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CHAOTIC BEHAVIOR OF COUPLED SHAPE MEMORY OSCILLATORS

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Abstract. *Shape memory and pseudoelastic effects are thermomechanical phenomena associated with martensitic phase transformations, presented by shape memory alloys. This contribution concerns with the response of coupled shape memory oscillators. Equations of motion are formulated assuming polynomial constitutive model to describe the restitution force of oscillators. Since equations of motion are associated with a five-dimensional system, the analysis is performed considering subspaces associated with each mass.*

Keywords: *Nonlinear Dynamics, Chaos, Shape memory alloys.*

1. INTRODUCTION

Shape memory alloys (SMAs) have been found in a great number of applications in different fields of sciences and engineering. They are ideally suited for use as fastener, seals, connectors and claps (van Humbeeck, 1999; Kibirkstis *et al.*, 1997; Borden, 1991). Self-actuating fastener, thermally actuator switches and several bioengineering devices are some examples of these applications (Duerig *et al.*, 1999; Lagoudas *et al.*, 1999). The use of SMAs can help solving many important problems in aerospace technology, in particular those concerning with space savings achieved by self-erectable structures, stabilizing mechanisms, solar batteries, non-explosive release devices and other possibilities (Denoyer *et al.*, 2000). Micromanipulators and robotics actuators have been built employing SMAs properties to mimic the smooth motions of human muscles (Garner *et al.*, 2001; Webb *et al.*, 1999; Fujita & Toshiyoshi, 1998; Rogers, 1995). Moreover, SMAs are being used as actuators for vibration and buckling control of flexible structures. In this particular field, SMAs wires embedded in composite materials have been used to modify mechanical characteristics of slender structures (Pietrzakowski, 2000; Birman, 1997; Rogers, 1995). The main drawback of SMAs is their slow rate of change.

Since the phenomena associated with martensitic transformation are intrinsically nonlinear, its dynamical response may present some characteristics not observed in linear systems. Chaotic motion is one of these possibilities, considering both proper mathematical and geometrical aspects.

The dynamical analysis of intelligent systems and structures that use SMA as actuators involves multi-degrees of freedom systems. High dimensional dynamical systems have intricate behavior either on temporal or on spatial evolution properties. In the past, most of the work on chaotic dynamics has been concentrated on temporal behavior of low-dimensional systems. Recently, spatiotemporal chaos has attracted much attention due to its theoretical and practical applications (Lai & Grebogi, 1999; Shibata, 1998; Barreto *et al.*, 1997; Thompson & Van der Heijden, 1997; Umberger *et al.*, 1989). The present contribution concerns with the nonlinear dynamics of coupled shape memory oscillators. The dynamical response of shape memory systems is also considered in other studies (Savi & Pacheco, 2002; Machado & Savi, 2001a,b). Here, the prospect of chaotic response is of concerned. Equations of motion are formulated using polynomial constitutive model to describe the restitution force of the oscillator. Since the equations of motion of the two-degree of freedom oscillator are associated with a five-dimensional system, the analysis is performed, considering two subspaces associated with each mass.

2. EQUATIONS OF MOTION

Consider a two-degree of freedom oscillator, depicted in Fig. 1. It consists of two masses, m_i (i = 1,2), supported by SMA elements and linear dampers with coefficient c_i (i = 1,2,3). Two forces excite the system harmonically $F_i = \bar{F}_i sin(\Omega_i t)$ (i = 1,2).

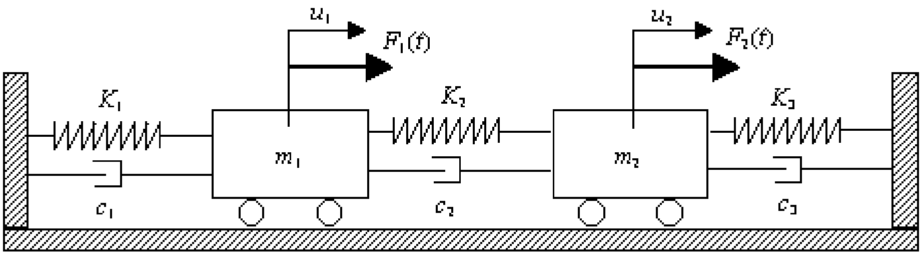

Figure 1 - Two-degree of freedom shape memory oscillator.

Shape memory behavior is described considering polynomial constitutive model (Falk, 1980). This is a one-dimensional model of which represents the shape memory and pseudoelastic effects considering a polynomial free energy that depends on the temperature and on the one-dimensional strain, E. Therefore, the restoring force of the oscillator is given by,

$$K = K(u,T) = \bar{a}(T - T_M)u - \bar{b}u^3 + \bar{e}u^5 \tag{1}$$

where $\bar{a}$, $\bar{b}$ and $\bar{e}$ are positive constants, while T_M is the temperature below which the martensitic phase is stable. Variable u represents the displacement associated with the SMA element. By establishing the equilibrium of the system, non-dimension equations of motion are presented as follows (Savi & Pacheco, 2002),

$$\begin{aligned}
y_0' &= y_1 \\
y_1' &= \delta_1 \sin(\varpi_1 \tau) - (\xi_1 + \xi_2 \alpha_{21} \mu) y_1 + \xi_2 \alpha_{21} \mu\, y_3 - [(\theta_1 - 1) + \alpha_{21}^2 \mu (\theta_2 - 1)] y_0 + \\
&\quad + \alpha_{21}^2 \mu (\theta_2 - 1) y_2 + \beta_1 y_0^3 - \varepsilon_1 y_0^5 - \beta_2 \alpha_{21}^2 \mu (y_2 - y_0)^3 + \varepsilon_2 \alpha_{21}^2 \mu (y_2 - y_0)^5 \\
y_2' &= y_3 \\
y_3' &= \alpha_{21}^2 \delta_2 \sin(\varpi_2 \tau) + \xi_2 \alpha_{21} y_1 - (\xi_2 \alpha_{21} + \xi_3 \alpha_{21} \alpha_{32}) y_3 + \alpha_{21}^2 (\theta_2 - 1) y_0 - \\
&\quad - [\alpha_{21}^2 (\theta_2 - 1) + \alpha_{21}^2 \alpha_{32}^2 (\theta_3 - 1)] y_2 + \beta_2 \alpha_{21}^2 (y_2 - y_0)^3 - \varepsilon_2 \alpha_{21}^2 (y_2 - y_0)^5 + \\
&\quad + \beta_3 \alpha_{21}^2 \alpha_{32}^2 y_2^3 - \varepsilon_3 \alpha_{21}^2 \alpha_{32}^2 y_2^5
\end{aligned} \tag{2}$$

3. NUMERICAL SIMULATIONS

Numerical simulations are performed employing a fourth-order Runge-Kutta scheme with time steps chosen to be less than $\Delta\tau = 2\pi/200\omega$. In all simulations, similar mechanical properties are regarded for all elements of the system. It is assumed a unitary mass and $\varpi_1 = \varpi_2 = 1$, $\xi_1 = \xi_2 = \xi_3 = 0.2$, $\beta_1 = \beta_2 = \beta_3 = 1.3e3$ and $\varepsilon_1 = \varepsilon_2 = \varepsilon_3 = 4.7e5$. These information allows one to conclude that $\alpha_{21} = \alpha_{32} = \mu = 1$ and $\theta_{A1} = \theta_{A2} = \theta_{A3} = 1.9$. Moreover, $\delta_2 = 0$, while δ_1 may vary on the analysis.

Since equations of motion are associated with a five-dimensional system, the visualization of the entire phase space became difficult. Therefore, the analysis of the phase space is performed splitting the space into two subspaces, each of which associated with each mass. The characterization of chaotic, hyperchaotic, quasi-periodic and periodic motion is done regarding Lyapunov exponents, and its estimation employs the algorithm proposed by Wolf *et al.* (1985).

The analysis is developed considering different temperature sets for shape memory elements. At first, consider a situation where all shape memory elements have a low temperature, *i.e.*, only martensitic phase is stable ($\theta_1 = \theta_2 = \theta_3 = 0.7$). After that, the temperature of the connection element, θ_2, is changed. Figure 2 shows the bifurcation diagrams for three different situations: $(\theta_1, \theta_2, \theta_3) = (0.7, 0.7, 0.7)$, $(\theta_1, \theta_2, \theta_3) = (0.7, 1.5, 0.7)$ and $(\theta_1, \theta_2, \theta_3) = (0.7, 3.5, 0.7)$.

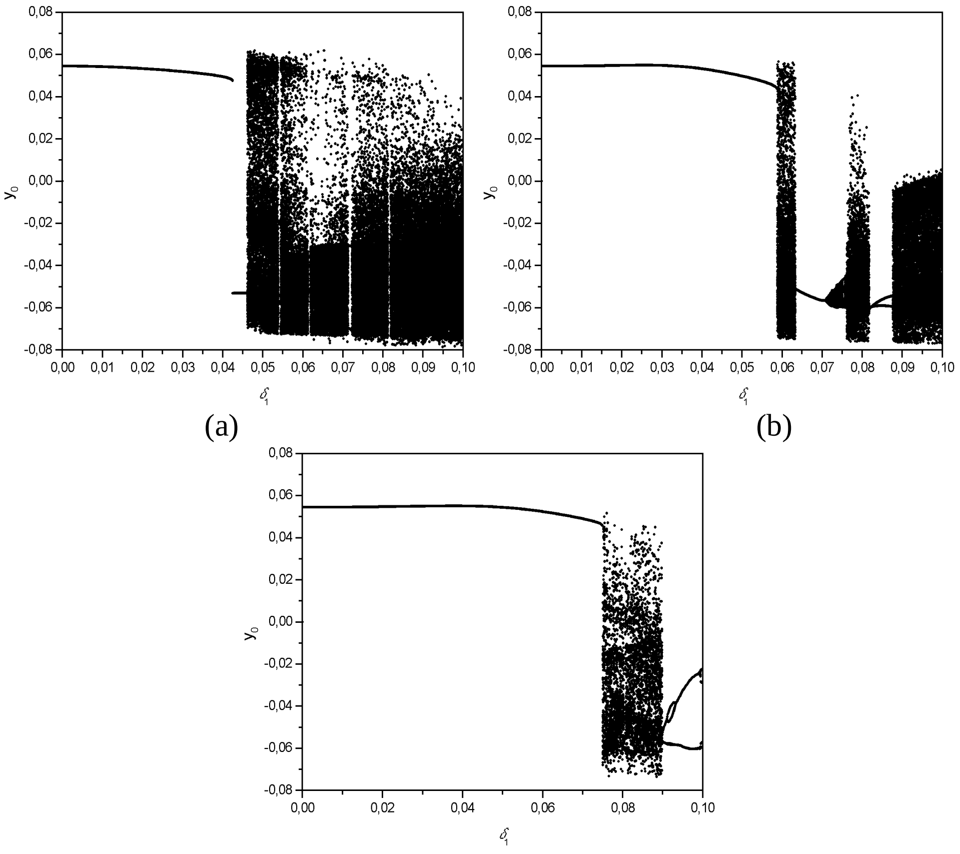

Figure 2 – Bifurcation diagrams for: (a) $(\theta_1, \theta_2, \theta_3) = (0.7, 0.7, 0.7)$; (b) $(\theta_1, \theta_2, \theta_3) = (0.7, 1.5, 0.7)$, and (c) $(\theta_1, \theta_2, \theta_3) = (0.7, 3.5, 0.7)$.

These diagrams of Fig. 2 show how the response of the system is sensitive to temperature changes. For $\delta_1 = 0.06$ and $(\theta_1, \theta_2, \theta_3) = (0.7, 0.7, 0.7)$, the system presents a chaotic response with Lyapunov exponents $\lambda_i = (+0.19, -0.02, -0.46, -0.86)$ (Figure 3). Increasing the temperature connection to $\theta_2 = 1.5$, the response becomes hyperchaotic with $\lambda_i = (+0.34, +0.05, -0.55, -1.0)$ (Figure 4). On the other hand, for $\theta_2 = 3.5$, the system presents a periodic response.

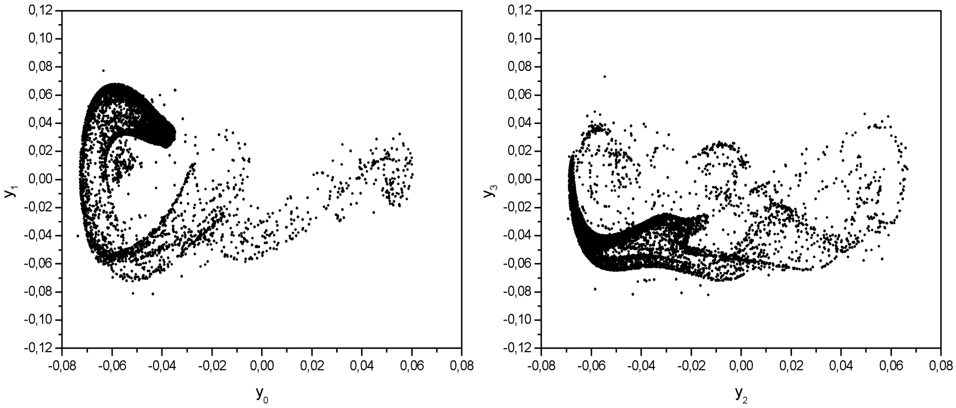

Figure 3 – Response for δ_1 = 0.06 and (θ_1, θ_2, θ_3) = (0.7, 0.7, 0.7).

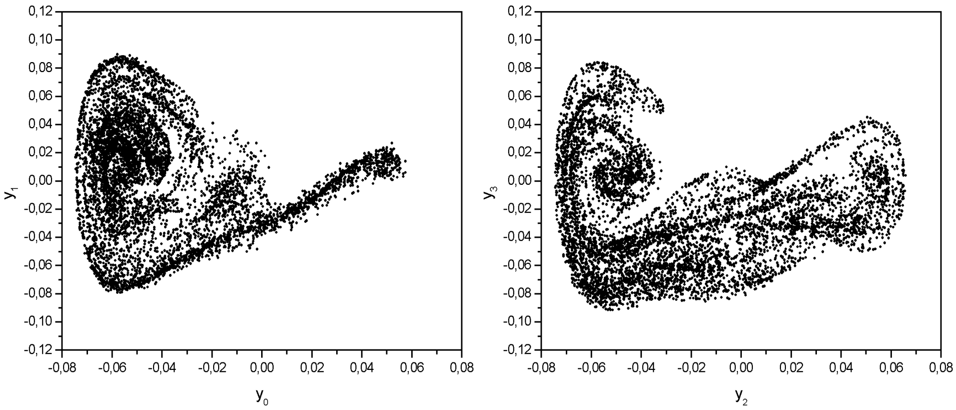

Figure 4 – Response for δ_1 = 0.06 and (θ_1, θ_2, θ_3) = (0.7, 1.5, 0.7).

Now, consider a situation where all shape memory elements have an intermediate temperature, where both martensitic and austenitic phases are stable ($\theta_1 = \theta_2 = \theta_3 = 1.5$). Likewise to the first example, the temperature of the connection element, θ_2, is changed. Figure 5 shows bifurcation diagrams for three different situations: (θ_1, θ_2, θ_3) = (1.5, 0.7, 1.5), (θ_1, θ_2, θ_3) = (1.5, 1.5, 1.5) and (θ_1, θ_2, θ_3) = (1.5, 3.5, 1.5).

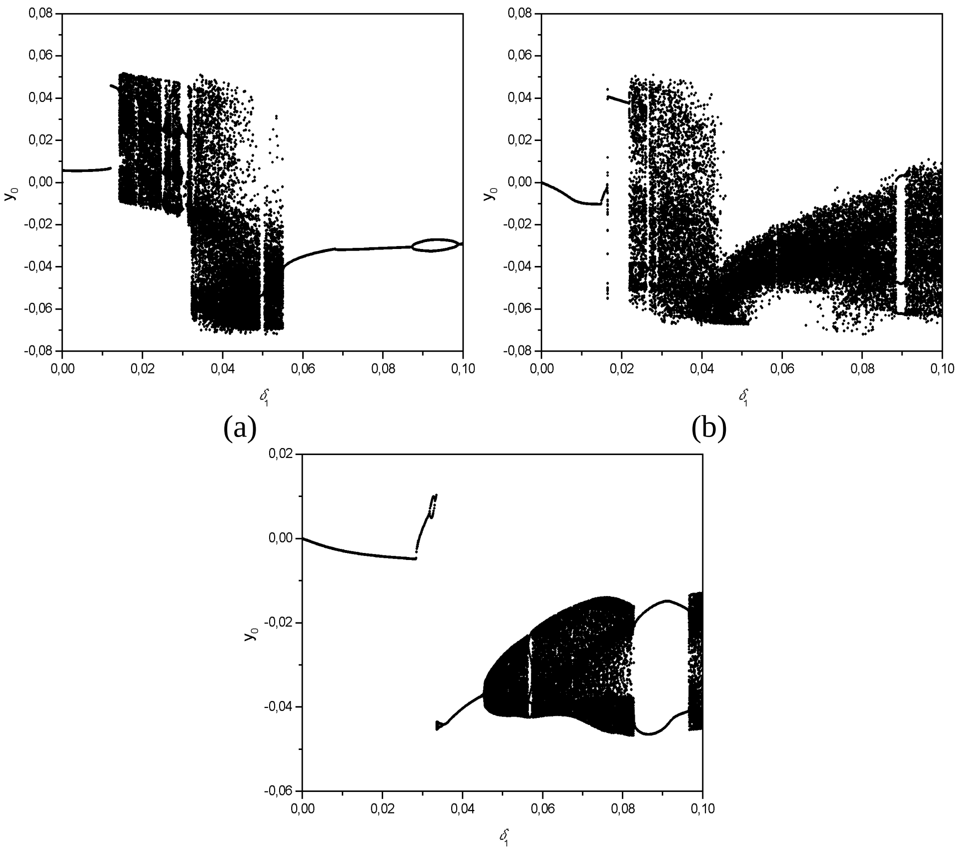

Figure 5 – Bifurcation diagrams for: (a) (θ_1, θ_2, θ_3) = (1.5, 0.7, 1.5); (b) (θ_1, θ_2, θ_3) = (1.5, 1.5, 1.5) and (c) (θ_1, θ_2, θ_3) = (1.5, 3.5, 1.5).

For δ_1 = 0.06 and (θ_1, θ_2, θ_3) = (1.5, 0.7, 1.5), the system presents a periodic response. Increasing the temperature connection to θ_2 = 1.5, the response becomes chaotic with λ_i = (+0.22, –0.13, –0.30, –0.94) (Figure 6). On the other hand, for θ_2 = 3.5, the system presents a quasi-periodic response with λ_i = (0.00, –0.15, –0.41, –0.66) (Figure 7).

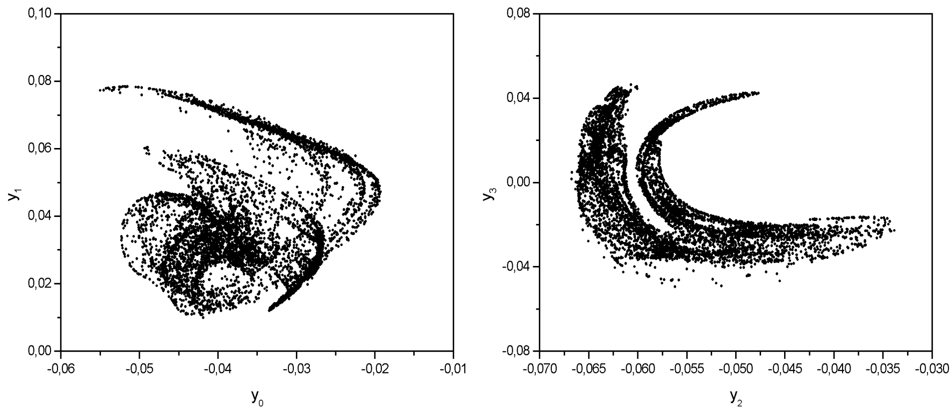

Figure 6 – Response for δ_1 = 0.06 and (θ_1, θ_2, θ_3) = (1.5, 1.5, 1.5).

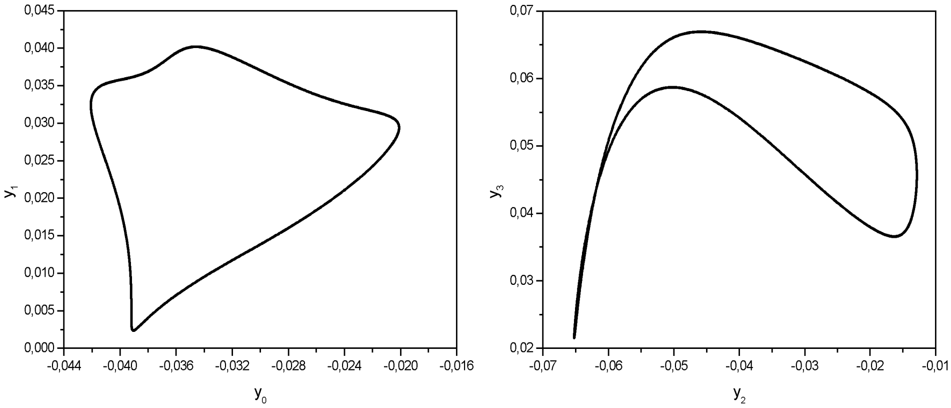

Figure 7 – Response for δ_1 = 0.06 and (θ_1, θ_2, θ_3) = (1.5, 3.5, 1.5).

Figure 8 shows the response for δ_1 = 0.02 and (θ_1, θ_2, θ_3) = (1.5, 0.7, 1.5). A chaotic attractor appears on the phase space, indicating the presence of chaotic motion. Lyapunov exponents for this situation, λ_i= (+0.23, –0.38, –0.44, –0.56), assure this conclusion. Increasing the temperature connection to both intermediate and high one, a periodic motion of period-1 appears to replace the chaotic one.

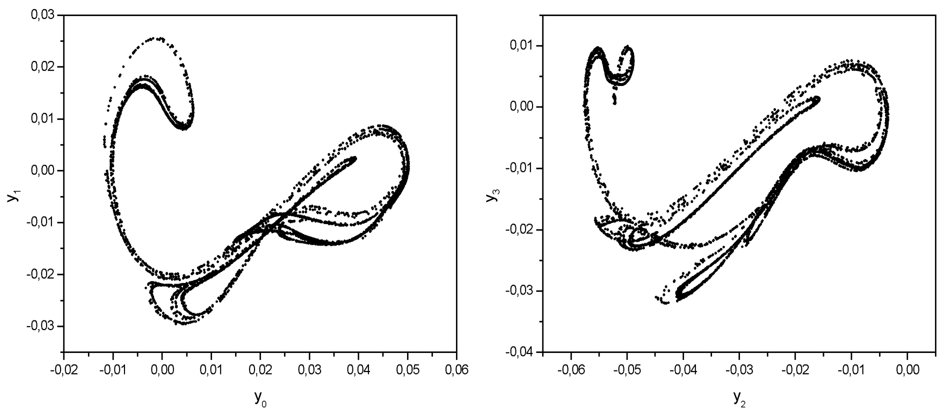

Figure 8 – Response for δ_1 = 0.02 and (θ_1, θ_2, θ_3) = (1.5, 0.7, 1.5).

4. CONCLUSIONS

This article reports an analysis of the response of a two-degree of freedom shape memory oscillator. A polynomial constitutive model was assumed to describe the constitutive behavior of the restitution force. The system response analysis was performed assessing Lyapunov exponents and phase spaces. Since the high dimension of the system, the phase space was split into two subspaces, each of which related to each mass. The analysis of the temperature of elements, through bifurcation diagrams, shows how its variation can modify the system response. Several routes of responses are observed just changing the temperature connection. Variations like hyperchaos → chaos → periodic and periodic → quasiperiodic → chaotic may occurs. These several routes show how intricate and rich is the system behavior.

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