonlinear feedback control law. At 1:2 internal resonance, the nonlinear coupling generates an energy link between the portal frame and the controllers. Thus, energy is transferred from the portal frame to the controllers where the active damping mechanisms subsequently dissipate it. Here the response of the structure is regulated with a single input torque applied to the portal frame coordinates.*

Keywords: *Internal resonance, Saturation phenomenon, Non-ideal system, Nonlinear control.*

1. INTRODUÇÃO

In the present work, the behavior of the control method applied to a portal frame foundation of an unbalanced rotating machine (limited power supply) is presented.

On the use of the saturation phenomenon and internal resonance have been investigated by many researchers (see, e.g., Nayfeh et al., 1973; Haddow et al., 1984; Nayfeh and Zavodney, 1988; Nayfeh and Balachandran, 1989).

Recently, the use of the saturation phenomenon and internal resonance on nonlinear oscillations of a portal frame under a single ideal harmonic excitation was studied by (Brasil and Balthazar, 2001). In the case of a portal frame under a non-ideal excitation, the saturation phenomenon appears in (Palacios, Balthazar and Brasil, 2001; Brasil, Garzeri and Balthazar, 2001)

On the active control strategies based on the saturation phenomenon and internal resonance to suppress the motions of the system have been investigated by the following researchers (Mook, 1985; Golnaraghi, 1991; Queini, Nayfeh and Golnaragui, 1997; Pai et al, 1998; Pai and Schulz, 2000).

The application of this control in the portal frame under a harmonic excitation has see theoretically studied by (Palacios, Balthazar and Brasil, 2001).

In this paper, we investigated the behavior of the non-ideal vibrating system (portal frame foundation and energy source with limited power supply) near of the resonance region (the frequencies of the first and second modes are in resonance with the average frequency of the energy source) and the physical and geometric properties of the frame are chosen to tune the natural frequencies of these two modes into a 1:2 internal resonance. In this case, we observe of the modal interactions of the foundation (the saturation appear in the energy transference from a higher frequency mode to a lower frequency mode) and interaction between foundation and energy source.

Considering a DC motor as energy source and its characteristic has been taken as linear.

Finally, we investigate the implementation of a control strategy based the saturation phenomenon due the internal resonance (modal and physical coupling between the non-ideal vibrating system and the controllers) to suppress the motions of a portal frame foundation of an unbalanced rotating machine (limited power supply). In the theoretical context of (Pai, et al., 1998) we may summarize non-linear control method as follow: The so-called saturation control method uses the saturation phenomenon to suppress system vibrations, and it can be described by the following two ordinary equations,

$$\begin{aligned} \ddot{u}_1 + 2\zeta_1\omega_1\dot{u}_1 + \omega_1^2u_1 &= \mathfrak{g}_{12}u_1u_2 \\ \ddot{u}_2 + 2\zeta_2\omega_2\dot{u}_2 + \omega_2^2u_2 &= \mathfrak{g}_{11}u_1^2 + F \cos(\Omega t) \end{aligned} \quad (1)$$

where u_1 denotes the response of a second-order controller, ω_1 is its natural angular frequency, and ζ_1 is its damping ratio, u_2 represents the response of the dynamical system to be controlled, ω_2 is its natural frequency and is close to $2\omega_1$, and ζ_2 is its damping ratio. Moreover, \mathfrak{g}_{11} and \mathfrak{g}_{12} are positive gain constants, $F \cos(\Omega t)$ is the external excitation force with amplitude F and frequency Ω . If the excitation frequency Ω is close to ω_2 and the excitation amplitude F is larger than a critical value F_c , the amplitude of u_2 saturates and all additional energy added to the system by increasing F flows into the controller u_1 due to the quadratic coupling terms u_1u_2 and u_1^2 , which act as an energy bridge to establish a state of exchange of energy between the system and the controller.

2. THE NON-IDEAL SYSTEM

The dimensionless equations of motion of the portal frame foundation of an unbalanced rotating machine model are similarly given by (Brasil and Balthazar, 2001)

$$\begin{aligned} q_1'' + \hat{\omega}_1q_1 &= -\hat{\alpha}_5q_1q_2 + \hat{\alpha}_1(q_3''\sin q_3 + q_3'^2 \cos q_3) - \hat{\mu}_1q_1' \\ q_2'' + \hat{\omega}_2^2q_2 &= -\hat{\alpha}_6q_1^2 + \hat{\alpha}_2(-q_3'' \cos q_3 + q_3'^2 \sin q_3) - \hat{\mu}_2q_2' - \hat{\alpha}_8 \\ q_3'' &= \hat{\alpha}_3q_1''\sin q_3 - \hat{\alpha}_4q_2'' \cos q_3 - \hat{\alpha}_7 \cos q_3 + F(q_3') \end{aligned} \quad (2)$$

where $q_1(\tau)$ is horizontal response with natural frequency $\hat{\omega}_1$, $q_2(\tau)$ is vertical response with natural frequency $\hat{\omega}_2$, $q_3'(\tau)$ is angular velocity response of the rotor, $\hat{\mu}_j$ are the non-dimensional damping coefficients, $\hat{\alpha}_j$ are the non-dimensional parameters.

Considering the characteristic of the motor of the form

$$F(q_3') = \hat{a} - \hat{b}q_3' \quad (3)$$

that has a corresponding regulating control in the energy source, in this case, the constant \hat{a} will be the control parameter and \hat{b} a fixed constant depending of the type of motor.

In the numerical simulation, the nonlinear ordinary differential equation solver selected is the MATLAB variable-step solver ODE45, which uses a fourth-five-order Runge-Kutta integration method from now on.

The physical values adopted are: $EI=128Nm^2$ for both the columns and the beam, $h=0.36m$, $L=0.52m$, $M=2.0Kg$, $m=0.5Kg$, $m_0=0.1Kg$, $I_m=0.00017Kg m^2$, $r=0.01m$, $c_1=1.55Ns/m$ and $c_2=2.14Ns/m$. Passage through resonance with the first and second natural frequency of the beam in the first vibration modes ($\omega_1=74rad/s$, $\omega_2=148.0rad/s$), is considered. These values were also chosen to allow for a 1:2 internal resonance condition for the foundation.

We consider, from now on, the following values for the dimensionless parameters in the SIMULINK solution corresponding to the system (2): $\hat{\mu}_1=0.01$, $\hat{\mu}_2=0.03$, $\hat{\omega}_1=1$, $\hat{\omega}_2=2$, $\hat{\alpha}_1=9.26 \times 10^{-4}$, $\hat{\alpha}_2=9.61 \times 10^{-4}$, $\hat{\alpha}_3=13.33$, $\hat{\alpha}_4=19.26$, $\hat{\alpha}_5=4.61$, $\hat{\alpha}_6=1.657$, $\hat{\alpha}_7=0.066$, $\hat{\alpha}_8=3.45 \times 10^{-3}$. Moreover, the initial conditions are chosen, from now on, to be $q_1(0)=0.0018$, $q_1'(0)=0.0$, $q_2(0)=0.0012$, $q_2'(0)=0.0$, and $q_3(0)=0.0$, $q_3'(0)=0.5$.

In the Fig. 1, we show the numerical results for the values of control parameter \hat{a} , namely, $\hat{a}=1.6$, $\hat{a}=3.4$ and $\hat{b}=1.5$.

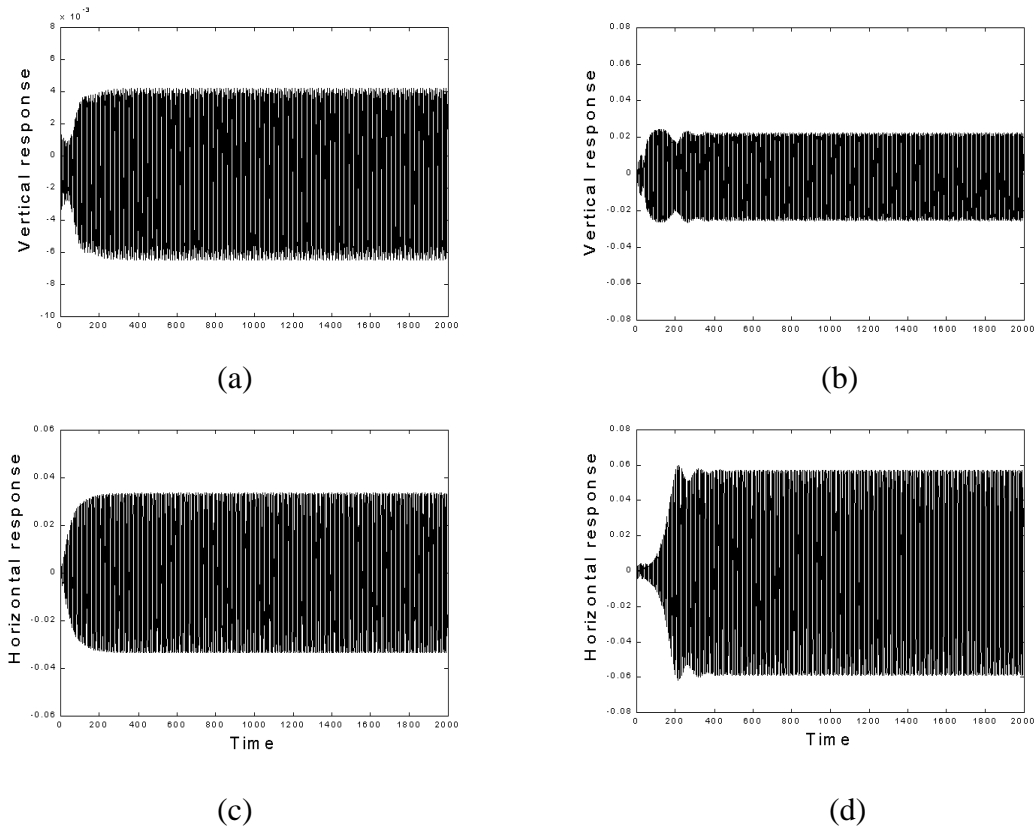


Figure 1. Time history of the vertical response $q_2(\tau)$ in (a),(c) and horizontal response $q_1(\tau)$ in (b),(d) with average $q_3'(\tau) \approx \omega_1$ ($\hat{a}=1.6$; in (a),(b)) and average $q_3'(\tau) \approx \omega_2$ ($\hat{a}=3.4$; in (c),(d)).

When the average angular velocity q_3' is in resonance with $\hat{\omega}_2$, one sees that the oscillation of q_2 stop of increasing and reaches its steady-state (see Fig 1c) at the same time the oscillation of q_1 is increasing and reaches its steady-state (see Fig 1d). In this case with the energy is transferred from a higher frequency mode to a lower frequency mode due to internal resonance, and with the effect of

the nonlinear coupling between the two modes is manifest the saturation phenomenon. The complete dynamics of this saturation phenomenon in a non-ideal system is analyzed through a frequency-amplitudes diagram as is shown in Fig. 2 (Palacios, 2002), this graph is estimated by numerical simulation defining the amplitudes as the maximum absolute value of the amplitudes of the first vibration modes of the portal plane frame, and the frequency as the mean value of the rotational speed of motor.

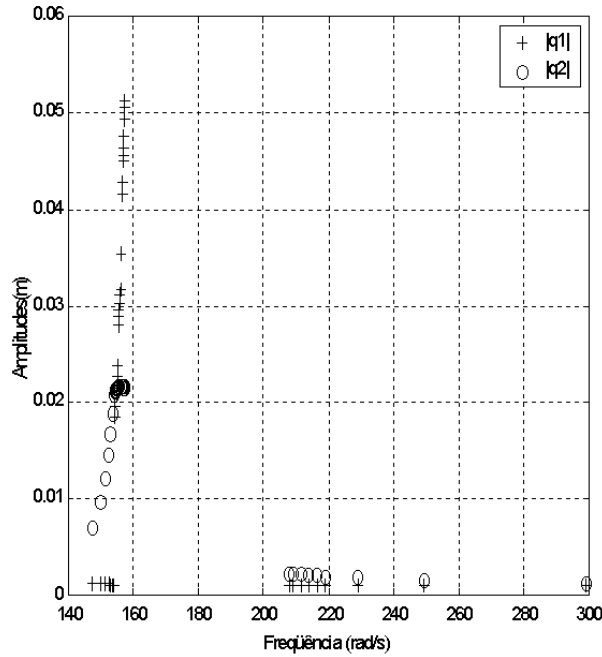


Figure 2. Frequency-Amplitudes diagram: Saturation phenomenon and jump in a non-ideal system. (Line +++) are the amplitudes of q_1 and (line ooo) are the amplitudes of q_2 in function of the average angular velocity \dot{q}_3 .

In the following section, the non-linear control technique will be applied in the passage through resonance and with the saturation phenomenon condition.

3. CONTROL OF THE PORTAL FRAME

In this section, we present the dynamic behavior of the controlled non-ideal vibrating system with the controllers acting on the first and second mode of the portal frame foundation. The plant under consideration consists of the portal frame foundation under a non-ideal excitation and we rewrite the equations of motion of this controlled system in the form,

$$\begin{aligned}
 q_1'' + \hat{\omega}_1^2 q_1 &= -\hat{\alpha}_5 q_1 q_2 + \hat{\alpha}_1 (q_3'' \sin(q_3) + q_3'^2 \cos(q_3)) - \hat{\mu}_1 q_1' - T_1 \\
 q_2'' + \hat{\omega}_2^2 q_2 &= -\hat{\alpha}_6 q_1^2 + \hat{\alpha}_2 (-q_3'' \cos(q_3) + q_3'^2 \sin(q_3)) - \hat{\mu}_2 q_2' - \alpha_8 - T_2 \\
 q_3'' &= \hat{\alpha}_3 q_1'' \sin(q_3) - \hat{\alpha}_4 q_2'' \cos(q_3) - \hat{\alpha}_7 \cos(q_3) + F(q_3')
 \end{aligned} \tag{4}$$

and we introduce the nonlinear controllers and its of control laws in the form

$$q_4'' + \mu_4 q_4' + \omega_4^2 q_4 = -g_{14} q_1 q_4 - g_{24} q_2 q_4 \text{ and } T_1 = g_{44} q_4^2$$

$$q_5'' + \mu_5 q_5' + \omega_5^2 q_5 = -g_{15} q_1 q_5 - g_{25} q_2 q_5 \text{ and } T_2 = g_{55} q_5^2 \quad (5)$$

where q_4 denotes the response of one of the two second-order controllers, ω_4 is its natural angular frequency with $\omega_4 = \frac{\hat{\omega}_1}{2}$, and μ_4 is its damping constant, q_5 denotes the response of the other second-order controller, ω_5 is its natural angular frequency with $\omega_5 = \frac{\hat{\omega}_2}{2}$, and μ_5 is its damping constant. Moreover, g_{ij} are positive gain constants, and T_1 and T_2 are the control signal applied to the mode first and second, respectively.

To perform numerical simulation of Eqs. (4) and (5) we built the SIMULINK model shown in Fig. 3. Hence the “ $G_{14} * q_1 * q_4$ ” and “ $G_{25} * q_2 * q_5$ ” are the direct excitations to “ q_4 ” and “ q_5 ” respectively. On the other hand, “ $G_{44} * q_4^2$ ” and “ $G_{55} * q_5^2$ ” are indirect excitation to “ q_1 ” and “ q_2 ” respectively. The definitions of the right-hand side of Eq. (5) enables the modal coupling between the non-ideal system and the controllers. The physical coupling required between the non-ideal system and the controllers are achieved by defining the torque by $g_{44} q_4^2$ and $g_{55} q_5^2$.

3.1 Control Applied to the Vertical Displacement

In this section, we present the dynamic behavior of the controlled non-ideal vibrating system when the controller q_5 is acting only in the vertical displacement q_2 .

We consider the following parameters values of the controllers in the SIMULINK solution: $g_{14}=0.0$, $g_{24}=0.0$, $g_{44}=0.0$, $\omega_4=0.5$, $g_{15}=0.0$, $g_{25}=10.0$, $g_{55}=10.0$, $\mu_5=0.01$, $\omega_5=1.0$ and $\hat{a}=3.4$.

The positive gain constants g_{mm} in Eqs. (4)-(5) are exactly the same as the positive gain constants G_{mm} in the SIMULINK block diagram (see Fig. 3).

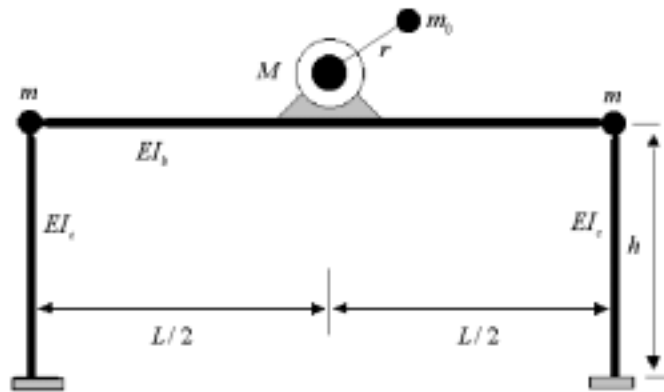
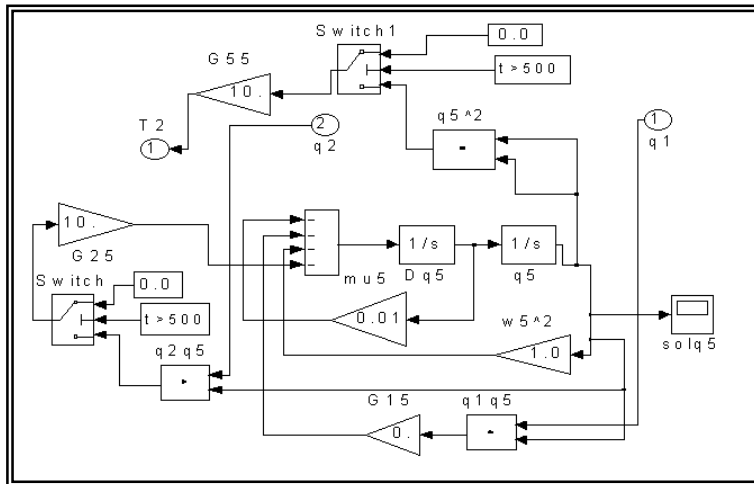
Moreover, the initial conditions are chosen to be

$$\{q_4(0) = 0.0010, q_4'(0) = 0.0, q_5(0) = 0.0014, q_5'(0) = 0.0\} \quad (6)$$

Figure 4 shows the numerical simulation of Eq. (4)-(5) when average angular velocity is near of the second natural frequency $\hat{\omega}_2$ (see Fig. 4a). When the controller is activated in $\tau=500$ one sees that the controller suppress the oscillation of the horizontal and vertical response (see Figs. 4b, c). Fig. 4d shows the controller response q_5 . In this case the energy is transferred from the plant to the controller q_5 , and the effect of the nonlinear coupling between the plant and controller is verified.

Figure 4c shows that the controller has more effective suppression on the response q_1 .

Controller q_5



Controller q_4

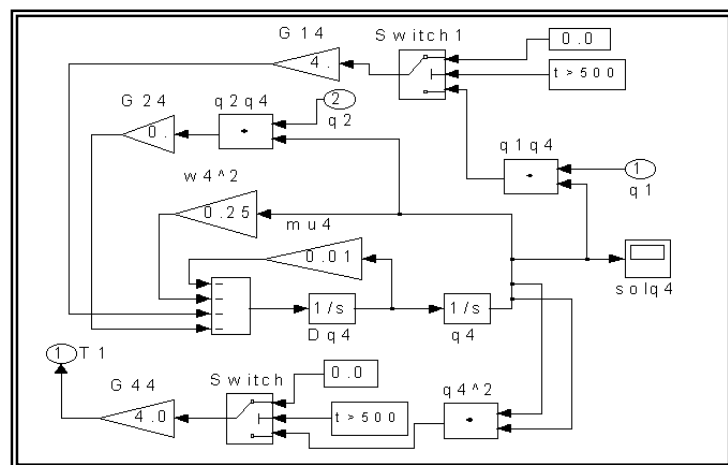


Figure 3. Controllers in SIMULINK block diagram applied to the non-ideal vibrating system

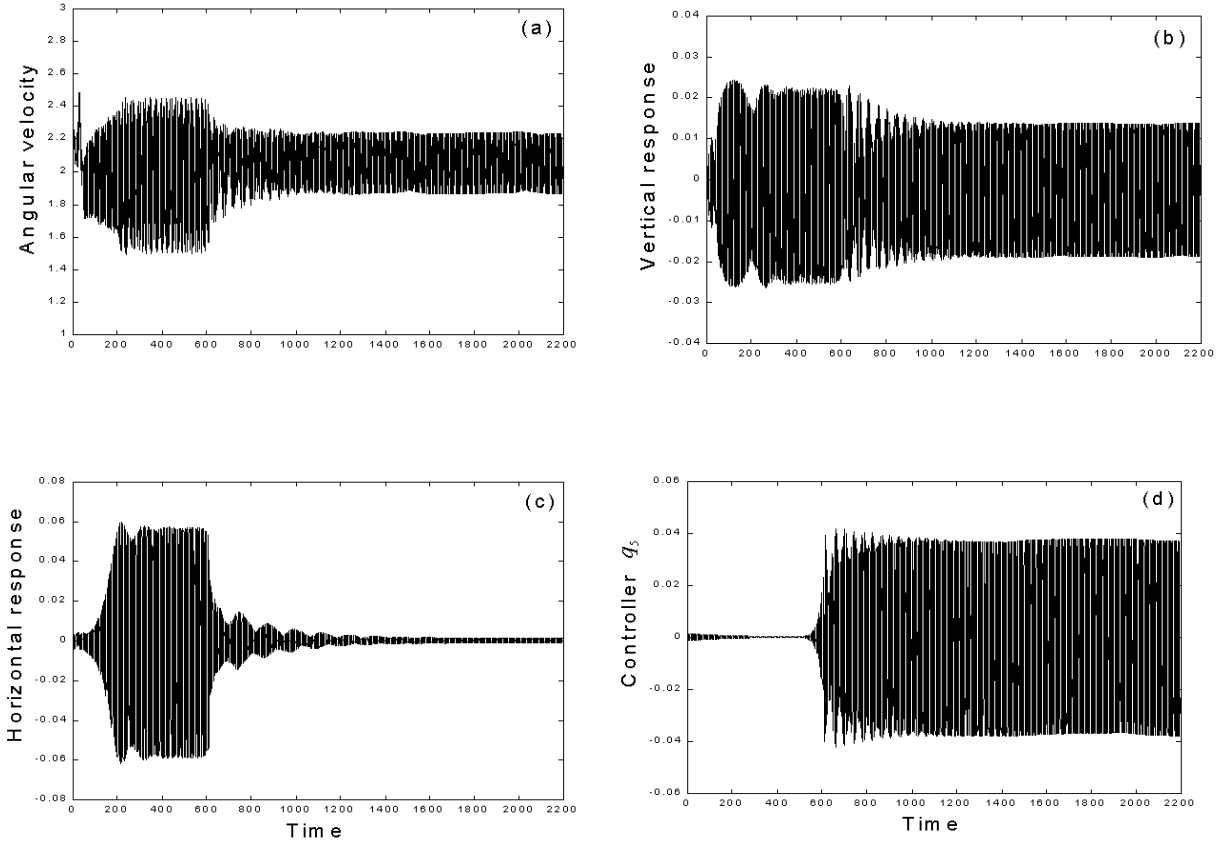


Figure 4. The response of Eqs. (4) and (5) with $g_{14}=0.0$, $g_{24}=0.0$, $g_{44}=0.0$, $\omega_4=0.5$, $g_{15}=0.0$, $g_{25}=10.0$, $g_{55}=10.0$, $\mu_5=0.01$, $\omega_5=1.0$, where the controller q_5 is activated on $\tau=500$,
(a) $q'_3(\tau)$, (b) $q_2(\tau)$, (c) $q_1(\tau)$, and (d) $q_5(\tau)$

3.2 Control Applied to the Horizontal Displacement

In this section, we present the dynamic behavior of the controlled system of Eqs. (4) and (5) when the controller q_4 is acting only of the first mode of the non-ideal portal frame. The parameters values adopted in the diagram of the SIMULINK configuration for this model are: $g_{14}=4.0$, $g_{24}=0.0$, $g_{44}=4.0$, $\mu_4=0.01$, $\omega_4=0.5$, $g_{15}=0.0$, $g_{25}=0.0$, $g_{55}=0.0$, $\omega_5=1.0$, and $\hat{a}=1.6$.

Moreover, the initial conditions are chosen to be Eq. (6).

Figure 5 shows the numerical simulations of Eqs. (4)-(5) when average angular velocity is near of the first natural frequency $\hat{\omega}_1$ (see Fig. 5a). When the controller is activated in $\tau=500$ one sees that the controller q_4 suppress the oscillation of the horizontal and vertical response (see Figs. 5b, c). Fig. 5d show the controller response q_4 . In this case the energy is transferred from the plant to the controller q_4 , and the effect of the nonlinear coupling between the plant and controller is verified. Fig. 5b shows that the controller has more effective suppression on the response q_2 .

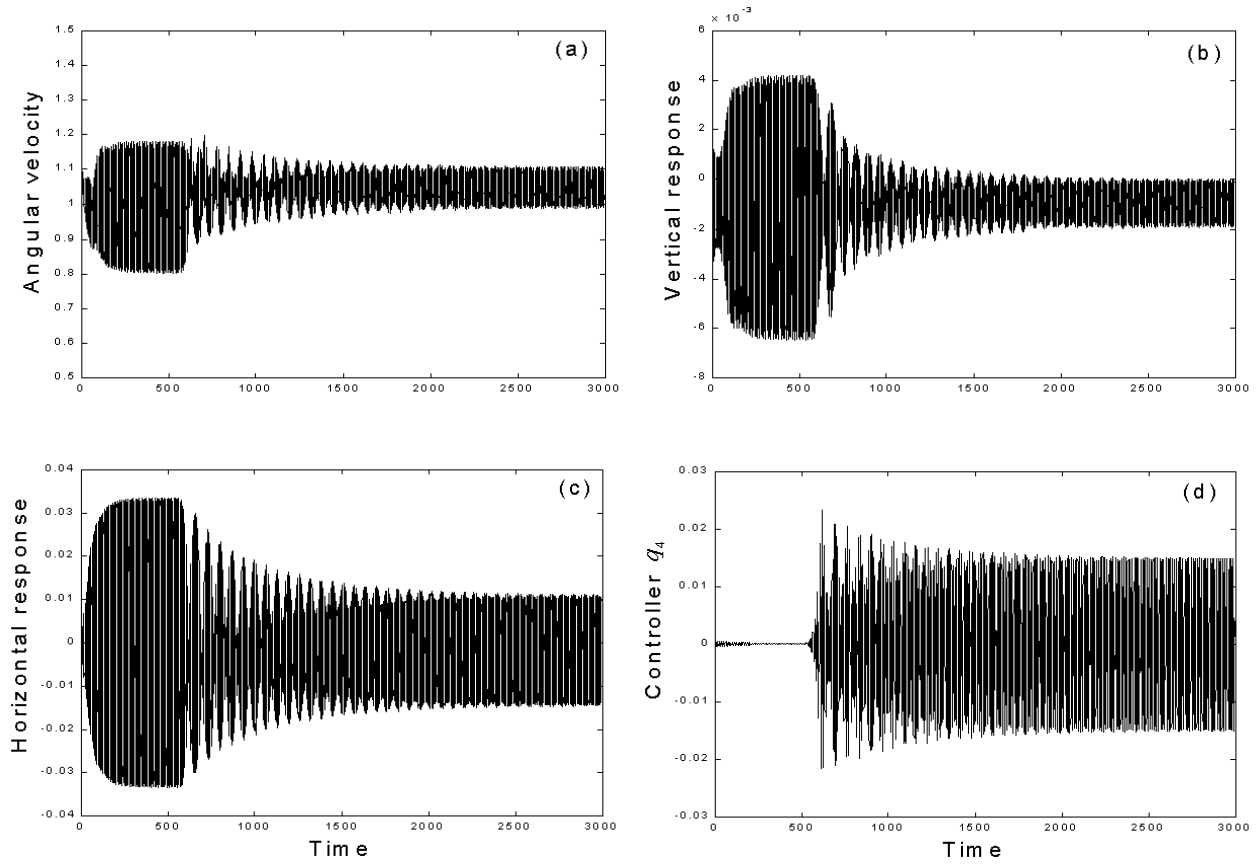


Figure 5. The response of Eqs. (4) and (5) with $g_{14}=4.0$, $g_{24}=0.0$, $g_{44}=4.0$, $\mu_4=0.01$, $\omega_4=0.5$, $g_{15}=0.0$, $g_{25}=0.0$, $g_{55}=0.0$, $\omega_5=1.0$, where the controller q_4 is activated on $\tau=500$,
(a) $q'_3(\tau)$, (b) $q_2(\tau)$, (c) $q_1(\tau)$, and (d) $q_5(\tau)$.

3.3 Control Applied To The Two Displacements

In this section, we present the dynamic behavior of the controlled system of Eqs. (4) and (5) when the controllers q_4 and q_5 are acting in the two modes of the portal frame foundation.

To perform numerical simulation of Eqs. (4) and (5) we built the SIMULINK model shown in Fig. 3.

We consider the following values for the parameters of the controllers in the SIMULINK solution: $g_{14}=4.0$, $g_{24}=0.0$, $g_{44}=3.0$, $\mu_4=0.0$, $\omega_4=0.5$, $g_{15}=0.0$, $g_{25}=17.0$, $g_{55}=7.0$, $\mu_5=0.0$, $\omega_5=1.0$.

Moreover, the initial conditions are chosen to be Eq. (6).

Figure 6 shows the results of a numerical simulation of Eq. (4)-(5) when average angular velocity is near of the second natural frequency (see Fig. 6a), one sees that the controllers suppress the oscillations of the horizontal and vertical displacement (see Figs. 5b, c). Figs. 5d and e shows the controllers response q_4 and q_5 . In this case the energy is transferred from the plant to the controllers, and the effect of the nonlinear coupling between the plant and controllers is verified.

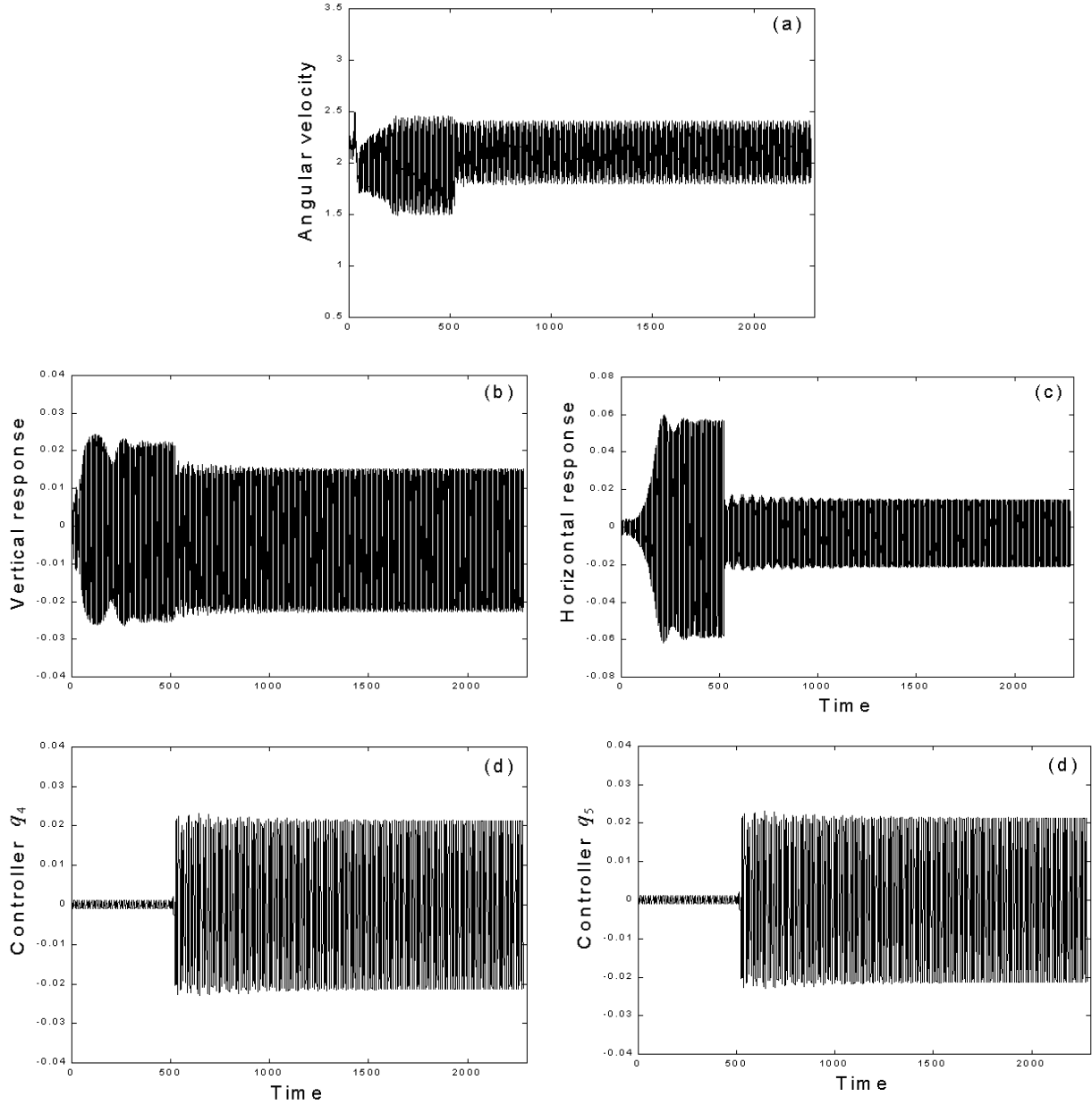


Figure 6. The response of Eqs. (4) and (5) with $g_{14}=4.0$, $g_{24}=0.0$, $g_{44}=3.0$, $\mu_4=0.0$, $\omega_4=0.5$, $g_{15}=0.0$, $g_{25}=17.0$, $g_{55}=7.0$, $\omega_5=1.0$, where the controllers q_4 and q_5 are activated on $\tau=500$, (a) $q'_3(\tau)$, (b) $q_2(\tau)$, (c) $q_1(\tau)$, and (d) $q_5(\tau)$.

3. CONCLUSIONS

We have investigated the dynamic behavior of the non-ideal vibrating system (portal frame foundation of an unbalanced rotating machine with limited power supply) in the resonance region $\Omega \approx \omega_2$ and condition internal resonance $\omega_2 \approx 2\omega_1$ and considering the saturation phenomenon.

A control technique based on the saturation phenomenon due the internal resonance was proposed for suppress the motion of the non-ideal vibrating system defining the modal and physical coupling between the generalized coordinates of the non-ideal vibrating system and the controllers.

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