



Elastic-plastic behavior of structures under random vibrations excited by the wind

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Abstract. *In this paper, we present a numerical model to analyze structures under random dynamic excitation induced by the wind. We also consider the structure to have nonlinear behavior due to the elastic-plastic constitutive law adopted for the material, structural steel. The members of the studied structures may experience formation or disappearance of plastic hinges, leading to a more economic design. We consider possible applications to telecommunication towers, offshore structures etc. Random vibrations of linear structures is a well established subject. In the other hand, very few results are available for the nonlinear case as the one we consider. To simulate random vibrations we use a Monte Carlo type analysis. Building Codes power spectral density functions for wind velocity are used to generate a large number of harmonic input functions. Their phase angles are generated via a pseudo-random algorithm. Numerical time integration using Newmark's method is performed for each input function. The large amount of response data obtained is statically treated to allow for useful engineering conclusions on the probability of occurrence of unfavorable events.*

Keywords. *synthetic wind, steel structures, nonlinear behavior, random vibrations.*

1. BEHAVIOR OF THE STRUCTURAL MATERIAL

Generally, as a start point of a standard structural analysis, it is assumed linear elastic behavior for the structural material, implying a reversible stress-strain relationship

However, the behavior of steel members in a structure involves phenomena that are far more complicated. When tested in compression or traction, a steel specimen behaves elastically until the stress reaches a well-defined yield limit. From this point on, large strains can be observed while the stress level keeps constant.

For the studies presented in this work, it will be adopted an ideal elastic-plastic material, presenting the stress-strain diagram of Fig. (1).

The load capacity of a steel frame under bending rarely is finished when the yield load is reached in one or in a certain number of sections. If the possibility of one of the elements of the structure to fail due to elastic instability is excluded, structural collapse will only occur if several sections present plastic behavior simultaneously, in a certain pattern, which is called a plastic collapse configuration.

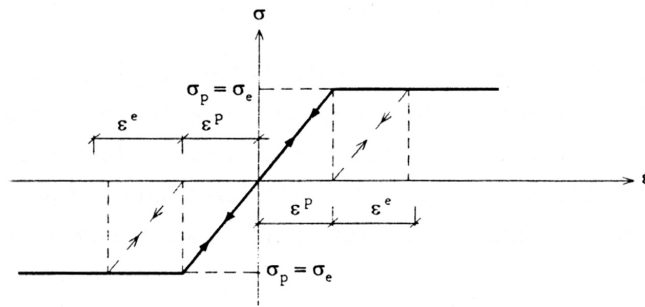


Figure 1. Behavior of an elastic-plastic ideal material, Mazzilli (2000)

In general, steel members present a stretch of plastic behavior after reaching the yield stress in a certain number of sections and collapse will occur for loads larger than the ones that provoke this situation. Thus, metallic frames have a spare resistance after the yield of a certain number of sections, due to the readjustment of the structural behavior and consequent redistribution of stresses.

This capacity to support loads that could produce stresses larger than the yield limit in some sections cannot be used in elastic designs. Therefore, it's unquestionable the fact that structures designed by elastic procedures are safer but not necessarily economic, as pointed by Pfeil (1995) and by the Brazilian code for design of steel structures of buildings, NBR-8800 (1986).

1.1. The plastic hinges

As described by Neal (1956), when a certain transversal section of the structure reaches the total plasticity due to bending, the action of a hinge will occur in this section. Additional section rotation may occur while the transmitted bending moments remain constant.

This kind of hinge is called a plastic hinge and the bending moment that is developed in the hinge is the fully plastic moment of the section. A basic hypothesis of the plastic theory is that, guaranteeing that the bending moment remains constant and equal to the fully plastic moment, a plastic hinge can suffer rotations of any magnitude.

For dynamic loading cases Neal (1956) quotes that when a hinge is formed the rotation direction coincides with the direction of the fully plastic moment of the section. If the bending moment decreases for a lesser value than the value of fully plastic, or if the direction of rotation is inverted, the elastic behavior is recovered in the section. As adopted by Fernandes (1995).

According to Baker (1969), the value of the fully plastic moment can be calculated as a function of the yield stress and the form and dimension of the transversal section.

2. CHARACTERIZATION OF THE WIND: THE PROCESS OF THE SYNTHETIC WIND

The process of the synthetic wind initially proposed by Franco (1993) and Franco; Isyumov (1997), divides the wind load, in the direction of the flow, in a fluctuating parcel and a mean parcel. According to the original method, the mean parcel is applied statically to the structure.

The fluctuating parcel is divided in a series of harmonic components with random phase angles. Initially, the method proposed the use of 11 harmonic components with one of them resonant with the fundamental frequency of the structure. The frequencies of the other components were defined as multiple or submultiples of this reference frequency.

The amplitude of each one of the harmonic components is obtained from the power spectrum of the wind.

The original process proposed that the fluctuating parcel corresponded to 52% of the total loading. In this work, we assume a correction that the floating percentage of the total varies as a function of the height, as considered by Carril Jr. (2000).

2.1. Mean and fluctuating parcel of the wind

As described by Blessmann (1995), one observes that the power spectrum of the wind speed presents low density in a range that varies from 5 cycles/hour to 0.5 cycles/hour. This interval defines clearly the macrometeorological components (frequencies below 0.5 cycles/hour) and the micrometeorological components (frequencies above 5 cycles/hour). This clear division of the spectrum allows the handling of the wind as the composition of a mean value plus gusts.

Civil engineering structures have natural frequencies for the first modes of vibration that can approach the zone of high frequency of the wind.

The mean speed of the wind conventionally is measured for time intervals varying from 10 min to 1 h. However, the Brazilian Code of Wind Load on Buildings, as several other international codes, defines values for the peak wind speed measured in very small time lags (2 to 5 s). From these practically instantaneous values it can be obtained, through empirical analytical expressions, the values of the average speeds for the desired time lag.

Using the expressions proposed by the Brazilian code of wind load on buildings, NBR-6123 (1988), we can get the following speeds and pressures:

$$v_{600} = 0,69b_{600}V_0\left(\frac{z}{10}\right)^{p_{600}} \quad (1)$$

$$v_3 = b_3V_0\left(\frac{z}{10}\right)^{p_3} \quad (2)$$

where v_{600} and v_3 represent the wind speed at height z for the period of 600 s and 3 s respectively. V_0 is the basic wind speed of the region, b and p are meteorological parameters as a function of the terrain class and the gust period, all of them defined by the Brazilian code (NBR-6123). The peak pressure (q_j) and the mean pressure (q_{est}) of the wind can be obtained:

$$q_j = 0,613v_3^2 \quad (3)$$

$$q_{est} = 0,613v_{600}^2 \quad (4)$$

Thus, the fluctuating pressure can be obtained as the difference between the peak and the mean pressure.

2.2. Spectrum of the floating wind speeds

The wind power as a function of the frequency range and height can be calculated by the expression:

$$dW = S(z, n)dn \quad (5)$$

where n is the frequency of the considered gust.

The function $S(z, n)$ is known as the power spectrum of wind speeds and using a logarithmic scale it can be conveniently exposed in a reduced form as:

$$S_r(z, n) = \frac{nS(z, n)}{u_*^2} \quad (6)$$

where u_*^2 is labeled as the friction speed of wind, which is a function of the terrain roughness.

The authors that started to measure the power spectrums of the wind didn't consider the influence of the height in it. Several empiric functions were proposed to determine the reduced power spectrum as a function of the frequency n and the average speed V_m at the height 10 m in open terrain.

One of the most important reduced spectrums is the one proposed by Davenport and slightly modified by Franco (1993). The expressions are:

$$\frac{nS(n)}{u_*^2} = 4 \frac{x^2}{(1+x^2)^{4/3}} \quad (7)$$

$$x = \frac{1220n}{V_m} \quad (8)$$

Franco (1993), quoting Davenport and Simiu/Scanlan, wrote the pressure spectrum as a function of the speed spectrum as:

$$S_{p'}(z, n) = (\rho c V_z)^2 S(z, n) \quad (9)$$

where ρ is the air density, c is the aerodynamic coefficient of the structure and V_z is the average speed at height z .

It indicates that at any point of the structure the power spectrum of pressures can be regarded as proportional to the power spectrum of wind speeds.

2.3 Decomposition of the floating pressures

According to Franco (1993), the floating pressures $p'(t)$ can be represented as a Fourier integral:

$$p'(t) = \int_{-\infty}^{\infty} C(n) \cos[2\pi nt - \theta(n)] dn \quad (10)$$

with

$$C(n) = \sqrt{A^2(n) + B^2(n)} \quad (11)$$

$$\theta(n) = \tan^{-1} \frac{B(n)}{A(n)} \quad (12)$$

$$A(n) = \int_{-\infty}^{\infty} p'(t) \cos 2\pi n t dt \quad (13)$$

$$B(n) = \int_{-\infty}^{\infty} p'(t) \text{sen } 2\pi n t dt \quad (14)$$

The mean square value of $p'(t)$, supposed defined in a long enough time lag T , is:

$$\sigma^2(p') = \frac{1}{T} \int_{T/2}^{T/2} p'^2(t) dt = \frac{2}{T} \int_0^{\infty} C^2(n) dn \quad (15)$$

and if $T \rightarrow \infty$, them:

$$\sigma^2(p') = \int_0^{\infty} S(n) dn \quad (16)$$

where $S(n)$ is the spectral density function of $p'(t)$ and $S(n)dn$ represents the contribution associated to the frequency range dn .

Instead of using an infinite number of functions, $p'(t)$ can be represented, as an approximation, by a finite number m of harmonic functions, conveniently chosen in a way that the frequency range adopted really contains the important range, which goes from approximately 1.7E-3 Hz (600 s) to 2 Hz (0.5 s) or more, in order to capture the higher modes.

As proposed by Franco (1993), eleven harmonic functions were adopted in a way that one of them has resonant frequency with the first vibration mode of the structure. Thus:

$$p'(t) = \sum_{k=1}^m C_k \cos\left(\frac{2\pi n_r}{r_k} t - \theta_k\right) \quad (17)$$

$$C_k = \sqrt{2 \int_{(k)} S(n) dn} \quad (18)$$

$$r_k = 2^{k-r} \quad (19)$$

The C_k values are calculated by integrating the spectral density function in the frequency range dn . This calculation can be developed either from the natural power spectrum of the wind or from the reduced spectrum of the wind, resulting the same values.

The maximum amplitude of the floating pressure can be written as a parcel of the total pressure, and then the amplitudes if the harmonic components can be written as:

$$p'_k = \frac{C_k}{\sum_{k=1}^m C_k} p' = c_k p' \quad (20)$$

The phase angles remain undetermined. Then the harmonic components are superposed with random combinations of these angles.

2.4 Spatial speed correlation

As it is well known, when the distance between two points of the structure grows, the correlation of the fluctuating wind speeds diminishes. In predominantly vertical structures, such as

slender buildings, towers and others, it is sufficient to regard only the vertical correlation of the wind speeds. Using the narrow band crossed correlation coefficient as proposed by Franco (1993) we have:

$$Coh(\Delta z, n_k) = \exp\left(-\frac{7\Delta z n_k}{V_m}\right) \quad (21)$$

It can be observed that the correlation coefficient varies from 1 when $\Delta z=0$ to 0 when $\Delta z \rightarrow \infty$. The shape of the function creates the concept of size of equivalent gust, meaning the size of a gust perfectly correlated that produces the same effect on the structure, as indicated in Fig. (3).

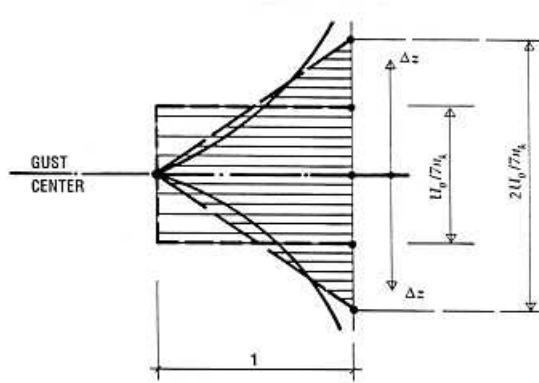


Figure 2. Spatial correlation of wind speed, Franco (1993)

The above consideration shows that a gust of frequency n_k can be approximately represented by a perfectly correlated gust with height Δz_{ok} and a rectangular shape, as adopted in the present paper.

2.5. Summary of the method

Possessing the wind data calculated in 2.1 the static load due to the wind can be defined as:

$$F = C_a A q_{est} \quad (22)$$

where C_a is the drag coefficient and A is the area of the vertical structure projection perpendicular to the wind. The dynamic components of the force can be written as:

$$F_{din} = C_a A q_f C_r c_k \cos(w_k t - \theta_k) \quad (23)$$

where C_r is the spatial correlation coefficient of speeds.

The approximation can be improved increasing m and observing the following instructions: m larger or equal to 11, the period of one harmonic function must be equal to the fundamental natural period of the structure and the periods of the other harmonic functions must be multiples or submultiples of the fundamental period by a factor 2.

Franco (1993) proposed that when $m=11$ the contribution of the resonant component is overestimated by a factor 2. Thus the amplitude of the resonant component is divided by 2 and, in order to keep the sum of the coefficients equal to 1, one quarter of the value of the non-corrected resonant coefficient is added to the immediately previous and posterior coefficients.

A series of loading is defined as the combination of the static loading and the eleven harmonic components with random phase angles. In this paper, we use 20 series of loading. The total time of analyses can be defined as an input data. However, values around 600 s are indicated. The structural response is obtained by a Newmark's incremental integration method, as in Clough (1975).

When the maximum displacement is obtained for each series of loading, a statistic analysis is performed considering a normal distribution. The value of the maximum characteristic displacement, with a confidence level of 95%, is obtained by:

$$u_{\max} = 1,96\sigma + \mu \quad (24)$$

where σ is the standard deviation and μ is the mean value of the maximum values series.

3. NUMERICAL EXAMPLE

We present the dynamic response of a hypothetic telecommunication tower. The following figure represents the structure and the simplified structural system adopted for the analysis.

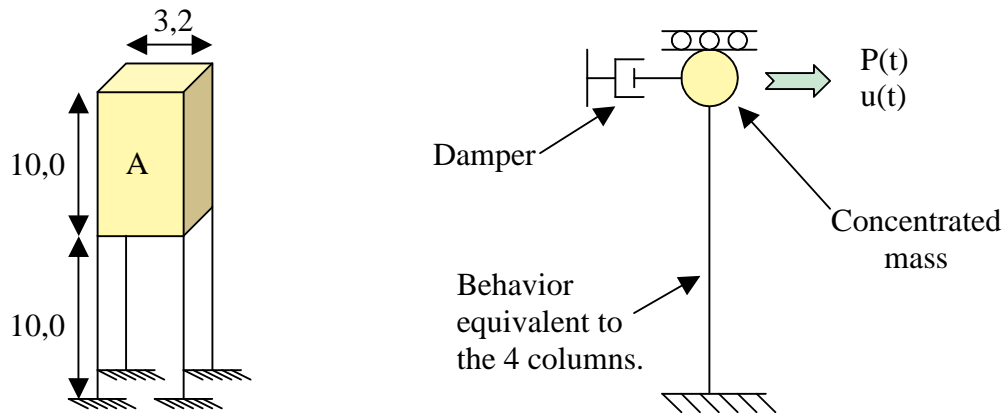


Figure 3. Structure Model and simplified structural system adopted

The wind velocity is perpendicular to side A. The concentrated mass is equal to 70000kg, and the columns have negligible mass. The damping rate is equal to 1% of the critical. The steel in the columns has yield stress equal to 250Mpa. The characteristic velocity of the wind is 45m/s, the aero dynamical coefficient of the structure is equal to 0,8 and the gust center was adopted at 15 meters high. Using the proposed formulation we can build the following table:

Table 1. Harmonic components and sizes of equivalent gusts

k	T_k (s)	n (Hz)	x	$\frac{n * S(n)}{u_*^2}$	C_k	c_k (%)	c_{k*} (%)	Δz_{0k} (m)
1	0,417	2,400	94,300	0,193	0,621	5,1	5,1	1,8
2	0,833	1,200	47,150	0,306	0,783	6,4	6,4	3,7
3	1,667	0,600	23,575	0,485	0,985	8,0	10,5	7,4
4	3,333	0,300	11,787	0,765	1,237	10,1	5,0	14,8
5	6,667	0,150	5,894	1,180	1,536	12,5	15,0	29,6
6	13,333	0,075	2,947	1,683	1,835	14,9	14,9	59,1
7	26,667	0,038	1,473	1,864	1,931	15,7	15,7	118,3
8	53,333	0,019	0,737	1,218	1,561	12,7	12,7	236,6
9	106,667	0,009	0,368	0,458	0,957	7,8	7,8	473,1
10	213,333	0,005	0,184	0,130	0,509	4,1	4,1	946,3
11	426,667	0,002	0,092	0,034	0,259	2,1	2,1	1892,6

Exciting the structure by the proposed method and making an incremental dynamic analysis, with integration step equal to 0.1s, we can obtain the dynamic response of the structure for all the 20 series of loading. We present the results for one of these 20 series.

Supposing the material behavior as always elastic for any loading magnitude, we have Fig. 4.

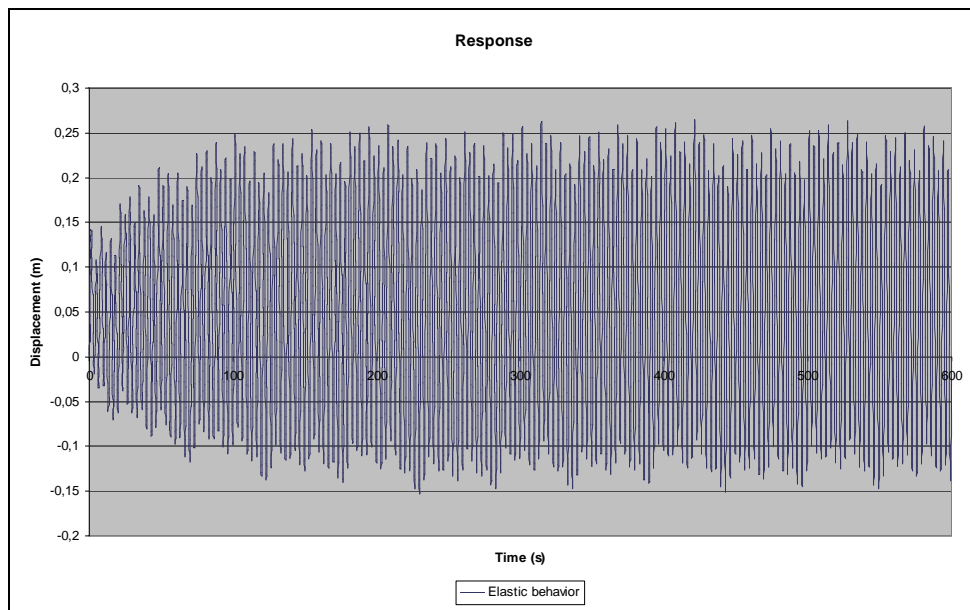


Figure 4. Dynamic response regarding the material as elastic

It's important to notice that Fig. 4 displays a complete hypothetical situation. After all, to present this kind of response, the material would be under higher stresses than the yield stress.

However, when we consider the elastic-plastic behavior of the material every time that a plastic hinge appears the structure suffers an irrecoverable damage.

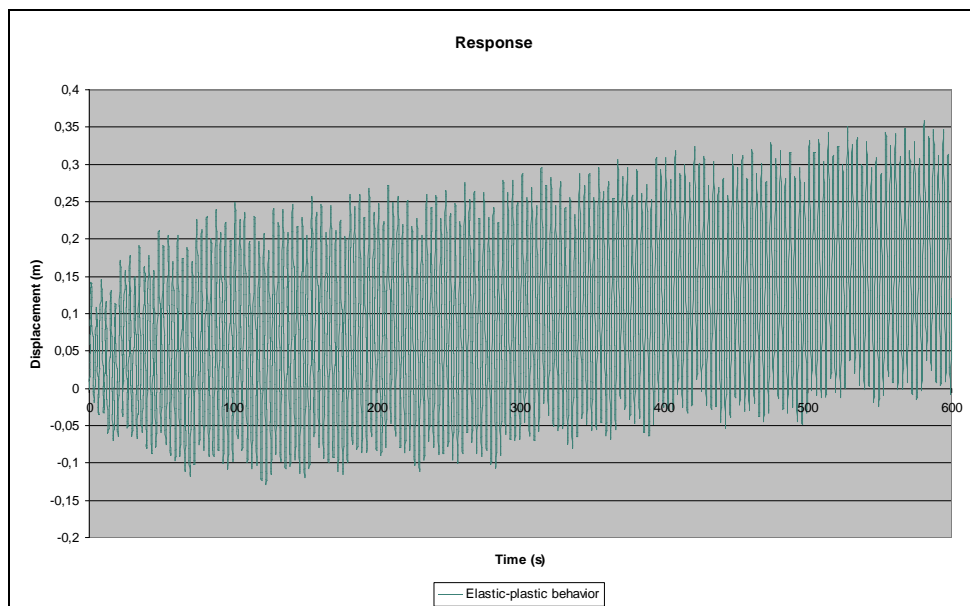


Figure 5. Dynamic response regarding the material as elastic-plastic

When we consider the elastic-plastic behavior of the material, the maximum displacement for each series of loading is strongly influenced by the total analysis time considered. It is

recommended considering a total analysis time equal to 600s obtaining the average wind speed for this period by Eq. (1).

The statistic analysis of the maximum displacements shows the differences found:

Table 2. Structural maximum responses

Maximum values and statistical proprieties			
Elastic behavior		Elastic plastic behavior	
Series	Displacement (m)	Series	Displacement (m)
1	0,26	1	0,36
2	0,27	2	0,40
3	0,25	3	0,29
4	0,26	4	0,33
5	0,25	5	0,26
6	0,27	6	0,35
7	0,27	7	0,38
8	0,27	8	0,34
9	0,26	9	0,32
10	0,26	10	0,30
11	0,26	11	0,33
12	0,26	12	0,34
13	0,27	13	0,37
14	0,26	14	0,32
15	0,26	15	0,32
16	0,27	16	0,35
17	0,26	17	0,32
18	0,26	18	0,35
19	0,26	19	0,28
20	0,26	20	0,33
Mean	0,2631	Mean	0,3336
Standard Deviation	0,0054	Standard Deviation	0,0326
Max. Charac. Value	0,2736	Max. Charac. Value	0,3975

4. CONCLUSIONS

First of all it's necessary to point out that for framed steel buildings the use of the structural capacity after plasticity as a way to resist to catastrophic or infrequent load, as strong winds, is perfectly justifiable and the method proposed in this paper constitutes a simple systematization method for these cases.

The dynamic analysis proposed is especially important for cases, as in the example, in which the structure has natural vibration frequencies under 1Hz.

For the integration steps in which the material behavior changes drastically, better results would be found refining the process in these points, diminishing the integration steps. It also can be observed that, adopting the same systematization used in the example, there will be harmonic loadings with periods equal to 1/8 of the natural period of the structure and to correctly represent these loadings it's necessary to diminish the integration step implying in a major computational effort.

The stochastic characteristic of the method can be observed in the table 2. When it's considered the elastic behavior of the material the maximum values obtained don't vary much from one series

to another. However, when it's considered the elastic plastic behavior of the material the values found vary in a larger range. This phenomenon occurs due to the fact that the way the harmonic loadings are combined strongly affects the number of times that a plastic hinge appears influencing the structural damage accumulated.

Finally it can be noticed that the definition of the gust center position doesn't have a systematization that can be applied easily, and the determination of the most unfavorable position to apply the load is a theme that needs more development.

5. ACKNOWLEDGEMENT

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