II CONGRESSO NACIONAL DE ENGENHARIA MECÂNICA
II NATIONAL CONGRESS OF MECHANICAL ENGINEERING
12 a 16 de Agosto de 2002 - João Pessoa – PB

IDENTIFICATION OF CONSTITUTIVE PARAMETERS OF VISCOELASTIC MATERIALS BASED ON A TIME DOMAIN TECHNIQUE

Daniel Alves Castello
Solid Mechanics Laboratory
Federal University of Rio de Janeiro, COPPE/PEM
P.O.Box 68503, Zip Code: 21945-970, Rio de Janeiro, RJ, Brasil
castello@mecsol.ufrj.br

Fernando Alves Rochinha
Solid Mechanics Laboratory
Federal University of Rio de Janeiro, COPPE/PEM
P.O.Box 68503, Zip Code: 21945-970, Rio de Janeiro, RJ, Brasil
faro@serv.com.ufrj.br

Abstract. *The present work approaches the problem of identification of the elastic and damping fields of a medium by means of a time domain technique. This technique is within the inverse problems scope, i.e., the solution of the problem is sought by means of the minimization of a suitable error function which includes data from both the system model and the experiment setup for the same input excitation. In order to assess the effectiveness of the proposed method, simulations on a bar-like structure have been performed under impact loading and considering the corrupting effects of noise.*

Keywords: *System Identification, Viscoelasticity and Inverse Problem.*

1. INTRODUCTION

Aiming at taking advantage of the dynamical properties of each material in a system design, it is required the fully understanding of the mechanical behavior of these materials. This behavior can be described by different models such that the designer has some freedom to choose the most suitable one for a certain type of application that the material will be part of. Once one has in hands the chosen model that will be used to describe the mechanical behavior of the material under study, the next step usually consists in determining the set of parameters that characterizes this model. The identification of these parameters provides a mathematical model which enables one to simulate and predict the response of the material when it is subjected to a certain excitation. In particular, the mechanical behavior of viscoelastic materials is of great interest in engineering sciences such as mechanical, civil, aerospace and biomechanical.

The technical literature concerned with the identification of viscoelastic materials is very extensive and it presents different approaches to the problem (Gavrus, 1996;Dietrich, 1998; Rusovici, 1999; Haupt, 2000; Mossberg, 2000; Sarron, 2000; Janno, 2001 and Mossberg, 2001) can be cited as the most recent ones.

The present work is built on the use of a constitutive equation for viscoelastic materials parameterized by a set of unknown constitutive parameters and makes use of a time domain technique to identify this set of parameters. The solution technique is within the inverse problems scope, i.e., the solution is sought by means of the minimization of a suitable error function which includes data from both the system model and the experiment. The technique takes into account the constraint associated to the system evolution equation as being part of an extended error function what naturally gives rise to the Lagrange multiplier variables which are obtained via solution of an adjoint problem. The effectiveness of the technique is assessed on simulations performed on a bar-like structure, where strains or displacements are measured at a subset of the system degrees of freedom. The simulated experiment consists on a bar under dynamic loading excitation and in order to furnish realism to the simulations, it is considered the corrupting effects of filtering, sampling and the analyses are performed for different levels of signal-to-noise ratio.

2. DIRECT PROBLEM

Consider an $n - DOF$ linear dynamic system such that its discretized evolution equation is given by

$$\begin{cases} \mathbf{M}\ddot{\mathbf{x}}(t) + \mathbf{D}\dot{\mathbf{x}}(t) + \mathbf{K}\mathbf{x}(t) = \mathbf{f}(t) \\ \mathbf{y}(t) = \mathbf{C}\mathbf{x}(t) \\ \mathbf{x}(0) = \mathbf{x}_0 \\ \dot{\mathbf{x}}(0) = \dot{\mathbf{x}}_0 \end{cases} \tag{1}$$

where $\mathbf{M}$, $\mathbf{D}$ and $\mathbf{K}$ are nxn matrices describing the mass, damping and stiffness properties respectively and the n dimensional vectors $\mathbf{x}$ and $\mathbf{f}$ correspond to the system displacement field and to the external loading applied to the system. The matrix $\mathbf{C}$ associates the system DOF to the measured observable variables $\mathbf{y}$, which in turn, can be displacements or strains. The direct problem consists basically in determining the transient displacement field $\mathbf{x}(t)$ when the external load is known. It should be emphasized that the direct analysis assumes a priori that the material behavior is known, fact that, for the present problem, means that one has in hands the constitutive equation between stress and strain for the material under study and moreover, the actual value of the parameters of this constitutive equation is available . In equation (1) it is implicity that the property matrices $\mathbf{D}$ and $\mathbf{K}$ are in some way functions of the parameters that characterize the material constitutive equation, viz.

$$\mathbf{K} = \mathbf{K}(\mathbf{p}) \qquad \text{and} \qquad \mathbf{D} = \mathbf{D}(\mathbf{p}) \tag{2}$$

where the vector $\mathbf{p}$ contains both elastic and damping parameters upon which the material constitutive equation is defined.

3. INVERSE PROBLEM

For the inverse problem, the elastic and damping parameters $\mathbf{p}$ are considered to be unknown. It is also assumed that there is set of experimental data available $\mathbf{y}^E(t)$, $t \in [0, t_f]$, which can be used as the additional information for the estimation of the parameters $\mathbf{p}$ and consequently the matrices $\mathbf{K}$ and $\mathbf{D}$. The idea is to minimize a suitable error function which consists basically of the norm of the difference between the measured data $\mathbf{y}^E(t)$ and the data obtained from the system model $\mathbf{y}(t)$ for the same input excitation. The error function $\hat{J}_1(\mathbf{p})$ is defined as follows

$$\hat{J}_1(\mathbf{p}) = \int_0^{t_f} [\mathbf{y} - \mathbf{y}^E]^T [\mathbf{y} - \mathbf{y}^E] \, dt \tag{3}$$

Therefore, the goal of the inverse problem step is to estimate the N_p-dimensional vector of unknown parameters $\mathbf{p}$ through the minimization of $\hat{J}_1(\mathbf{p})$. The search step size determination will be presented later.

3.1. Parameter Estimation

The technique used for parameter estimation is the Conjugate Gradient Method, which is a powerful iterative technique for solving linear and nonlinear inverse problems of parameter estimation (Özisic, 2000). In the iterative procedure of the conjugate gradient method, at each iteration a suitable step size is taken along a direction of a descent in order to minimize the error function as follows

$$\mathbf{p}^{(k+1)} = \mathbf{p}^{(k)} - \beta^{(k)}\mathbf{d}^{(k)} \tag{4}$$

where k indicates the current iteration, $\beta^{(k)}$ is the search step size, $\mathbf{d}^{(k)}$ is the direction of descent which is defined as follows

$$\mathbf{d}^{(k)} = \nabla\hat{J}^{(k)} + \gamma^{(k)}\mathbf{d}^{(k-1)} \tag{5}$$

For the conjugation coefficient $\gamma^{(k)}$, among some possibilities, one has chosen

$$\gamma^{(k)} = \frac{\nabla\hat{J}^{(k)} \cdot [\nabla\hat{J}^{(k)} - \nabla\hat{J}^{(k-1)}]}{\nabla\hat{J}^{(k-1)} \cdot \nabla\hat{J}^{(k-1)}} \tag{6}$$

Further details about the previous choice may be found in (Daniel, 1971).

3.2. Sensitivity Problem

The sensitivity function $\Delta\mathbf{x}(t)$, which is the solution of the sensitivity problem, is defined as the directional derivative of the displacement field $\mathbf{x}(t)$ in the direction of the perturbation of the unknown parameter vector $\mathbf{p}$ (Özisic, 2000). The presentation of the sensitivity problem is required in order to obtain the search step size $\beta^{(k)}$. Aiming at obtaining the sensitivity problem one assumes that the displacement field $\mathbf{x}(t)$ is perturbed by an amount $\Delta\mathbf{x}(t)$ when the unknown vector of parameters $\mathbf{p}$ is perturbed by $\Delta\mathbf{p}$ such that

$$\begin{cases} \mathbf{x}(t,\mathbf{p}) \mapsto \mathbf{x}(t,\mathbf{p}+\Delta\mathbf{p}) = \mathbf{x}(t,\mathbf{p}) + \Delta\mathbf{x}(t,\mathbf{p}) \\ \mathbf{D}(\mathbf{p}) \mapsto \mathbf{D}(\mathbf{p}+\Delta\mathbf{p}) = \mathbf{D}(\mathbf{p}) + \Delta\mathbf{D}(\mathbf{p}) \\ \mathbf{K}(\mathbf{p}) \mapsto \mathbf{K}(\mathbf{p}+\Delta\mathbf{p}) = \mathbf{K}(\mathbf{p}) + \Delta\mathbf{K}(\mathbf{p}) \end{cases} \tag{7}$$

The evolution equation for the system under this new set of parameters casts as follows

$$\begin{cases} \mathbf{M}[\ddot{\mathbf{x}} + \Delta\ddot{\mathbf{x}}] + [\mathbf{D} + \Delta\mathbf{D}][\dot{\mathbf{x}} + \Delta\dot{\mathbf{x}}] + [\mathbf{K} + \Delta\mathbf{K}][\mathbf{x} + \Delta\mathbf{x}] = \mathbf{f} \\ \mathbf{y} + \Delta\mathbf{y} = \mathbf{C}[\mathbf{x} + \Delta\mathbf{x}] \end{cases} \tag{8}$$

Where $\mathbf{x} = \mathbf{x}(t,\mathbf{p})$ e $\mathbf{y} = \mathbf{y}(t,\mathbf{p})$. Rearranging the equation in a more suitable form one gets

$$\begin{cases} \mathbf{M}\Delta\ddot{\mathbf{x}} + \mathbf{D}\Delta\dot{\mathbf{x}} + \mathbf{K}\Delta\mathbf{x} = (\mathbf{f} - \mathbf{M}\ddot{\mathbf{x}} + \mathbf{D}\dot{\mathbf{x}} + \mathbf{K}\mathbf{x}) - (\Delta\mathbf{D}\dot{\mathbf{x}} + \Delta\mathbf{K}\mathbf{x}) - (\Delta\mathbf{D}\Delta\dot{\mathbf{x}} + \Delta\mathbf{K}\Delta\mathbf{x}) \\ \Delta\mathbf{y} = [\mathbf{C}\mathbf{x} - \mathbf{y}] + \mathbf{C}\Delta\mathbf{x} \end{cases}$$

(9)

Applying the initial conditions to the new solution $\mathbf{x}(t, \mathbf{p} + \Delta\mathbf{p})$ leads to

$$\begin{cases} \mathbf{x}(0, \mathbf{p} + \Delta\mathbf{p}) = \mathbf{x}_0 = \mathbf{x}(0, \mathbf{p}) + \Delta\mathbf{x}(0, \mathbf{p}) \\ \dot{\mathbf{x}}(0, \mathbf{p} + \Delta\mathbf{p}) = \dot{\mathbf{x}}_0 = \dot{\mathbf{x}}(0, \mathbf{p} + \Delta\mathbf{p}) = \dot{\mathbf{x}}(0, \mathbf{p}) + \Delta\dot{\mathbf{x}}(0, \mathbf{p}) \end{cases} \tag{10}$$

Since the solution $\mathbf{x}(t, \mathbf{p})$ also fulfills the initial conditions of the problem, the previous equations lead to the following initial conditions of the sensitivity problem

$$\Delta\mathbf{x}(0, \mathbf{p}) = \mathbf{0} \qquad \Delta\dot{\mathbf{x}}(0, \mathbf{p}) = \mathbf{0} \tag{11}$$

Hence, disregarding the second order terms of the evolution equation and considering that the first terms on the right in both equations presented in (9) are automatically satisfied, inasmuch as they constitute the direct problem itself, the sensitivity problem is described as

$$\begin{cases} \mathbf{M}\Delta\ddot{\mathbf{x}} + \mathbf{D}\Delta\dot{\mathbf{x}} + \mathbf{K}\Delta\mathbf{x} = -(\Delta\mathbf{D}\dot{\mathbf{x}} + \Delta\mathbf{K}\mathbf{x}) \\ \Delta\mathbf{y} = \mathbf{C}\Delta\mathbf{x} \\ \Delta\mathbf{x}(0) = \mathbf{0} \qquad \Delta\dot{\mathbf{x}}(0) = \mathbf{0} \end{cases} \tag{12}$$

3.3. Adjoint Problem

The adjoint problem naturally appears when one considers that the displacement field $\mathbf{x}(t)$ needs to satisfy the evolution equation described in (1), which is the solution of the direct problem. Therefore, instead of considering the evolution equation as an additional constraint of the minimization problem, one may consider it naturally inherent to the own functional to be minimized. The price that has to be paid is the inclusion of a new set of variables into the problem under study, which are simply the well known Lagrange Multipliers $\boldsymbol{\lambda}(t)$. The Lagrange multipliers $\boldsymbol{\lambda}$ here belong to the n-dimensional vector space. So, the new functional that has to be minimized $\hat{J}(\mathbf{p})$ encompasses the one defined in (3) and a new one $\hat{J}_2(\mathbf{p})$ which is defined as follows

$$\hat{J}_2(\mathbf{p}) = \int_0^{t_f} \boldsymbol{\lambda}^T [\mathbf{M}\ddot{\mathbf{x}} + \mathbf{D}\dot{\mathbf{x}} + \mathbf{K}\dot{\mathbf{x}} - \mathbf{f}]\,\mathbf{dt} \tag{13}$$

Therefore the identification problem becomes the minimization of the functional $\hat{J}(\mathbf{p})$ which casts as

$$\hat{J}(\mathbf{p}) = \hat{J}_1(\mathbf{p}) + \hat{J}_2(\mathbf{p}) \tag{14}$$

The concrete definition and presentation of the adjoint problem will be possible only after the determination of the functional variation which is addressed in the next subsection.

3.4. Functional Variation

In order to perform the iterative process of parameter updating described in (4) it is clear that one has to determine the gradient of the functional $\nabla\hat{J}(\mathbf{p})$ at each iteration. The point is that the gradient determination is not an easy task since the functional depends on the system response $\mathbf{y}(t)$ which, in general, does not possess an analytic expression as a function of time t and the parameters $\mathbf{p}$. Aiming at overcoming this drawback one may determine the variation of functional $\Delta\hat{J}(\mathbf{p})$ when the parameter vector $\mathbf{p}$ suffers a variation of $\Delta\mathbf{p}$ and based on some suitable assumptions, try to extract, if it is feasible, the gradient out of this functional variation. The functional variation demands the calculation of the functionals $\hat{J}_1$ and $\hat{J}_2$ evaluated at $\mathbf{p}+\Delta\mathbf{p}$. For the functional $\hat{J}_1$ one has

$$\hat{J}_1(\mathbf{p}+\Delta\mathbf{p}) = \int_0^{t_f} [(\mathbf{y}+\Delta\mathbf{y}) - \mathbf{y}^E]^T[(\mathbf{y}+\Delta\mathbf{y}) - \mathbf{y}^E]\,dt = \int_0^{t_f} [\mathbf{y}-\mathbf{y}^E]^T[\mathbf{y}-\mathbf{y}^E]\,dt + \\ + \quad 2\int_0^{t_f} [\mathbf{y}-\mathbf{y}^E]\Delta\mathbf{y}\,dt \quad (15)$$

where the second order terms have been disregarded. For the second functional $\hat{J}_2$ one has

$$\hat{J}_2(\mathbf{p}+\Delta\mathbf{p}) = \int_0^{t_f} \boldsymbol{\lambda}^T[\mathbf{M}(\ddot{\mathbf{x}}+\Delta\ddot{\mathbf{x}}) + (\mathbf{D}+\Delta\mathbf{D})(\dot{\mathbf{x}}+\Delta\dot{\mathbf{x}}) + (\mathbf{K}+\Delta\mathbf{K})(\dot{\mathbf{x}}+\Delta\mathbf{x}) - \mathbf{f}]\,dt \quad (16)$$

such that after expanding the terms and disregarding the second order ones, one obtains the following resulting equation

$$\hat{J}_2(\mathbf{p}+\Delta\mathbf{p}) = \int_0^{t_f} \boldsymbol{\lambda}^T[\mathbf{M}\ddot{\mathbf{x}} + \mathbf{D}\dot{\mathbf{x}} + \mathbf{K}\mathbf{x} - \mathbf{f}]\,dt + \int_0^{t_f} \boldsymbol{\lambda}^T[\mathbf{M}\Delta\ddot{\mathbf{x}} + \mathbf{D}\Delta\dot{\mathbf{x}} + \mathbf{K}\Delta\mathbf{x}]\,dt + \\ + \int_0^{tf} \boldsymbol{\lambda}^T\Delta\mathbf{D}\dot{\mathbf{x}}\,dt + \int_0^{tf} \boldsymbol{\lambda}^T\Delta\mathbf{K}\mathbf{x}\,dt \quad (17)$$

Subtracting $\hat{J}(\mathbf{p})$ from $\hat{J}(\mathbf{p}+\Delta\mathbf{p})$ and integrating by parts the terms containing the time derivatives of the variation $\Delta\mathbf{x}$,one reaches to the variational of the functional $\hat{J}$

$$\Delta\hat{J}(\mathbf{p}) = \int_0^{t_f} [\mathbf{M}\ddot{\boldsymbol{\lambda}} + \mathbf{D}\dot{\boldsymbol{\lambda}} + \mathbf{K}\boldsymbol{\lambda}]\Delta\mathbf{x}\,dt + \int_0^{tf} 2(\mathbf{y}-\mathbf{y}^E)^T\Delta\mathbf{y}\,dt + \int_0^{tf} \boldsymbol{\lambda}^T\Delta\mathbf{D}\dot{\mathbf{x}}\,dt + \mathring{A} + \\ + \quad \int_0^{tf} \boldsymbol{\lambda}^T\Delta\mathbf{K}\mathbf{x}\,dt \, (18)$$

where $\mathring{A}$ corresponds to

$$\mathring{A} = \boldsymbol{\lambda}(t)^T\mathbf{M}\Delta\dot{\mathbf{x}}(t) - \dot{\boldsymbol{\lambda}}^T(t)\mathbf{M}\Delta\mathbf{x}(t) + \boldsymbol{\lambda}(t)^T\mathbf{D}\Delta\mathbf{x}(t)\Big|_{t=0}^{t=tf} \quad (19)$$

It is clear that the term of $\mathring{A}$ associated to $t=0$ is null due to the initial conditions of the sensitivity problem and one may choose the Lagrange Multipliers such that it is null at $t=t_f$ as well its first time derivative inasmuch as the user has this freedom in hands. Hence the term $\mathring{A}$ containing data at the final and initial instants of time disappears from equation (18).

Considering that the variation of the output $\Delta\mathbf{y}$ has a straightforward relation with the variation of the displacement vector $\Delta\mathbf{x}$ as shown in equation (1) one may write rewrite equation (18) as follows

$$\Delta\hat{J}(\mathbf{p}) = \int_0^{t_f} [\mathbf{M}\ddot{\boldsymbol{\lambda}} - \mathbf{D}\dot{\boldsymbol{\lambda}} + \mathbf{K}\boldsymbol{\lambda} + 2\mathbf{C}^T(\mathbf{y} - \mathbf{y}^E)]\Delta\mathbf{x}\,dt + \int_0^{t_f} \boldsymbol{\lambda}^T \Delta\mathbf{D}\dot{\mathbf{x}}\,dt + \int_0^{t_f} \boldsymbol{\lambda}^T \Delta\mathbf{K}\mathbf{x}\,dt \quad (20)$$

As it has already been mentioned, the goal is to obtain an expression for $\Delta\hat{J}(\mathbf{p})$ as a straightforward function of the parameter variation $\Delta\mathbf{p}$ and it is clear that it cannot be achieved in equation (20) since there is one term containing the variation $\Delta\mathbf{x}$ which is likely to have a very complicated relation with $\Delta\mathbf{p}$. In order to obtain a simpler relation between $\Delta\hat{J}(\mathbf{p})$ and $\Delta\mathbf{p}$ one can make use of an adjoint problem defined as follows

$$\mathbf{M}\ddot{\boldsymbol{\lambda}} - \mathbf{D}\dot{\boldsymbol{\lambda}} + \mathbf{K}\boldsymbol{\lambda} = 2\mathbf{C}^T(\mathbf{y}^E - \mathbf{y}) \quad (21)$$

under the following conditions

$$\boldsymbol{\lambda}(t_f) = \mathbf{0} \qquad \dot{\boldsymbol{\lambda}}(t_f) = \mathbf{0} \quad (22)$$

It should be emphasized that the problem stated by equations (21) and (22) can be changed to a problem with initial conditions rather than with final conditions with a suitable change of variables.

3.5. Gradient Equation

To obtain the gradient of the functional $\hat{J}(\mathbf{p})$ it is necessary to obtain the matrices $\mathbf{D}$ and $\mathbf{K}$ as functions of the variations of the parameters $\Delta\mathbf{p}$. This task is accomplished by expressing the damping and stiffness matrices as functions of the parameters and then evaluating their variations as follows

$$\mathbf{K} = \mathbf{K}(\mathbf{p}) \qquad \Rightarrow \qquad \Delta\mathbf{K}(\mathbf{p}) = \sum_{j=1}^{j=N_p} \frac{\partial\mathbf{K}}{\partial p_j}\Delta p_j \quad (23)$$

and

$$\mathbf{D} = \mathbf{D}(\mathbf{p}) \qquad \Rightarrow \qquad \Delta\mathbf{D}(\mathbf{p}) = \sum_{j=1}^{j=N_p} \frac{\partial\mathbf{D}}{\partial p_j}\Delta p_j \quad (24)$$

Hence, the variation of the functional $\hat{J}$ casts as

$$\Delta\hat{J}(\mathbf{p}) = \sum_{j=1}^{j=N_p} \Delta p_j \int_0^{tf} \boldsymbol{\lambda}^T(t)\Big[\frac{\partial\mathbf{K}}{\partial p_j}\mathbf{x}(t) + \frac{\partial\mathbf{D}}{\partial p_j}\dot{\mathbf{x}}(t)\Big]dt = \Delta\mathbf{p}^T\,\nabla\hat{J}(\mathbf{p}) \quad (25)$$

where each component of the gradient vector is given by

$$[\nabla\hat{J}(\mathbf{p})]_j = \int_0^{tf} \boldsymbol{\lambda}^T(t)\Big[\frac{\partial\mathbf{K}}{\partial p_j}\mathbf{x}(t) + \frac{\partial\mathbf{D}}{\partial p_j}\dot{\mathbf{x}}(t)\Big]\,dt\ ; \qquad j = 1, 2, ..., N_p \quad (26)$$

where N_p is the number of parameters that characterize the constitutive equation of the material.

3.6. Search Step Size

The search step size $\beta^{(k)}$ that appears in equation (4) is obtained through the minimization of the functional $\hat{J}_1$ at the iteration $k+1$, viz.

$$\min_{\beta^{(k)}} \hat{J}(\mathbf{p}^{(k+1)}) = \min_{\beta^{(k)}} \int_0^{t_f} [\mathbf{y}(t, \mathbf{p}^{(k+1)}) - \mathbf{y}^E]^T [\mathbf{y}(t, \mathbf{p}^{(k+1)}) - \mathbf{y}^E] \, dt \tag{27}$$

such that $\mathbf{p}^{(k+1)}$ is defined in equation (4). After linearizing the measured field $\mathbf{y}(t, \mathbf{p}^{(k+1)})$ around the point $\mathbf{p}^{(k)}$ and performing the corresponding minimization one gets

$$\beta^{(k)} = \frac{\int_0^{t_f} \Delta\mathbf{y}^T(t, \mathbf{p}^{(k)})[\mathbf{y}(t, \mathbf{p}^{(k)}) - \mathbf{y}^E(t)] \, dt}{\int_0^{t_f} \Delta\mathbf{y}^T(t, \mathbf{p}^{(k)}) \Delta\mathbf{y}(t, \mathbf{p}^{(k)}) \, dt} \tag{28}$$

4. CONSTITUTIVE EQUATION

It should be emphasized that the starting point of the present technique is the constitutive equation of the material, i.e., it is out of it that one is able to define the matrices $\mathbf{K}$ and $\mathbf{D}$ as being functions of the parameters $\mathbf{p}$ that characterize the constitutive equation. For the first trial one may consider a material which possesses a simple one-dimensional localized constitutive relation between stress σ and strain ϵ that is given by

$$\sigma(x, t) = E(x)\epsilon(x, t) + G(x)\dot{\epsilon}(x, t) \tag{29}$$

where $E(x)$ and $G(x)$ represent the elastic and the damping parameters over the entire body respectively. A material whose constitutive equation is given as a funcion od parameters that vary over its domain corresponds to a Functionally Graded Material (Paulino, 2001). Diferent types of constitutive equations could have been used, or even proposed, (Rusovici, 1999), (Haupt, 2000) and (Janno, 2001), to cite some recent articles.

5. NUMERICAL ILLUSTRATIONS

5.1. Noise

In order to introduce a more realistic scenario to the simulation one may introduce some Gaussian noise to the experimental data. The level of noise in the analyzed signal can be quantified by means of the signal-to-noise ratio, which is defined as follows

$$SNR = 10 \log \frac{{\sigma_s}^2}{{\sigma_n}^2} \tag{30}$$

where σ_s and σ_n are the variances of the signal and the noise respectively (Oppenhein, 1999). In the present work the signals considered as being noise are stationary random signals with zero mean.

5.2. Examples

In order to assess the effectiveness of the proposed approach to identify mechanical system properties from a certain set of experimental data, a bar-like structure will be considered.

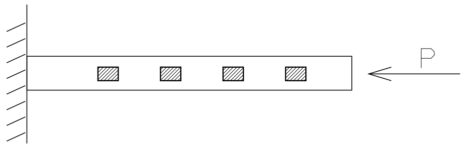

Figure 1: Virtual experiment sketch and its 4 strain sensor locations.

The virtual experiment consists basically of a bar instrumented with four strain sensors (strain gages) along its length and which is subjected to a dynamic loading such as an impact.

A brief sketch of the virtual experiment is depicted in figure (1) and it has been chosen four equally spaced positions at which strain measures will be taken during the experiment. The properties of the bar has been chosen as follows: cross-section area $A = 2.84\ 10^{-4}\ m^2$, length $L = 2.03\ m$, specific mass $\rho = 4408.2\ kg/m^3$. The simulation data have been obtained from a finite element element model of the bar. The one-dimensional finite element model has 82 elements and it has been considered that a compressive force $P(t)$ has been applied at the boundary $x = L$ as shown in picture (2). The force $P(t)$, in Newtons, is defined as follows

$$P(t) = 125\,[1 - \cos(\Omega t)] \qquad t \in [0, T_{imp}]; \tag{31}$$

where $\Omega = 2.52\ 10^5\ rad/s$ and $T_{imp} = 2.50\ 10^{-5}\ s$ and the impact force is zero for $t \in (T_{imp}, t_f]$.

All the experimental data possess 8192 points and the sampling frequency $4\,MHz$. For the first example (S1) it has been considered that the elastic field is a linear distribution defined by its values at the nodes 1 (0.00 m), 20 (0.48 m), 40 (0.98 m), 60 (1.48) and 82 (2.03 m), which were set to be 113.8, 92.2, 72.8, 55.7 and 40.9 respectively, in GPa. The damping field is defined similar to the elastic field and at the same nodes such that its nodal values were set to be $3.71\,10^3$, $3.18\,10^3$, $2.65\,10^3$, $2.12\,10^3$ and $1.59\,10^3$ in Ns/m^2. The strain sensors are located at the nodes 20 (0.45 m), 40 (0.95 m), 60 (1.45 m) and 80 (1.95 m).

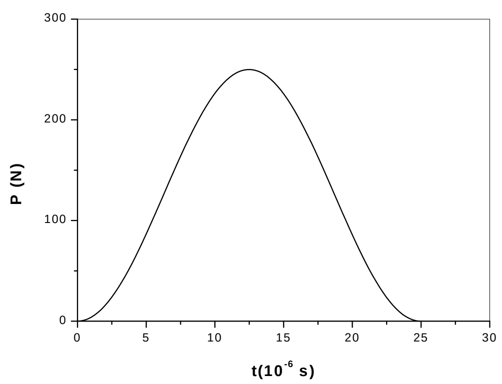

Figure 2: Impact force applied to the bar.

It is assumed for the iteration process that the initial damping field is null and that the elastic field is uniform over the bar and its value is equal to a characteristic value that is assumed to be obtained by means of a static test on the bar. The signal-to-noise ratio adopted here is 30 dB. The result obtained for the elastic and damping fields are depicted in Fig.(3) on the left and on the right respectively and the term "original" refers to the original finite element model of the system. It is clear from the picture on the left in Fig.(3) that the elastic field has been determined quite accurately and that although the obtained damping has some oscillations it is also an effective result. The number of iterations for this case is 75.

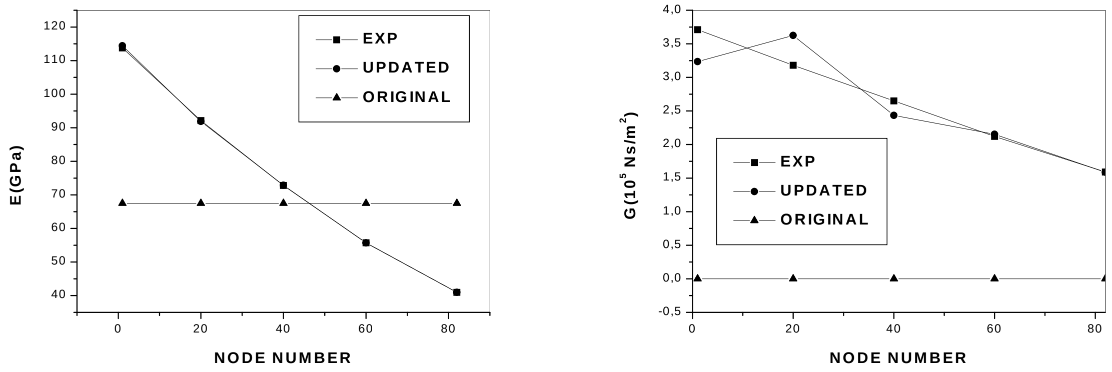

Figure 3: Elastic and damping field for case S1.

For the second case to be analyzed (S2) everything has been maintained equal to the first case (S1) but the damping field. The damping field has been defined at the same set of nodes as the previous example but its nodal values have been changed to: $1.59\,10^3$, $2.12\,10^3$, $2.65\,10^3$, $3.18\,10^3$ and $3.71\,10^3$ in Ns/m^2. The result obtained for the elastic field is graphed in the picture on the left in Fig.(4) and the obtained damping field is graphed in the picture on the right in Fig.(4) on the right. As in the previous example the elastic field has been perfectly determined and the obtained damping field presented some oscillations and it was effective. The number of iterations for this case is 56. It should be remarked that the results presented for the two examples have been determined taking into account real-like limitations such as few measurement sensors and measured signals polluted with noise.

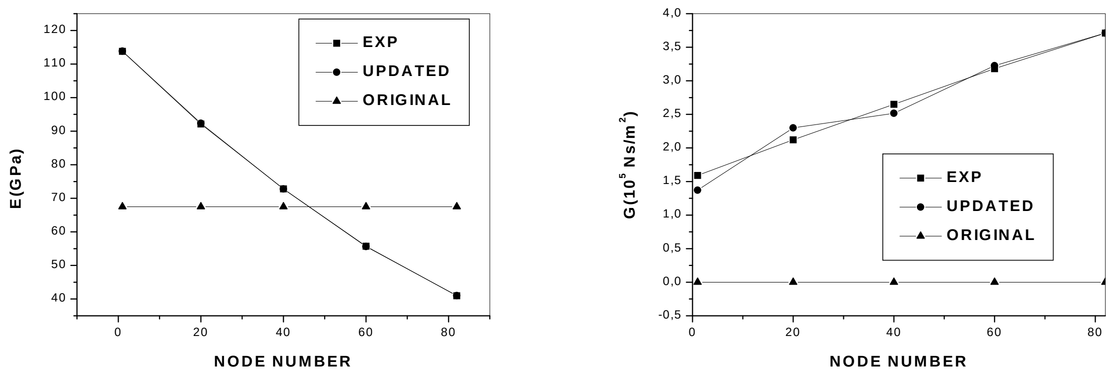

Figure 4: Elastic and damping field for the case S2.

6. CONCLUDING REMARKS

A time domain technique aiming at identifying the unknown parameters that characterize a viscoelastic model for a certain material has been presented. In order to assess the effectiveness of the approach some simulations have been performed on a bar-like structure subjected to an impact loading. It was considered that the constitutive law of the material for stress and strain is characterized by distributed elastic and damping fields. The measured signals have been polluted with white noise to furnish more realism to the simulations and the results provided by the present approach has shown to be effective for the examples that have been analysed.

7. REFERENCES

Daniel, J.W., 1971, "The Approximate Minimization of Functionals", Prentice Hall Inc.

Dietrich, L., Lekszycki, T. and Turski K., 1998,"Problems of Identification of Mechanical Characteristics of Viscoelastic Composites", Acta Mechanica, Vol. 126, pp. 153-167.

Gavrus, A., Massoni, E. and Chenot, J.L., 1996, "An Inverse Analysis Using a Finite Element Model for the Identification of Rheological Parameters", Journal of Materials Processing Technology, vol. 60, pp. 447-454.

Haupt, P., Lion, A. and Backhaus, E., 2000,"On the Dynamic Behaviour of Polymers under Finite Strains: Constitutive Modelling and Identification of Parameters", International Journal of Solid and Structures, Vol. 37, pp. 3633-3646.

Janno, J. and von Wolfersdorf, L., 2001, "An Inverse Problem for Identification of a Time-and Space-Dependent Memory Kernel in Viscoelasticity",Inverse Problems, Vol.17, pp.13-24.

Mossberg, M., 2000, "Identification of Viscoelastic Materials and Continuous-Time Stochastic Systems", Ph.D. Dissertation, Uppsala University.

Mossberg, M., Hillström, L. and Söderström T., 2001, "Non-Parametric Identification of Viscoelastic Materials from Wave Propagation Experiments", Automatica, Vol. 37, pp. 511-521.

Oppenheim, A.V. and Schafer, R.W., " Discrete-Time Signal Processing", Second Edition, Prentice Hall Signal Processing Series, 1999.

Özisic, M.N. and Orlande, H.R.B, 2000, " Inverse Heat Transfer: Fundamentals and Applications", Taylor and Francis.

Paulino, G.H. and Jin, Z.-H., "Viscoleastic Funcionally Graded Materials Subjected to Antiplane Shear Fracture", Transactions of the ASME, 68, 284-293, 2001.

Rusovici, R., 1999, "Modelling of Shock Wave Propagation and Attenuation in Viscoelastic Structures", Ph.D. Dissertation, Virginia Polytechnic Institute and State University.

Sarron, J.C., Blondeau, C., Guillaume, A. and Osmont, D., 2000, "Identification of Linear Viscoelastic Constitutive Models, Journal of Biomechanics, Vol. 33, pp. 685-693.