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THE INFLUENCE OF THE WOUNDING ANGLE ON THE BEHAVIOR OF COMPOSITE ROTORS

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Abstract. *The purpose of this work is to investigate the dynamic behavior of simply-supported rotors in composite material. The shaft is in carbon/epoxy or in glass/epoxy. The finite element method is used and the rotor is modelled as a beam element with four degrees of freedom to analyse bending motion and one degree of freedom to analyse torsional motion. An equivalent Young's modulus and a damped equivalent Young's modulus is used to represent the composite materials proprieties. It is observed the influence of the wounding angle on the prediction of the natural frequencies as a function of the rotation of the rotor as well as the response to an unbalance mass. The optimal position of critical speeds was determined by using an optimization technique considering the wounding angle, the stiffness of the bearings and the position of the disk as design variables.*

Keywords: *rotors, composite materials, optimization, finite element.*

1. INTRODUCTION

Rotordynamic problems have been exhaustingly studied for several decades for the case in which the shaft is in isotropic material Zorzi et al. (1977), Özgüven et al. (1984), Melanson et al. (1998), Ku et al. (1998). The problems most frequently analysed involve response to an excitacion and stability of the rotor, Pereira et al. (2000). Sometimes it is used optimization techniques to search an optimal design of the machine using as parameters: the stiffness of the bearing, the geometric proprieties and position of the compounds, Steffen et al. (1987) and Pereira et al. (2001).

The reason for introducing composite materials in rotordynamics is the anisotropy of the materials and the high capacity of damping, Wettergren (1996), Gupta et al. (1998) and Silveira (2001). The first propriety of the composite materials can be used in a convenient way in order to take a critical speed far from the working speed. The second propriety can be used in order to reduce the amplitude of vibration when passing through one critical speed. Nevertheless, this propriety should be used with care to avoid instability regions, Pereira et al. (2000). As observed in an experimental measurements the internal damping is not viscous but hysteretic, nevertheless the internal damping can be treated as a viscous damping by using an equivalence between the energy dissipated by both mechanisms, Singh et al. (1994).

In this work we purposed to investigate the behavior of composite rotors using the finite element method. The shaft is obtained by winding several plies of embbebed fibers over a mandrel. The Euler-Bernoulli beam with five degrees of freedom is used to represent bending and torsional modes. The disks are supposed to be rigids and the assembly is supported by flexible bearings. An equivalent modulus approach is used in order to represent the orthotropic proprieties of the composite shaft. The effect of the damping in composite materials is made by introducting a model proposed by Adams et al. (1973). The strain stress relation that include the internal damping on the strain energy in bending developed in Silveira (2001) is used. It will be shown the effect of the wounding angle on the natural frequencies and on the response to an excitacion for rotors in bending and in torsion, and the effect of the wounding angle on the stability for rotors in bending. It will be also shown how the wounding angle can be used as a parameters in order to search an optimal design.

2. THE FINITE ELEMENT MODEL

The finite element model of a rotor is composed by beam elements and rigid elements to represent the shaft and the disks respectively. The rotor is supposed to be simple-supported and the wounding angle of each layer of the shaft is j.

As shown by Lalanne et al. (1998), the kinetic energy of a disk can be expressed by:

$$T_D = \frac{1}{2} M_D \left(\dot{u}^2 + \dot{w}^2\right) + \frac{1}{2} I_{Dx} \left(\dot{q}^2 + \dot{y}^2\right) + I_{Dy} \Omega \dot{y} q + \frac{1}{2} I_{Dy} \Omega^2 \tag{1}$$

and the kinetic energy of a disk in torsion as:

$$T_{Dt} = \frac{1}{2} I_{Dy} \dot{F}^2 \tag{2}$$

where M_D is the mass of the disk, u and w are the coordinates of the center of inertia of the disk on the inertial axes, I_{Dx} and I_{Dy} are the moments on the principal directions of inertia, F is the torsional angle. The rotation speed of rotor is W and $\dot{y}$ and $\dot{q}$ are instantaneous velocities.

For an element of the shaft, the kinetic energy in bending can be expressed by:

$$T_S = \frac{r S}{2} \int_0^L \left(\dot{u}^2 + \dot{w}^2\right) dy + \frac{r I_{xx}}{2} \int_0^L \left(\dot{q}^2 + \dot{y}^2\right) dy + r I_{xx} L \Omega^2 + 2 r I_{xx} \Omega \int_0^L \dot{y} q \, dy \tag{3}$$

and for the shaft in torsion, the kinetic energy can be given as:

$$T_{St} = \frac{r J}{2} \int_0^L \dot{F} \, dy \tag{4}$$

where r is the volumetric mass, S is the area of the cross section, I_{xx} is the inertia moment of the cross section, J is the inertia polar moment and L is the lenght of the element.

The general expression for the strain energy of the shaft in bending is:

$$U = \frac{1}{2} \int_V \{e\}^t [s] \, dV \tag{5}$$

As proposed by Silveira (2001), the stress-strain relation for a composite beam, including the effect of hysteretic damping can be given as:

$$s = E_{eq}e + E_{eq}^{y}\dot{e} \tag{6}$$

where E_{eq} is the equivalent Young's modulus and E_{eq}^{y} is the equivalent damped Young's modulus. Considering small deformations, the longitudinal strain and the longitudinal strain rate can be expressed as:

$$e = -x\frac{\partial^2 u^*}{\partial y^2} - z\frac{\partial^2 w^*}{\partial y^2}; \qquad \dot{e} = -x\frac{\partial^2 \dot{u}^*}{\partial y^2} - z\frac{\partial^2 \dot{w}^*}{\partial y^2} \tag{7}$$

where u^* and w^* are displacements of the geometric center measured on the rotating axes of the shaft, Lalanne et al. (1998). Considering the relation between the displacements u^* and w^* and the displacements u and w, and using Eqs. (5)-(6), we obtain the strain energy in bending:

$$\begin{aligned} U = {} & \frac{1}{2}E_{eq}I_{xx}\int_0^L\left[\left(\frac{\partial^2 w}{\partial y^2}\right)^2 + \left(\frac{\partial^2 u}{\partial y^2}\right)^2\right]dy + \frac{1}{2}E_{eq}^{y}I_{xx}\int_0^L\left[\left(\frac{\partial^2 u}{\partial y^2}\frac{\partial^2 \dot{u}}{\partial y^2}\right) + \left(\frac{\partial^2 w}{\partial y^2}\frac{\partial^2 \dot{w}}{\partial y^2}\right)\right]dy \\ & + \frac{1}{2}E_{eq}^{y}I_{xx}\int_0^L\left[-\Omega\left(\frac{\partial^2 u}{\partial y^2}\frac{\partial^2 w}{\partial y^2}\right) + \Omega\left(\frac{\partial^2 w}{\partial y^2}\frac{\partial^2 u}{\partial y^2}\right)\right]dy \end{aligned} \tag{8}$$

The second term and the third term of Eq. (6) are related to the hysteretic damping named $[H_b]$ and $[H_c]$. The equivalence between the hysteretic damping and the viscous damping is made by:

$$[K_b] = \frac{[H_b]}{p\,[w]}; \qquad [K_c] = \frac{[H_c]}{p\,[w]} \tag{9}$$

where $[w]$ is the diagonal frequencies matrix.

For the shaft in torsion, the expression for the strain energy can be written as:

$$U_t = \frac{1}{2}\int_V tg\, dV \tag{10}$$

Similarly at Eq. (6), the shear stress can be given as:

$$t = G_{eq}g + G_{eq}^{y}\dot{g} \tag{11}$$

where G_{eq} is the equivalent shear modulus and G_{eq}^{y} is the equivalent damped shear modulus. Substituting Eq. (11) into Eq. (10), we obtain the strain energy for the shaft in torsion as being:

$$U_t = \frac{1}{2}\ G_{eq}J\int_0^L\left(\frac{\partial F}{\partial y}\right)^2 dy + \frac{1}{2}G_{eq}^{y}J\int_0^L\left(\frac{\partial \dot{F}}{\partial y}\frac{\partial F}{\partial y}\right)dy \tag{12}$$

where the second term represents the dissipassion matrix of the element in torsion.

The equation of motion of the rotor is obtained by applying Lagrange's equations, on the kinetic energy and on the strain energy of the elements, and can be written as:

$$[M]\{\ddot{u}\} + [K_b + \Omega G]\{\dot{u}\} + [K + \Omega K_c]\{u\} = \{F\} \tag{13}$$

where $[M]$, $[G]$ and $[K]$ are global mass, global Coriolis and global stiffness matrices. $[K_b]$ and $[K_c]$ are global dissipation matrix and global circulation matrix. Vectors $\{\ddot{u}\}$, $\{\dot{u}\}$, and $\{u\}$ are nodal acceleration, nodal velocity and nodal displacement respectively and $\{F\}$ is the generalised force vector due to the unbalanced mass. The elementaries matrices are obtained according to the Euler-Bernoulli equation for beams and are presented in Zorzi et al. (1977) and Silveira (2001).

3. The equivalent modulus and the internal damping model

Considering that the shaft is thin walled and slender, and the laminate is symmetric and balanced, the equivalent Young's modulus and the equivalent shear modulus is found as:

$$E_{eq} = \frac{\left[4\left(U_1 - U_5\right)\left(U_5 + U_3 g_g\right) - b^2 U_2^2\right]}{U_1 - bU_2 + g_g U_3}; \qquad G_{eq} = U_5 - U_3 g_g \tag{14}$$

where $U_{1\text{-}5}$ are the laminate invariants and, Tsai et al. (1980):

$$g_g = \sum_{k=1}^{N} \frac{h_k}{h} \cos(4j_k) \qquad b = \sum_{k=1}^{N} \frac{h_k}{h} \cos(2j_k) \tag{15}$$

where h_k is the thickness of layer k, h is the laminate thickness and j_k the wounding angle.

On the prediction of damping on multi-layer shell structures the model proposed by Adams in 1973 is used and the specific damping capacity is defined as:

$$y = \frac{\Delta U}{U} \tag{16}$$

From Eq. (5) and Eq. (16) and assuming plane stress state, the dissipative energy for a single layer of unidirectionally fiber on the orthotropic axis is:

$$\Delta U = \frac{1}{2}\int_V \{e\}^t [y]\{s\} dV \tag{17}$$

where $[y]$ is the specific damping capacity matrix in the form:

$$[y] = \begin{bmatrix} y_{11} & 0 & 0 \\ 0 & y_{22} & 0 \\ 0 & 0 & y_{12} \end{bmatrix} \tag{18}$$

and y_{11}, y_{22}, y_{12} are the specific damping capacities of a layer on longitudinal, transverse and shear direction. From Eq. (17) and considering the constitutive relation, we obtain:

$$\Delta U = \frac{1}{2}\int_V \{e\}^t [y][Q]\{e\} dV \tag{19}$$

Using the same procedure to derive E_{eq} and G_{eq}, the equivalent damped Young's modulus E_{eq}^{y} and the equivalent damped shear modulus G_{eq}^{y} are determined from Eq. (19) as being a function of the specific damping capacities.

4. THE RESPONSE TO AN EXCITATION

The more commom source of synchronous excitation is an unbalance mass m_u situated at a distance d from the geometric center of the shaft. The general equation of the rotor with this excitation become, Lalanne et al. (1998):

$$[m]\{\ddot{q}\}+[c]\{\dot{q}\}+[k]\{q\}=\{f_1\}\sin\Omega t+\{f_2\}cos\Omega t \tag{20}$$

where $\{f_1\}$ and $\{f_2\}$ are related to m_u, d and W^2, and $[m]$, $[c]$ and $[k]$ are the modal matrices obtained from Eq. (13) by using the pseudo-modal method. Solutions for this problem are sought as:

$$\{q\}=\{p_1\}\sin\Omega t+\{p_2\}cos\Omega t \tag{21}$$

and the identification of the terms in *sinW t* and *cosW t* gives the equation:

$$\begin{bmatrix} k-m\Omega^2 & -\Omega c \\ \Omega c & k-m\Omega^2 \end{bmatrix}\begin{Bmatrix} p_1 \\ p_2 \end{Bmatrix}=\begin{Bmatrix} f_1 \\ f_2 \end{Bmatrix} \tag{22}$$

The system of the Eq. (22) is solved and the unbalance response is determined for any values of W. The response to an unsynchronous excitation is solved on this same way where the frequency of the excitation is sW for $s\neq 1$.

5. INSTABILITY IN ROTORDYNAMICS

The natural frequencies and the zones of instability can be determined from the solution of the eigenvalue problem as a result of the homogeneous equation:

$$[m]\{\ddot{q}\}+[c]\{\dot{q}\}+[k]\{q\}=0 \tag{23}$$

where $[m]$, $[c]$ and $[k]$ are modal matrices obtained from Eq. (13) by using the pseudo-modal method, Lalanne et al. (1998). Solutions for this problem are sought as:

$$\{q\}=[P]\{e^{rt}\} \tag{24}$$

Substituting Eq. (24) in the Eq. (23) we obtain:

$$\left[\{r\}^2[m]+\{r\}[c]+[k]\right][P]=0 \tag{25}$$

Equation (25) can be rearranged as follow:

$$\begin{bmatrix} [0] & [I] \\ -[m]^{-1}[k] & -[m]^{-1}[c] \end{bmatrix}\begin{Bmatrix} [P] \\ \{r\}[P] \end{Bmatrix}=\{r\}\begin{Bmatrix} [P] \\ \{r\}[P] \end{Bmatrix} \tag{26}$$

where $[I]$ is the identity matrix. The eigenvectors of Eq. (26) are obtained in the complex form:

$$\{r\}=\{l\}\pm i\{w\} \tag{27}$$

where $\{w\}$ is the natural frequencies vector and $\{l\}$ is the vector that determine the stability.

6. APPLICATION

Firstly, it will be shown the influence of the wounding angle on the behavior of composite rotors: on the natural frequencies as well on the response to a synchronous excitation for a rotor supported by isotropic bearings. In this case, the rotor is composed by a wounding shaft, two disks equidistants from the ends and the assembly is supported by flexible bearings, Fig. (1).

The wounding shaft has length of 1.2m, inner radius of 0.04m, outer radius of 0.048m, with eight plies of 0.001m thickness in a balanced and symmetric configuration such as $[\pm j]_s$. The disks have inner radius of 0.048m, outer radius of 0.15m and thickness of 0.05m. The stiffness of the bearings are $K_{xx} = K_{zz} = 10^7$N/m. A mass of 10^{-4}kg was placed at 0.15m from the center of the first disk and the response is taken on the node corresponding to the first disk. The material data are given in Tab.(1).

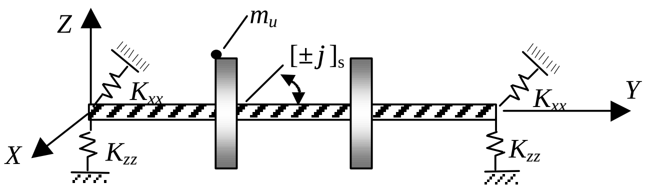

Figure 1. Rotor in wounding shaft with two disks.

Table 1. Material data of the shaft and the disks.

	E_1 (GPa)	E_2 (GPa)	G_{12} (GPa)	ρ (kg/m^3)	ψ_{11} (%)	ψ_{22} (%)	ψ_{12} (%)	ν_{12}
Shaft (carbon/epoxy)	172.7	7.20	3.76	1446.2	0.45	4.22	7.05	0.3
Shaft (glass/epoxy)	37.78	10.90	4.91	1813.9	0.87	5.05	6.91	0.3
Disks (steel)	-	-	-	7800	-	-	-	-

Figures (2a), (2b) and (2c) show the influence of the wounding angle on the position of the natural frequencies, as well as the influence of the internal damping on the instability regions for rotors in carbon/epoxy supported by isotropic bearings. In all figures, the legend is as follow: --- synchronous exicitation; —— stable natural frequency; —●— unstable natural frequency; —×— response to an unbalance mass. As it can be noted, as higher is the wounding angle, lower is the equivalent stiffness of the shaft and consequently, lower is the natural frequency. According to others authors that has used shaft in conventional material, Zorzi et al. (1977), the instability begin also at the first critical speed. Anisotropic bearings, coupled terms and external damping can change quite the threshold speed of instability, Pereira et. al. (2000).

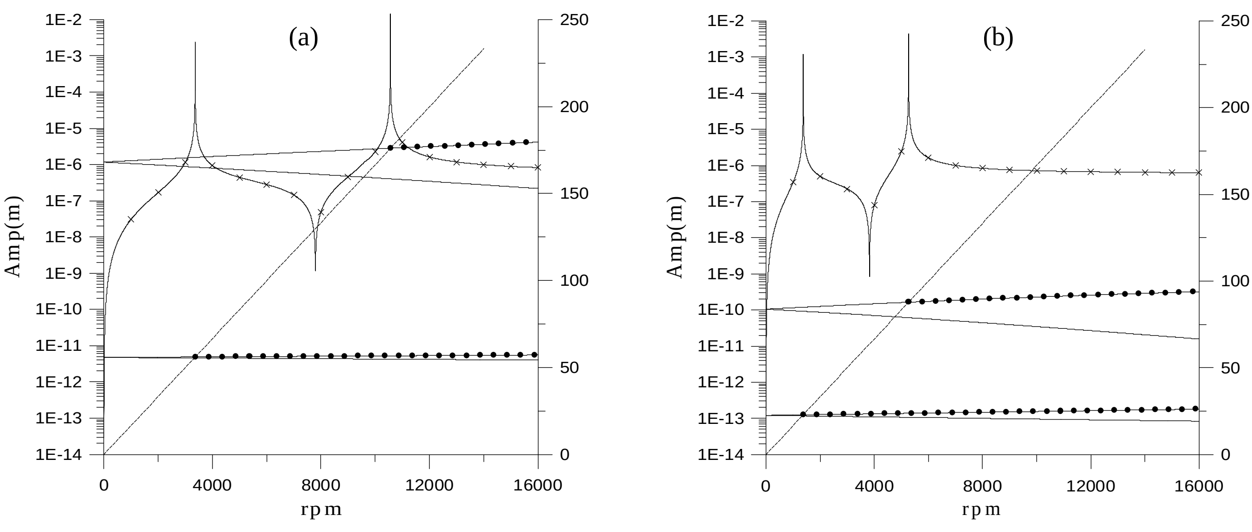

Figure 2. Campbell diagram and response to an unbalance mass for rotor supported by isotropic bearings with (a) $j = 15°$; (b) $j = 45°$.

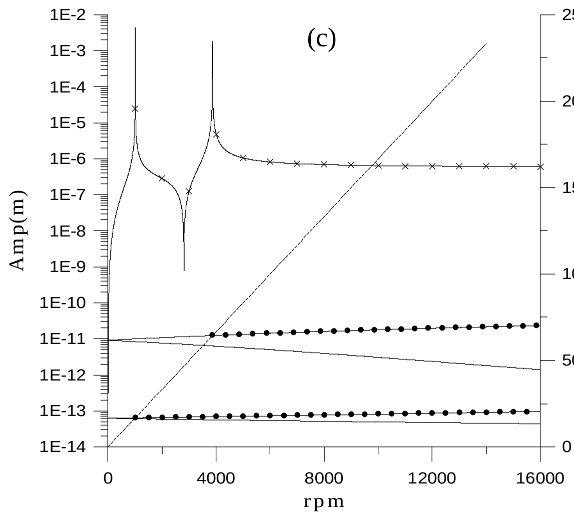

Figure 2c. Campbell diagram and response to an unbalance mass for rotor supported by isotropic bearings and j = 75º.

Figure (3) shows the influence of the wouding angle on the first two vibration modes for a rotor in carbon/epoxy supported by isotropic bearings. It can be seen that the wounding angle can affect considerably the vibration mode, and consequently, the influence of the external damping on the rotor. For lower wounding angles, the equivalent stiffness is higher, and consequently the influence of the bearings on the behavior of the rotor is more pronounced.

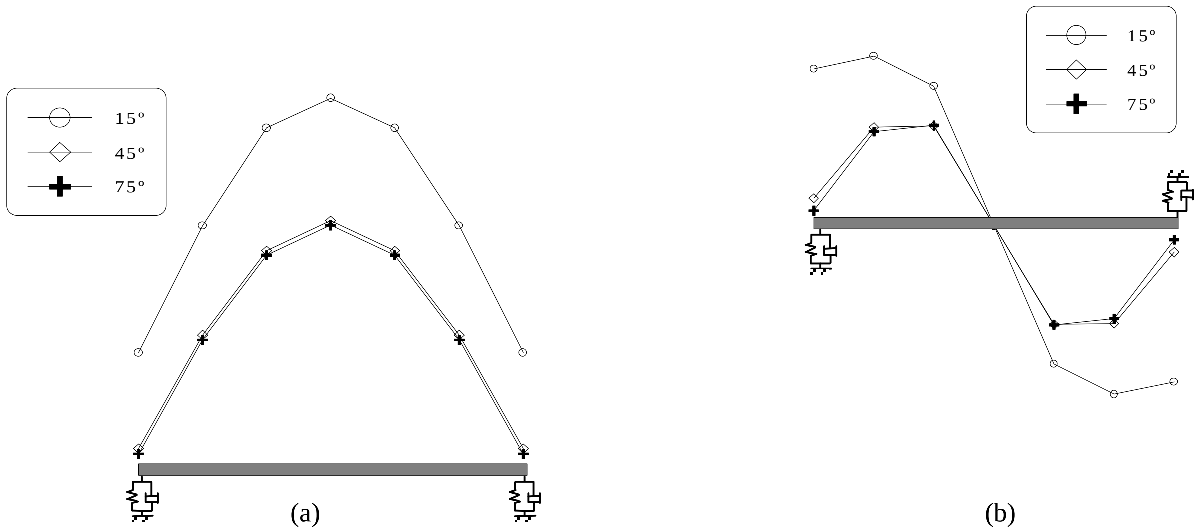

Figure 3. (a) First vibration mode and (b) second vibration mode for rotor in carbon/epoxy.

Figures (4) and (5) show the Campbell diagram for torsional modes as well as the response on frequency for a rotor in carbon/epoxy submitted to a torque in a form: $T_r = T_{constant} + T\cos(Wt)$. The maximum stiffness in torsion and consequently the maximum frequency in torsion is for j = 45º. In this model, the internal damping causes no instability on the rotor but can reduce considerably the amplitude of vibration.

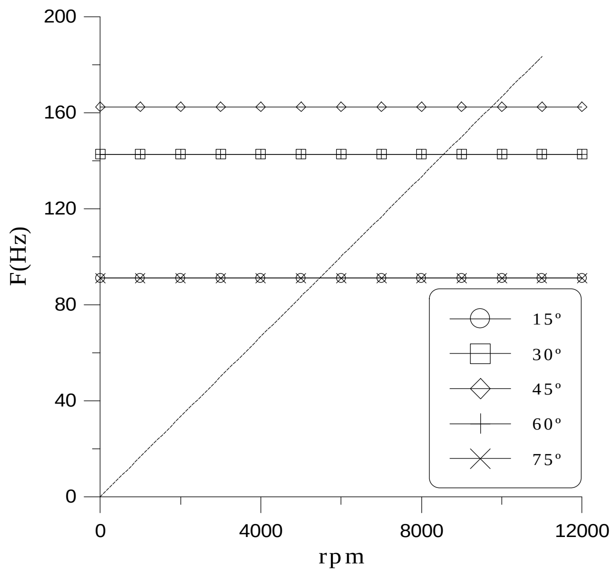

Figure 4. Campbell diagram for torsional modes for rotors in carbon/epoxy.

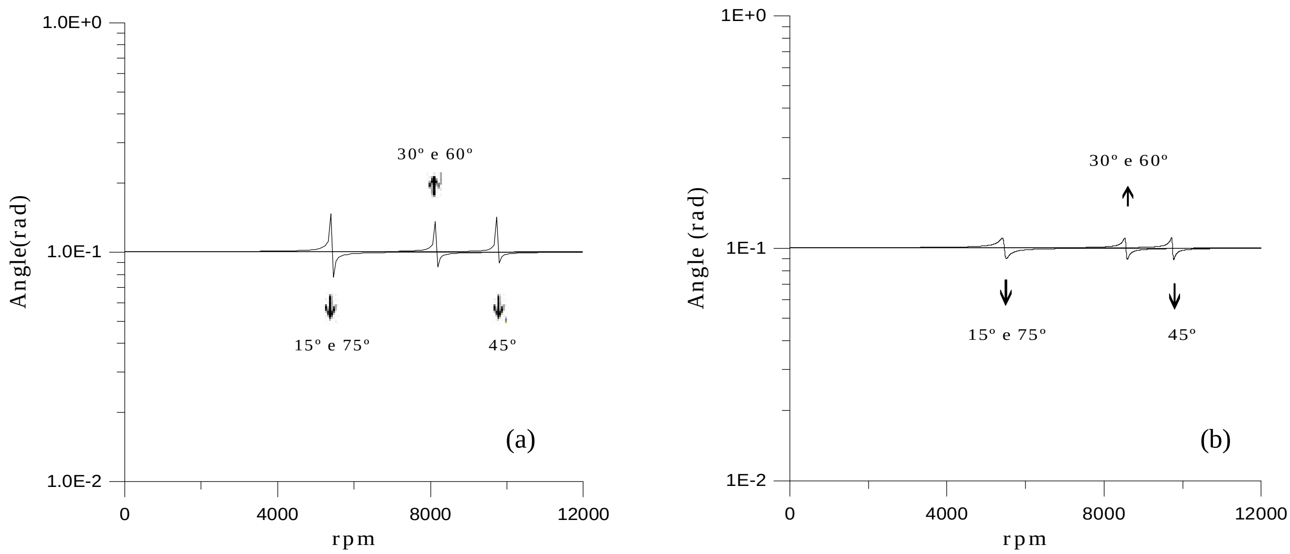

Figure 5. Response on frequency for torsional modes for rotors in carbon/epoxy, (a) without internal damping; (b) including internal damping

7. OPTIMIZATION OF THE NATURAL FREQUENCIES

A non-linear unconstrained optimization technique was used to determine the optimal position of the natural frequencies. In an analogous way as used by Steffen et al. (1987), the goal was to increase the distance between the first and the second critical speed, at a constant rotation W = 8000rpm. It was used the Quasi-Newton method, in which the approach of the Hessian is made by the BFGS, Arora (1989), and the gradient of the objective function was determined by the forward finite difference.

The problem of optimization was formulated as follow:

Max $w(4) - w(2)$

$15^o \leq j_1 \leq 75^o$

$15^o \leq j_2 \leq 75^o$

$1.10^6 \leq K_{xx} \leq 1.10^7$

$1.10^6 \leq K_{zz} \leq 1.10^7$

$0.2 \leq y_c \leq 0.4$

where y_c is the position of the disk on the shaft.

The rotor is composed now with only one disk, located at 0.33m of the left end. The wounding shaft has lenght of 1m, inner radius of 0.031m, outer radius of 0.039m, with eight lies of 0.001m thickness in a balanced and symmetric configuration, such as $[\pm j_1, \pm j_2]_s$. The disk has inner radius of 0.039m, outer radius of 0.15m and thickness of 0.03m.

The initial and optimal configurations for rotors in carbon/epoxy and in glass/epoxy are shown in Tab. (3) and the results plotted on Fig. (8). For both carbon/epoxy and glass/epoxy shafts, it can be observed that the optimal configuration is for the case when the bearings are on the upper limit of the stiffness.

Table 3. Initial and optimal configurations of the rotor.

	j_1	j_2	K_{xx} (MPa)	K_{zz} (MPa)	y_c (m)
Initial configuration (carbon/epoxy and glass/epoxy)	30.0º	30.0º	5.10^6	5.10^6	0.333
Optimal configuration (carbon/epoxy)	15.0º	15.0º	1.10^7	1.10^7	0.4
Optimal configuration (glass/epoxy)	15.0º	15.0º	1.10^7	1.10^7	0.4

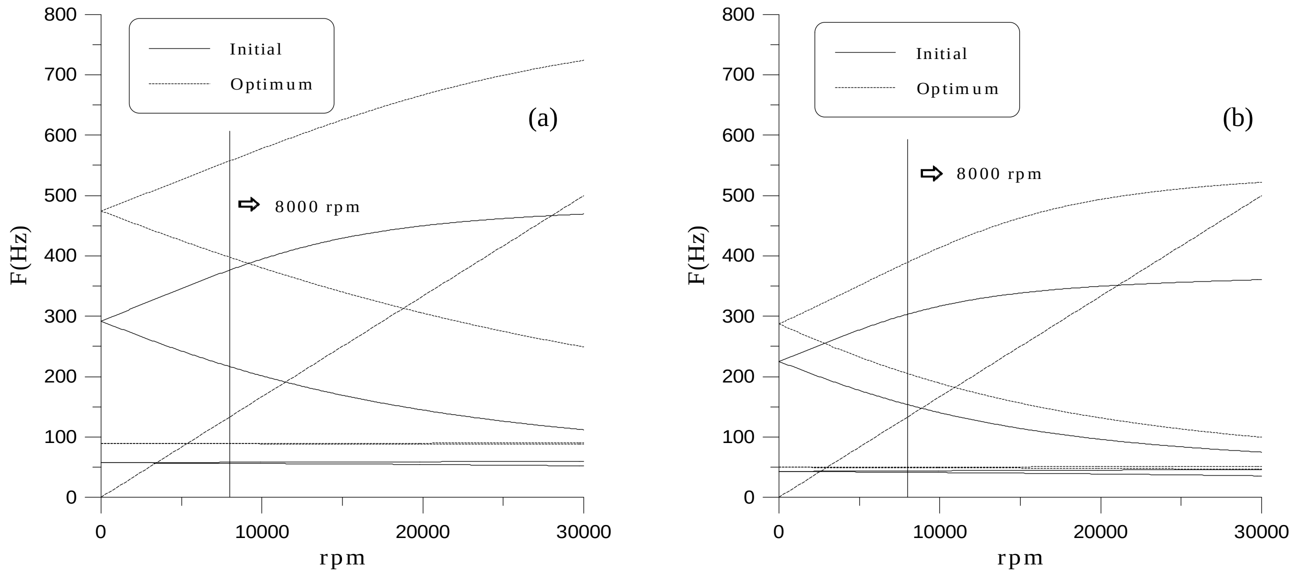

Figure 8. Initial and optimal Campbell diagram for a rotor in (a) carbon/epoxy and (b) glass/epoxy.

8. CONCLUSION

In this work it was investigated the effect of the wounding angle on the behavior of carbon/epoxy and glass/epoxy rotors. Bending modes and torsional modes were analysed considering different rotordynamic problems. We can conclude that composite materials can be used in shaft of rotors in advantage in comparasion the conventional materials. Considering the stability of the system, we can conclude that the internal damping offered by the composite materials to the rotors should be used with care if we consider the stability, nevertheless, the internal damping associated with others parameters can be explored in rotordynamics analysis in order to search the optimal design.

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