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ON THE IDENTIFICATION OF ACOUSTIC MODAL AND RESPONSE MODES FOR REACTIVE FILTERS

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Abstract: *Acoustic modal analysis employs concepts of the traditional structural modal analysis in the study of resonance phenomena in acoustic systems. The major task here is to obtain the so called acoustic mode shapes, that result from the resonance of the medium fluid in a given acoustic system. In the present work the three-dimensional reduced wave equation (Helmholtz equation) is solved using finite element and variational techniques in order to obtain the equivalent mass and stiffness matrix that will be used in the solution of the eigenvalue problem for the acoustic natural frequencies and mode shapes in filters. This theoretical framework is used in numerical simulations in order to provide a comparison basis for the results obtained via experiments. Reactive filters can be constructed with simple changes in the acoustical system. This work presents the theory, simulation and experimental analysis for one type of filter high-pass in a piping system. Although the modal characteristics of low-pass filters have already been discussed in recent publication, there is still a lack of research on the modal characterization of high-pass filters. This paper focuses on this type of filter, discussing how the acoustic modal parameters of such filters are altered when the characteristics of branches is varied. The eigenvalue is formulated and solved for the system without fluid resistance, the natural frequencies and mode shapes are compared. The results obtained experimentally and via simulation are presented and discussed.*

Keywords: *acoustic, modal analysis, high-pass, filter, pipe.*

1. INTRODUCTION

There are two ways of attenuating frequencies in a pipe system: adding sections or branches and introducing acoustic absorptive materials. The filter generated through the addition of sections is called reactive and the filter obtained through the introduction of absorptive materials is called absorptive.

The ability of a side branch to attenuate the sound energy transmitted in a pipe is the basis of a class of acoustic filters. Depending on the input impedance of the side branch, such systems can act as *low-pass, high-pass* or *band-pass* filters. In this paper we study several types of high-pass filters. This type of device is used for attenuate the sound generated by explosion motors without introduce excessive pressure in the motor. Another application is the study of wind musical instruments, whose behavior is the same of a high-pass filter.

Arruda (Arruda, 1999) studied theoretically and experimentally the modal characteristics of a low-pass filter but it has been found that there is a lack of research on the modal characterization of high-pass filters. The presence of a single orifice converts a pipe into a high-pass filter. As the radius of such orifice is increased, the attenuation of the low frequencies is increased. The filtering action of an orifice does not result from the transmission of acoustic energy out of the pipe, but rather from the reflection of energy back toward the source (Kinsler, 1982). Employing combinations of branches, we have a filter network. The design of such networks is made by using a combination of reactances of different types of impedances in line. Davis et alli, 1948 performed a complete study about mufflers and Munjal, 1987 summarized the theory for low-pass and high-pass filters.

2. THEORETICAL FORMULATION

2.1. Lumped Formulation

The resonance frequencies for a pipe driven at $x=0$ and open-ended at $x=L$ are given as:

$$f_n = \frac{n}{2}\frac{c}{L} \tag{1}$$

where $n=1,2,\ldots$

The cutoff frequency of a high-pass filter is given as:

$$f = \frac{1}{4\pi\sqrt{MC}} \tag{2}$$

where M is called the fluid inertance, and C is called the fluid capacitance. They are given by:

$$M = \frac{\rho L}{A} \quad \text{and} \quad C = \frac{AL}{\rho c^2} \qquad (3), (4)$$

and A is the area of the cross-section of the tube, L is the length of the pipe, ρ is the density of the fluid and c is the sound speed.

Through these fluid characteristics and using the conservation of mass and neglecting resistance (friction) influence, we can obtain the lumped model of the system, but this theoretical formulation is very limited. For the high-pass filter studied, with two branches, we can find only the first two natural frequencies of the system, while through finite element analysis we can obtain a much larger number of natural frequencies. Therefore, we will adopt the finite element formulation to develop the major theoretical concepts concerned with this paper.

2.2. Finite Element Formulation

For the simulation of modal analysis of the high-pass filter, we use the finite element formulation. For didactic purposes, the two-dimensional discretization of the wave equation, which is sufficient for the solution of the problem is adopted here. However, for visualization purposes, the results are presented in a three-dimensional shape. The two-dimensional discretization can be easily expanded to the three-dimensional formulation. The two-dimensional wave equation is given as

$$\nabla(k\nabla u(x,y,t)) - \rho\frac{\partial^2 u}{\partial t^2} + f(x,y,t) = 0 \tag{5}$$

on a domain D. This equation has the following initial and boundary conditions:

$$\begin{cases} u = g(s,t) \ \ on \ \ \Gamma_1 \\ k\dfrac{\partial u}{\partial n} + \alpha(s,t)u = q(s,t) \ \ on \ \ \Gamma_2 \\ u(x,y,0) = c(x,y) \ \ in \ \ D \\ \dfrac{\partial u(x,y,0)}{\partial t} = d(x,y) \ \ in \ \ D \end{cases} \tag{6}$$

Developing the weak formulation by multiplying the differential Eq. (5) by an arbitrary test function $v(x,y)$ and manipulating the Eq. (5) using the divergence theorem described by Bickford (1990), we obtain:

$$u_e(x,y) = [N]^T\{u_e\} = \{u_e\}^T[N] \tag{7}$$

where $[N]$ is the so-called matrix of shape. The finite element model can then be expressed as:

$$\begin{aligned} &[B]\{\ddot{u}\} + [A]\{u\} = \{F\} \\ &\{u(0)\} = \{u_0\} \\ &\{\dot{u}(0)\} = \{\dot{u}_0\} \end{aligned} \tag{8}$$

where:

$$[A] = \sum_e [k_G] + \sum_e{}' a_G \quad \text{and} \quad [B] = \sum_e [r_G] \tag{9}$$

and

$$F = \sum_e f_G + \sum_e{}' q_G \tag{10}$$

with

$$[k_e] = \iint_{A_e}\left[\frac{\partial N}{\partial x}k\frac{\partial N^T}{\partial x} + \frac{\partial N}{\partial y}k\frac{\partial N^T}{\partial y}\right]dA \tag{11}$$

$$[r_e] = \iint_{A_e} [N]\rho[N]^T dA \quad \text{and} \quad [a_e] = \int_{\gamma 2e} [N]\alpha[N]^T ds \qquad (12), (13)$$

$$[f_e] = \iint_{A_e} [N] f dA \quad \text{and} \quad [q_e] = \int_{\gamma 2e} [N] q(s) ds \qquad (14), (15)$$

For the present problem, we neglect the terms $[a_e]$, $[f_e]$ and $[q_e]$. In the above equations, the index e means elemental formulation and G global formulation. For the discretization of the three-dimensional wave equation, we use the element FLUID30, 3D Acoustic Fluid, with 8 nodes, 4 degrees of freedom (u_x, u_y, u_z and pressure), with structure present at interface, reference pressure 20e6. The model has a total of 602 nodes and 2172 elements for the first type of filter and 955 nodes and 3437 elements for the second type. It was used the unsymmetric method for parameters extraction. Boundary conditions are: zero pressure in the ends of the filter and non-zero pressure in the ends of the filter. We use sound velocity of $343 m/s$ and mass density by $1.21 kg/m^3$ (density at $20^o C$). The results obtaining with the simulation will be compared with the practical results presented in next section.

3. EXPERIMENTAL SETUP

Figure 1 presents the experimental setup that was built for acoustic modal tests. The filter is built from PVC pipes that have a nominal diameter of 38 mm. The acoustic excitation system is composed of a nylon piston that is positioned close to the left end of the filter and that is driven by an electrodynamic vibration exciter (MB Dynamics Modal 50 A). The force applied to the piston as well as its acceleration is measured by an B&K impedance head model 8001 (100 mV/g and 340 pC/N) that is mounted through a stinger between the piston and the exciter head.

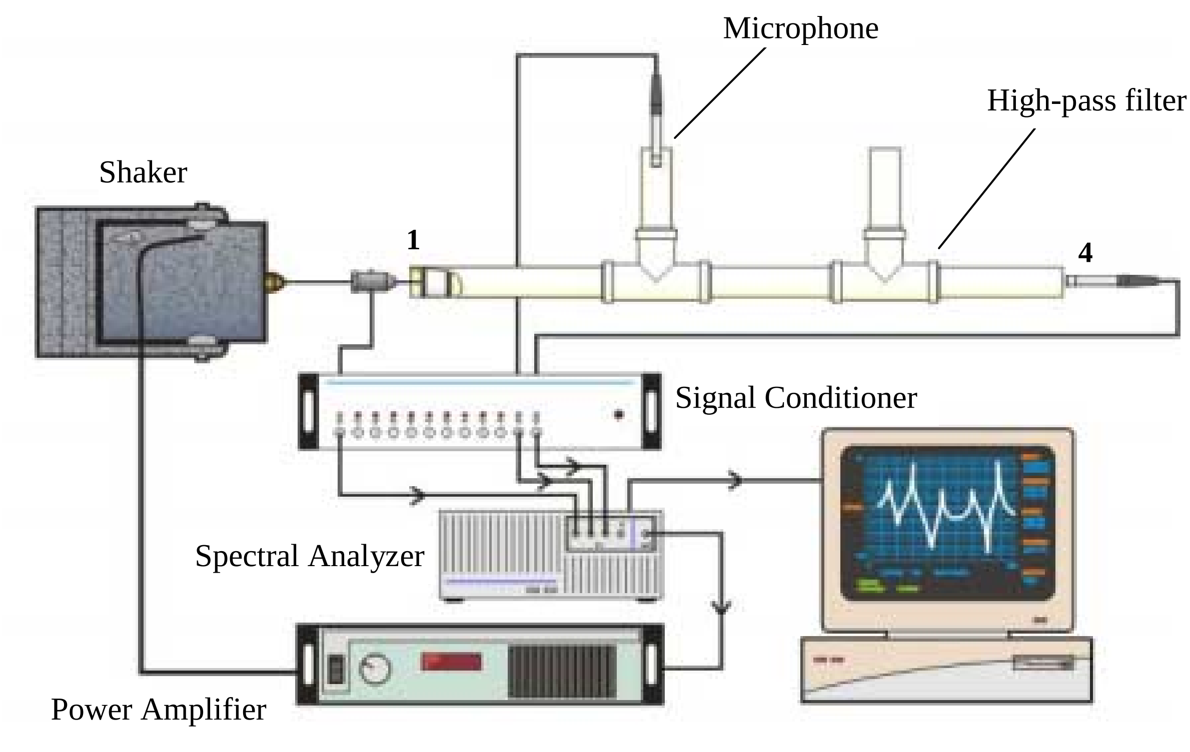

Figure 1 – Experimental Setup

The piston acceleration signal is further used along with the piston cross sectional area in order to get an estimate of the acceleration of the fluid volume, that represents an important acoustic parameter in the acoustic FRF calculations for the filter.

The exciter was driven by a white noise random signal and the data was processed by a 2630 Tektronix analyzer in the 0-2000 Hz frequency range. Hanning windows were used in all acquisition channels in order to reduce filter leakage. The response signals were gathered at different locations by G. R. A. S., ½" prepolarized free field, type 26 CA microphones (49,49 mV/Pa). The acoustic FRF were estimated by considering the acoustic pressure from the microphones as the output variable and the acceleration of the fluid volume as the input variable. The piston was considered to be light enough so that the acceleration signal from the impedance head could be directly used. The microphones were positioned inside the branches, as illustrated in Fig. (1) in three different locations, respectively at the upper end, mid point and opposite lower end. Two filters were tested: one with short branches and open ends and the second with long branches and closed ends. The corresponding finite element models of each filter were generated as it will be described in the next section.

4. RESULTS AND DISCUSSION

Figure 2 presents the experimental results obtained for the first filter, which has the following characteristics: Length of 316 mm, branches with length equal to 20 mm, the first one located at 30 mm of the end and the second at 178 mm of the same end, diameter 38 mm. The acoustic FRF presented in Fig. (2) relates the output acoustic pressure at location 4 to the acceleration of the fluid volume at 1. It can be seen from Fig. (2) that the filter has the capability of attenuating frequencies up to 600 Hz since the first acoustic resonant frequency occurs at 799 Hz.

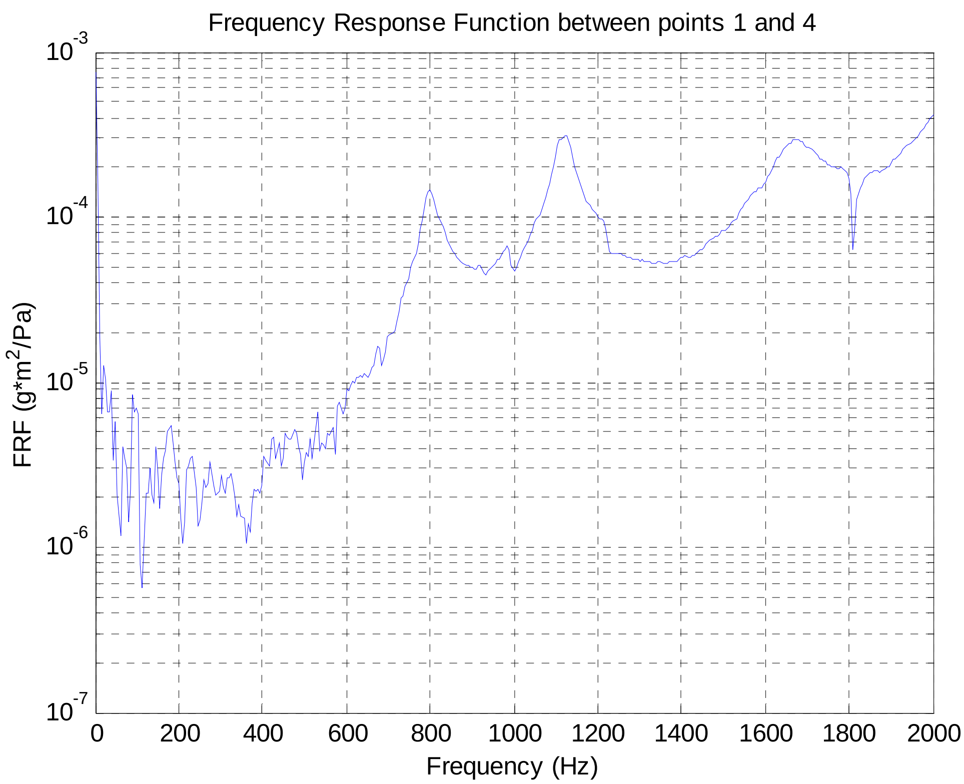

Figure 2 – Experimental FRF result for the first filter

Figure 3 shows the results obtained via the Finite Element simulation for the same filter and illustrates the first acoustic mode or the pressure distribution inside the filter.

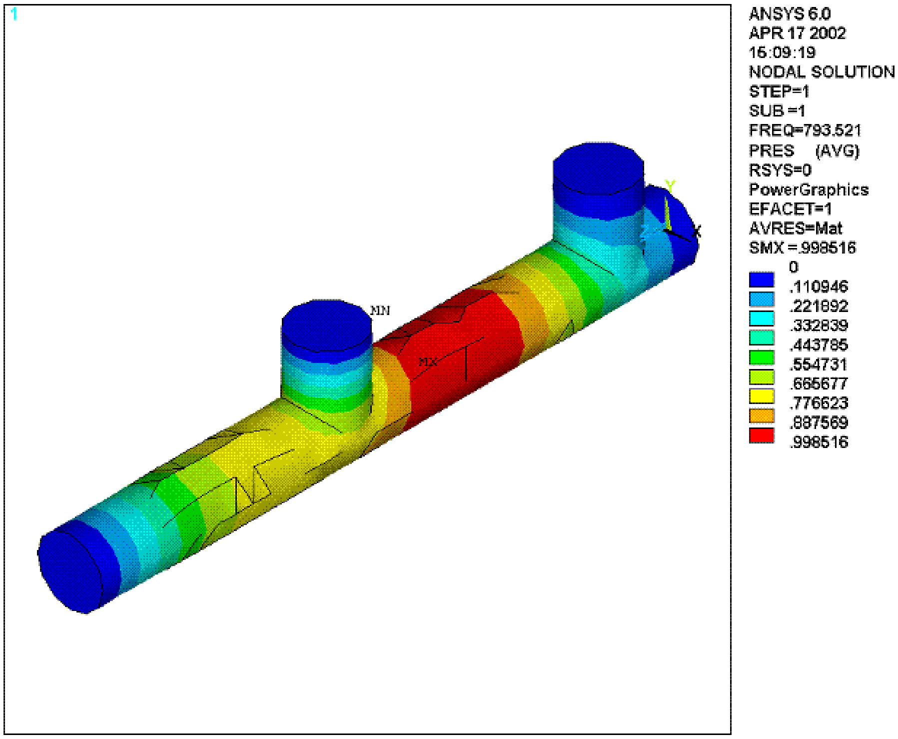

Figure 3 – Finite Element results for the first type of filter

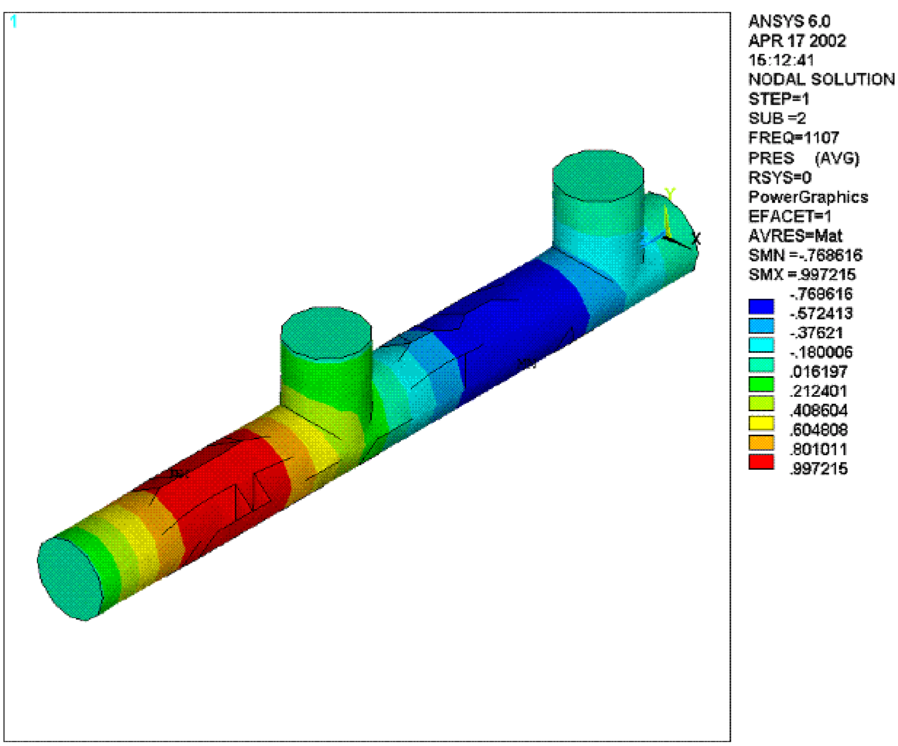

Figure 4 – Finite Element results for the first type of filter

From Fig. (2), we can see that the three first experimental natural frequencies of the filter are 799 Hz, 1122 Hz and 1666 Hz. The natural frequencies obtained by finite element formulation, according to Figs. (3) and (4) are 794 Hz, 1107 Hz and 1639 Hz. Thus, we can conclude that there is a reasonable agreement between the FE results and the experimental data. Using equation (1), we can find the natural frequencies of the pipe without branches: 542 Hz and 1085 Hz.

Figure (5) presents the experimental result for the second type of filter, which has the following characteristics: Length – 316 mm; branches with length equal to 200 mm, the first one located at 30 mm of the end and the second at 178 mm of the same end, diameter 38 mm. Again, the acoustic FRF presented in Fig. (5) relates the output acoustic pressure at location 4 to the acceleration of the fluid volume at 1.

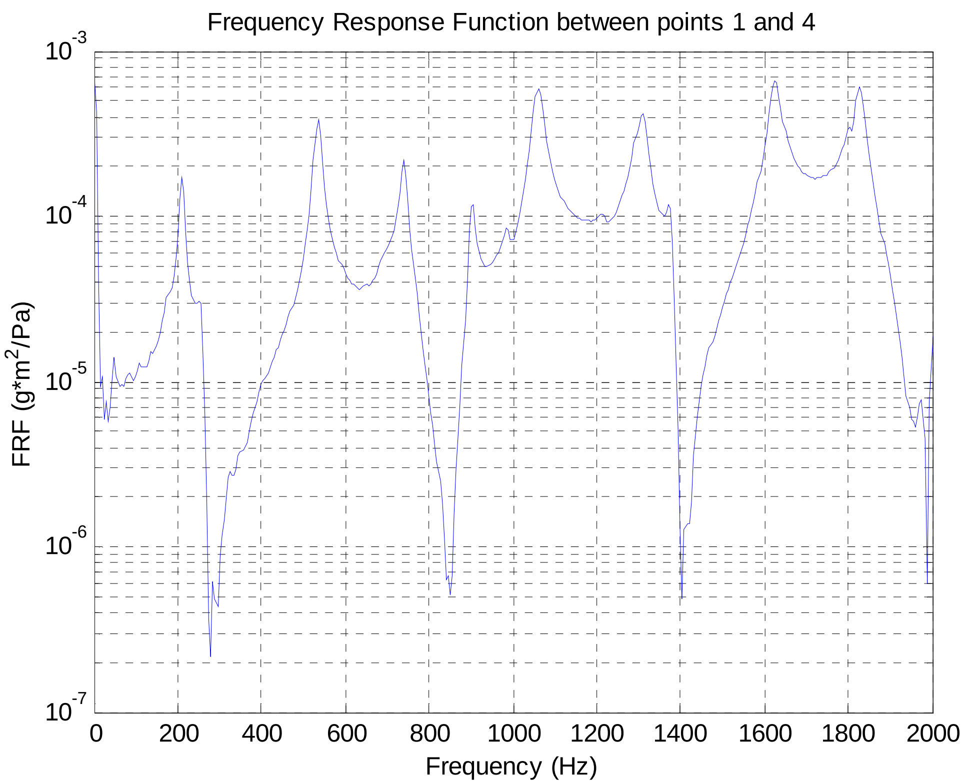

Figure 5 – Experimental result for the second type of filter – FRF

From Fig. (5), we can see that the experimental natural frequencies obtained for the second high-pass filter are: 209 Hz, 536 Hz and 744 Hz. We present only the three first natural frequencies because, as it is well known, one filter with two branches will change only the first two natural frequencies. If the filter has three braches, the first three natural frequencies will be modified and so on.

Figure (6) presents the Finite Element results for the second high-pass filter obtained via Ansys® software for the first acoustic mode.

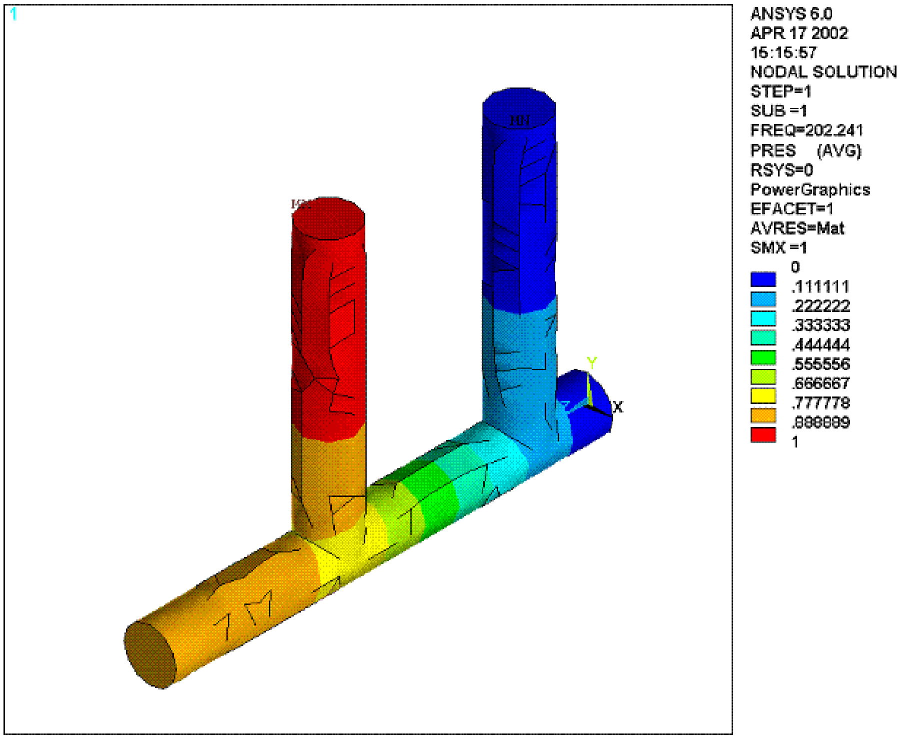

Figure 6 – Finite Element results for the second type of filter

Figure 7 shows the second acoustic modal shape for the same filter.

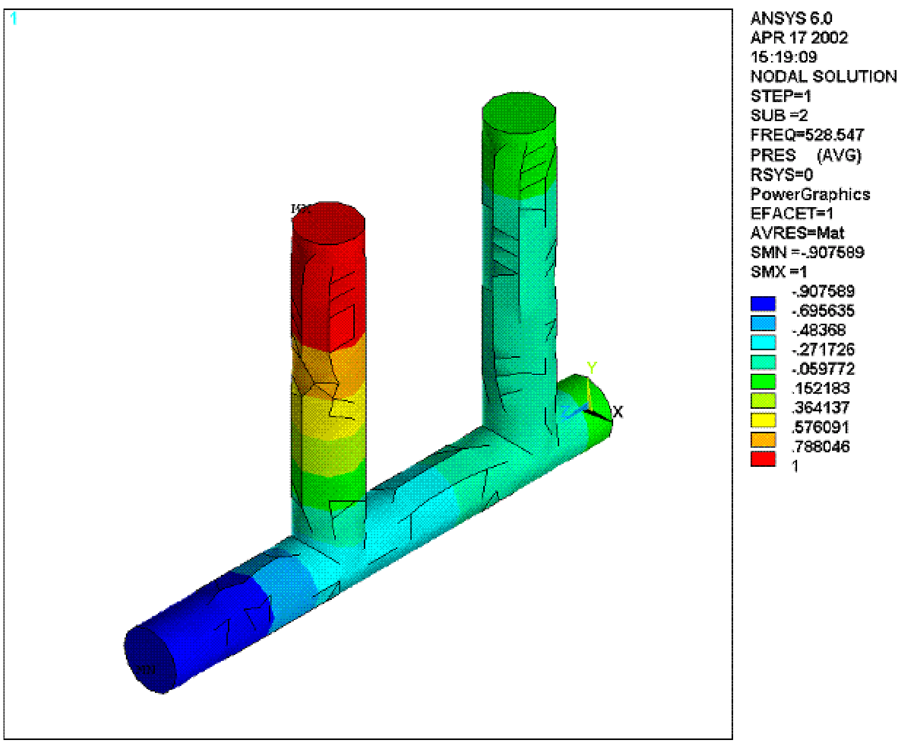

Figure 7 – Finite Element results for the second type of filter

The first type of filter works as a high-pass filter, increasing the first natural frequency, from 542 Hz to 794 Hz. The second frequency has a small change, from 1085 Hz to 1107 Hz. This filter has all open ends, with zero pressure. The second filter has closed ends, and works as a low-pass filter. This happens also because of the length of the branches. A reasonable agreement was found between the results obtained from the experimental results and the FE simulation.

Comparing the two types of high-pass filter, the pressure distribution inside the first filter differs from the second. Observing Figures (3) and (6), we can notice, for example, that the largest values of the acoustic pressure migrates to a branch of the filter, instead of being located in the pipe. The same behavior can be seen if comparing the second acoustic mode, Figures (4) and (7).

In the experimental case, it was used an actuator to obtain the acceleration of the fluid volume, which is an acoustic variable, and some remarks could be made. If we simulate a low-pass filter (Arruda, 1999), the piston is located in a cavity. Here, considering the high-pass filter, the piston was located in an end of the filter. If the piston is inserted inside the pipe, then we have the closed-end boundary condition. For the condition with both ends open (null pressure), the piston must be located in a certain distance from the end of the pipe and not be inserted inside the pipe in order to satisfy the corresponding boundary condition.

Finally, Figure (8) depicts experimental results for the acoustic FRF between points 1 and 4 for the two types of filters studied. We can observe the superior efficiency of the first filter since its first natural frequency is higher than the corresponding frequency for the second filter.

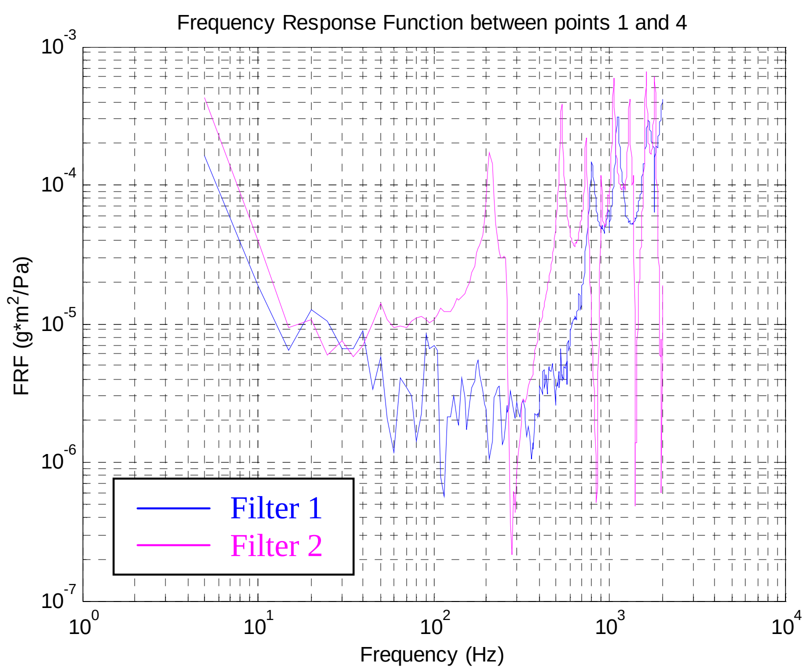

Figure 8 – Comparison between the two types of low-pass filter

5. CONCLUSIONS

The natural frequencies of the filter and consequent attenuation depend of the length and the distance between the branches, but it is more sensitive to the distance between the branches. We can measure the pressure in any point to obtain the FRF of the system since it resulted essentially the same, with small variations on the natural frequency values.

The smallest the length of the branch, the more effective is the filter and the two first natural frequencies occur at higher values. The branch behaves as a capacitance and not as an inertance, as desired in the high-pass filter, if we maintain the diameter constant. According to theory, if we decrease the length of the branches, we also decrease the capacitance and consequently the natural frequency of the filter increases, increasing also the efficacy of this filter. The location of the microphone is not essential, but it can not be located in nodal points, as it is also required in standard modal testing procedures, if we wish to fully assess the dynamics of the system.

6. REFERENCES

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