II CONGRESSO NACIONAL DE ENGENHARIA MECÂNICA
II NATIONAL CONGRESS OF MECHANICAL ENGINEERING
12 a 16 de Agosto de 2002 - João Pessoa – PB

ON THE ESTIMATION OF ANGULAR FRF IN MODAL TESTING

Paulo S. Varoto , Assistant Professor
varoto@sc.usp.br

Melina Lofrano, Graduate Student
melinal@sc.usp.br

Leopoldo P. R. Oliveira, Graduate Student
leopro@uol.com.br

Laboratório de Dinâmica
Departamento de Engenharia Mecânica
Escola de Engenharia de São Carlos
Universidade de São Paulo
Av. Trabalhador São-Carlense 400
13566-590 São Carlos SP

Abstract. *In order to get a reliable and accurate response model for a structure under investigation, the structure's Frequency Response Functions (FRF) must be precisely measured. Although the measurement technology currently available in terms of piezoelectric sensors have grown significantly lately, there still persists the problem of a lack of an adequate instrumentation for the accurate measurements of angular FRFs. Rotational degrees of freedom play an important role in the field of experimental structural dynamics. When the structure is vibrating due to some external excitation, there are locations on the structure that may present significant levels of angular motions. These motions, if not properly measured and accounted for in the FRF determination, can cause significant distortions on the structure's modal and response models. An immediate consequence of these incompleteness is a difficulty in correlating the actual measured data and the results from computational simulations. The present works aims to perform an investigation on experimental techniques for determining angular FRF. One of these techniques consists of exciting the structure with a shaker and using a rigid T-block to measure the linear accelerations and then calculating the angular FRFs. Simple experiments were conducted with different configuration of the T-block, and the results obtained so far reveal that, depending on the level of angular vibrations exhibited by the structure and the fixture of the T-block, the resulting angular FRFs can suffer some significant changes. Additional tests are performed with a new commercially available angular piezoelectric accelerometer.*

Keywords: *Angular FRF, T-Block, Angular acceleration, Experimental techniques*

1. INRODUCTION

Standard modal testing procedures usually employ a single excitation source applied to the structure under test in order to measure the structure's Frequency Response Functions (FRF). The excitation signals can be obtained from an electrodynamic vibration exciter or through an impact hammer. On the other hand, the response signals are gathered through one or several accelerometers mounted at selectred locations on the structure. This test arrangement issues the so called linear FRF since the excitation and the response are given in terms of a linear force and a linear motion. However, McConnell has shown that between two structural points there exists a total of 36 FRFs, since the excitation and response vectors contain both linear and angular quantities. Thus, a more refined mathematical model of the structure under test must also contain information from rotational degrees of freedom, either from the analytical and the experimental viewpoints. In practice, it is rather difficult to obtain reliable measurements of angular quantities, and thus, mathematical models derived from experimental data are usually confined in the linear FRF.

This lack of information in terms of angular FRF causes some difficulties in the application of the experimental data to further analysis such as: response prediction due to external moments, dynamic prediction by the substructuring approach, finite element model updating, and to achieve the complete model. For these reasons, the accurate measurement of rotational FRFs plays an important role in the field of structural dynamics, so that the applicability of experimental modal data can be expanded (Yoshimura, 2000).

The present paper aims to tackle the difficult problem of obtaining accurate estimates of the angular FRF by making use of a T-shaped rigid block combined to matched linear accelerometers attached to the structure under test. This technique also makes use of conventional exciters and force transducers. Two similar accelerometers are mounted on the T-block's arms and then connected to point of interest on the structure under test. The structure is then excited by an electrodynamic shaker, and the signals generated by the matched accelerometers are acquired simultaneously and saved for further calculation that allows the translational and angular measurement estimation for the point of interest.

The structure chosen for these testes was a free-free beam, as a consequence of its well known dynamic behavior so that the accuracy of the technique could be better assessed. The results obtained from the T-block have been compared to the ones given by a piezoelectric angular accelerometer that was recently made commercially available.

2. REVIEW OF THEORY

The measurement of input moments and angular response has been accepted as a difficult issue when experimental modal analysis is concerned mostly due to the practical difficulties to generate and measure moments to be applied to the structure under test. As a result , the experimental models do not carry all the information about the structure and sometimes the missing part represents almost 70 % of the complete model (Maia, 1997). However, the increasing necessity of accurate models has been encouraging the development of new equipment and techniques for measuring input moments and angular responses.

In the subsequent section the technique for measurement angular acceleration based on conventional type of transducers is presented.

2.1. Angular acceleration measurement

The angular acceleration measurement technique exhibited in this paper is the one that makes use of two translational accelerometers mounted on a rigid T-block separated by a distance *2s* as shown in

Fig. (1). For measuring the acceleration at point P, the T-block must be rigidly attached to this point (McConnell, 1995).

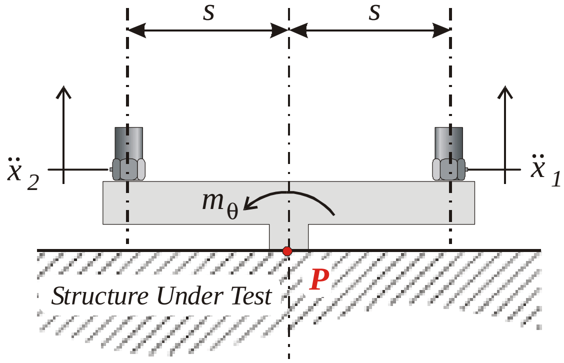

Figure 1. T-Block mounting scheme

The acceleration signal generated by the couple of accelerometers can be converted in the translational and rotational accelerations of point P as follows:

$$\ddot{x}_P = \frac{\ddot{x}_1 + \ddot{x}_2}{2} \tag{1}$$

$$\ddot{\theta}_P = \frac{\ddot{x}_1 - \ddot{x}_2}{2s} \tag{2}$$

One of the challenges of this approach is that we have to deal with the noise and sometimes calibration mismatches in our measured accelerations. Despite of this difficult, this technique has been applied with reasonable success in a wide range of situations. It might be reinforced that the assumption of rigid body behavior of T-block in the frequency range of interest must be assumed so that the equations shown above can be used.

The measurement of rotational degrees of freedom (RDOF) solves a part of the identification problem. Although, a reliable technique to apply a pure moment in a structure remains as an unsolved problem, it might be approached by the natural extension of the technique showed above as shown in Fig. (2), where a forces are applied separately in each of the T-block's arms (Maia, 1997).

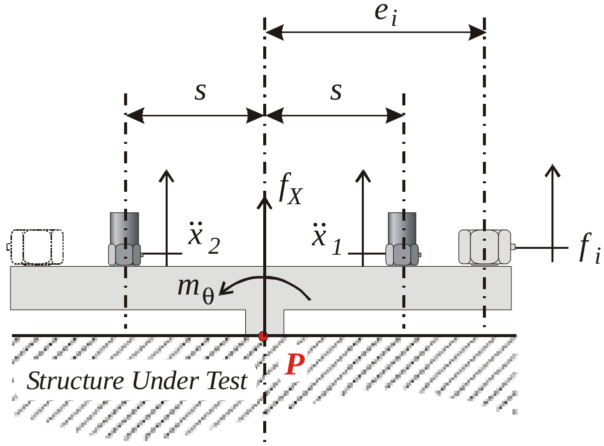

Figure 2. Alternative mounting for input force and moment.

It is basically a matter of relating the force f_x and moment m_θ with the responses $\ddot{x}_P$, and $\ddot{\theta}_P$. So performing a first test run with force f_1 applied to one arm of the T-block you may write:

$$f_x = f_1 - m(\ddot{x}_P)_1 \tag{3}$$

$$m_\theta = e_1 f_1 - I_P(\ddot{\theta}_P)_1 \tag{4}$$

where m is the block's mass and Ip is the inertia about P. The acceleration values $(\ddot{x}_P)_1$ and $(\ddot{\theta}_P)_1$ at P may be related to the measured values as:

$$\begin{Bmatrix}(x_P)_1\\(\theta_P)_1\end{Bmatrix} = \begin{bmatrix}\alpha_{xx} & \alpha_{x\theta}\\ \alpha_{\theta x} & \alpha_{\theta\theta}\end{bmatrix}\begin{Bmatrix}f_x\\ m_\theta\end{Bmatrix} = [\alpha(\omega)]\begin{Bmatrix}f_1 + \omega^2 m(x_P)_1\\ e_1 f_1 + \omega^2 I_P(\theta_P)_1\end{Bmatrix} \tag{5}$$

Dividing by f_1:

$$\begin{Bmatrix}\dfrac{(x_P)_1}{f_1}\\ \dfrac{(\theta_P)_1}{f_1}\end{Bmatrix} = [\alpha(\omega)]\begin{Bmatrix}1+\omega^2 m\dfrac{(x_P)_1}{f_1}\\ e_1 + \omega^2 I_P\dfrac{(\theta_P)_1}{f_1}\end{Bmatrix} \tag{6}$$

Performing the second test in the same condition, but now applying the force f_2 on the other arm of the T-block you have:

$$\begin{Bmatrix}\dfrac{(x_P)_2}{f_2}\\ \dfrac{(\theta_P)_2}{f_2}\end{Bmatrix} = [\alpha(\omega)]\begin{Bmatrix}1+\omega^2 m\dfrac{(x_P)_2}{f_2}\\ -e_2 + \omega^2 I_P\dfrac{(\theta_P)_2}{f_2}\end{Bmatrix} \tag{7}$$

Combining Eqs. (6) and (7) and taking into account Eqs. (1) and (2) for $(x_P)_1$ and $(\theta_P)_1$ and equivalently for $(x_P)_2$ and $(\theta_P)_2$ the receptance matrix is:

$$[\alpha(\omega)] = -\frac{1}{\omega^2}[T][G][[\Pi]-[M][T][G]]^{-1} \tag{8}$$

where

$$[T] = \begin{bmatrix}0.5 & 0.5\\ (2s)^{-1} & -(2s)^{-1}\end{bmatrix} \tag{9}$$

$$[\Pi] = \begin{bmatrix}1 & 1\\ e_1 & -e_2\end{bmatrix} \tag{10}$$

$$[G]=\begin{bmatrix}\left(\frac{\ddot{x}_A}{f}\right)_1 & \left(\frac{\ddot{x}_A}{f}\right)_2\\ \left(\frac{\ddot{x}_B}{f}\right)_1 & \left(\frac{\ddot{x}_B}{f}\right)_2\end{bmatrix} \tag{11}$$

$$[M]=\begin{bmatrix} m & 0\\ 0 & I_P\end{bmatrix} \tag{12}$$

Therefore, the relevant elements of [$\alpha(\omega)$] can be calculated by knowing the T-block geometry and after measuring the elements of [G], that are directly given by the measuring system.

The inertia properties of the T-block may be neglected if they are small compared to the structure under test. Then Eq. (8) becomes:

$$[\alpha(\omega)]=-\frac{1}{\omega^2}[T][G][\Pi]^{-1} \tag{13}$$

where

$$[\Pi]^{-1}=\begin{bmatrix}\frac{e_2}{e_1+e_2} & \frac{1}{e_1+e_2}\\ \frac{e_1}{e_1+e_2} & -\frac{1}{e_1+e_2}\end{bmatrix} \tag{14}$$

Although the equations previously derived are relative to point measurement, it quite simple to derive similar expressions for transfer measurements. Note that, in this case the inertia properties of the T-block cannot be taken into account unless a different theory is followed (Maia, 1997). It is also important to mention that the results showed in this paper makes use only of a simple force input in the beam's tip, regardless of tests with input moment has been performed and will be shown in a near future.

3. A BRIEF REMARK ON THE T-BLOCK ATTACHMENT TO THE STRUCTURE

The theory presented in the last section assumes that the T-block behaves as a rigid mass attached to the structure under test. Therefore, in principle this assumption cannot be violated, at least in the frequency range covered by the tests. A simple check of the validity of this rigid body assumption was performed by mounting the T-block on the exciter´s armature and applying an input signal at the mounting point, as seen in Fig. (3).

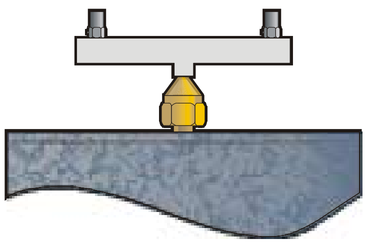

Figure 3. T-block test

The accelerations at the accelerometers locations were measured simultaneously in two different mounting conditions. The basic difference between these mounting conditions was the contact area between the T-block and the exciter head. Both tests were performed in the 0-500 Hz frequency range and the results are depicte in Figs. (4) and (5).

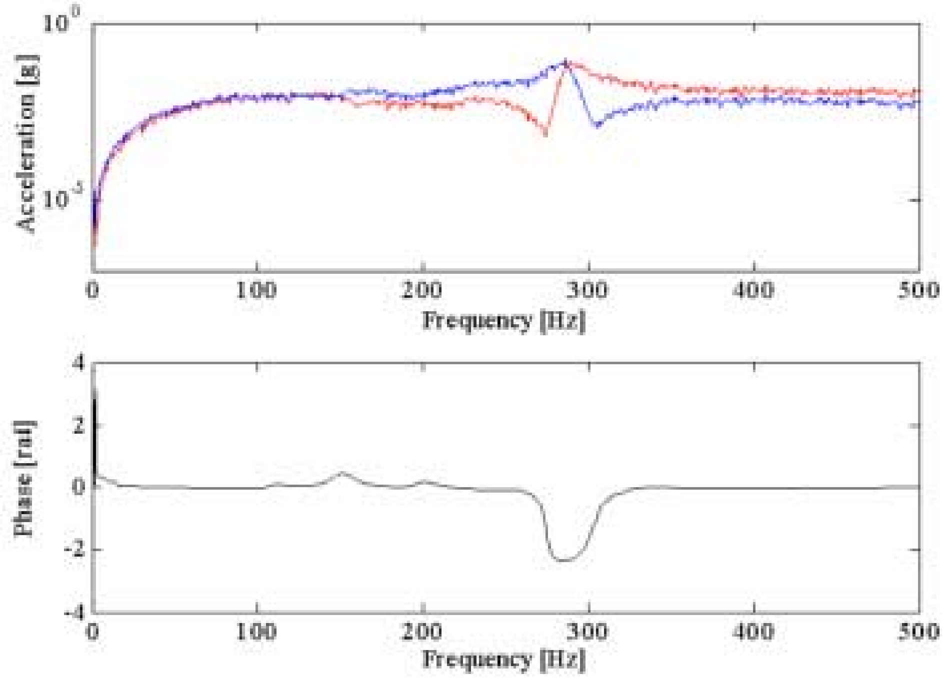

Figure 4. T-Block Accelerometers amplitudes and phase

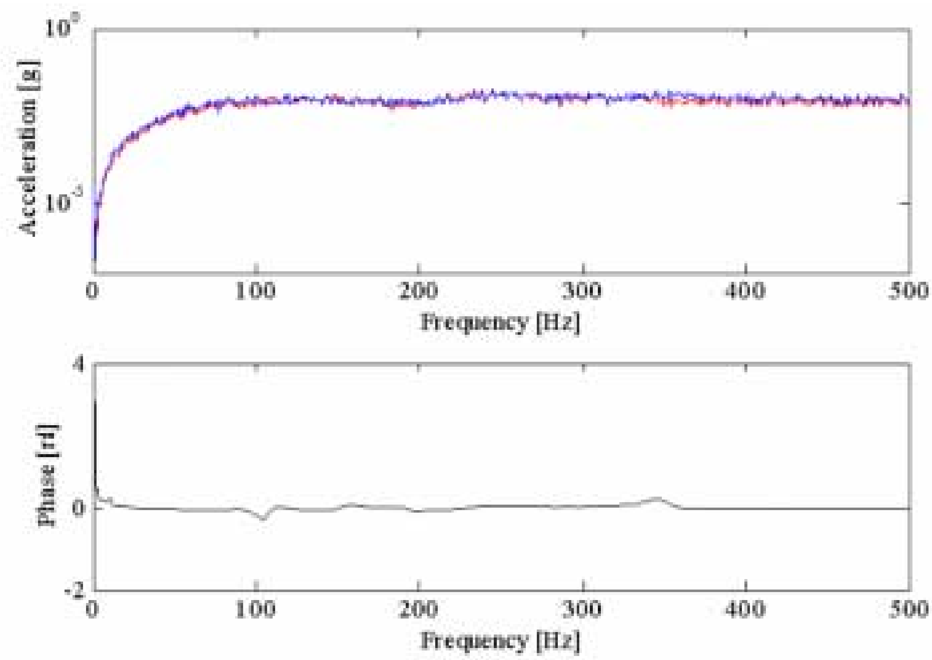

Figure 5. T-Block Accelerometers amplitude and phase

It can be seen from Fig. (4) that the accelerometers responses reveal a resonance close to 287 Hz what essentially represents a rigid body resonance belonging to the system formed by the T-block and accelerometers. This resonance was caused by a rocking motion of the T-block due to an inadequate mounting condition on the exciter head. It can be also seen a phase inversion on the phase angle plot in Fig (4), what reinforces the resonant phenomenon observed. The acceleration results depicted in Fig. (5) show a much better dynamic behavior presented by the T-block, this time mounted on the exciter head through a more adequate fixture. These results are important to illustrate the importance of a correct fixture system to connect the T-block to the structure under test in order to avoid problems as the one shown in Fig. (4).

4. EXPERIMENTAL RESULTS

The experimental setup used in the tests are shown in Fig. (6). The free free beam was suspend by flexible nylon strings in order to better simulate the free free suspension condition. The T-block isconnected to the beam and the acceleration responses are measured at four different locations. The beam is excited by an electromagnetic exciter that is attached to the beam at one of its free ends, as illustrated in Fig. (6). The excitation signal was random and covered the 0-400 Hz frequency range. Hanning windows were employed in all acquisition channels in order to reduce filter leakage. The signals measured by the accelerometers mounted on the T-block were measured simultaneously and processed mathematically in order to get estimates of the angular FRF. The results obtained through this procedure were compared with the angular FRF directly measured by a commercially available angular piezoelectric accelerometer.

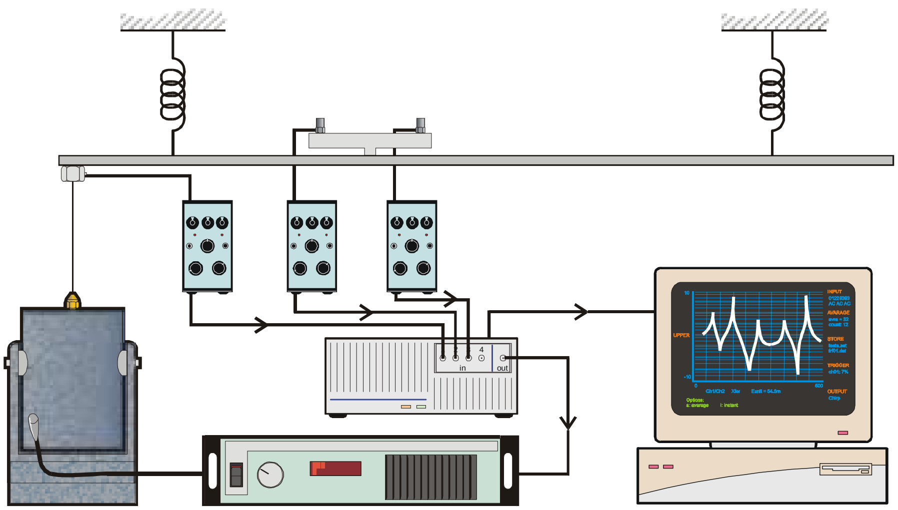

Figure 6. Experimental set-up.

Figure (7) shows the results obtained for the driving point angular FRF, that was obtained by attaching the T-block to the left end of the free free beam. It can be seen that the T-block system issued a reasonable estimate of the driving point angular FRF in comparison with the directly measured FRF through the angular accelerometer. For frequencies in the 0-100 Hz the T-block estimate presents some noise problems especially in the vicinity of the anti-resonances. This behavior is primarily due to the calculation scheme previously shown where the accelerometer signals are subtracted one from another in order to get the angular motion. Also, it can be noticed a mismatch between the two measurement results in the vicinity of all natural frequencies in the 0-400 Hz frequency range. For frequencies in the 0-100 Hz the natural frequencies values obtained through the T-block approach are slightly higher that those obtained from the angular sensor, whereas for frequencies above 100 Hz the natural frequencies values obtained through the T-block are lower than the those obtained through the angular sensor. There are also some amplitude mismatches in the vicinities of the natural frequencies between these two estimates. It should be emphasized that the measurements shown in Fig. (7) correspond to the FRF relating the angular acceleration to the input linear force applied by the excitation mechanism.

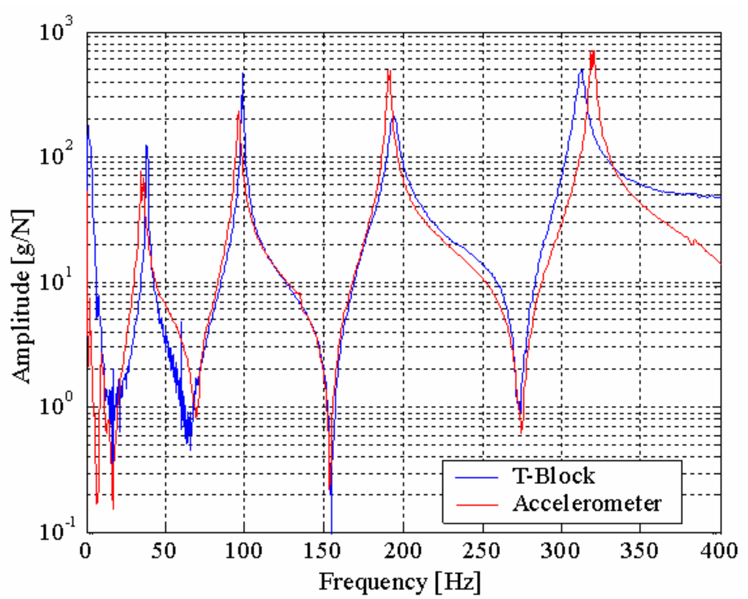

Figure 7. Driving point measurements

In the second test, the T-block system was attached to a location on the beam that is very close to the beam's node of the first bending mode shape. The reason for this choice is that in theory this point has no linear motion, so that the beam presents only angular motion at this location. The measured signals were processed, and the transfer angular FRF was calculated. The results are shown in Fig. (8).

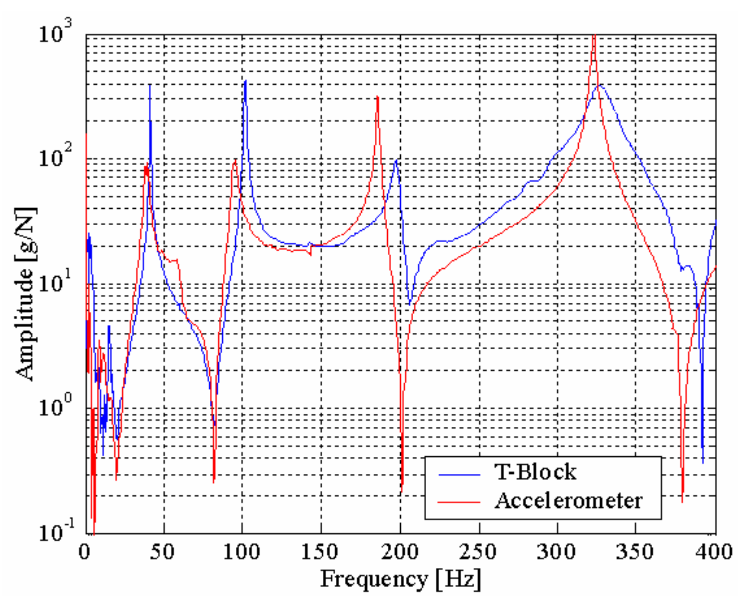

Figure 8. Transfer FRF for beam's node point

It can be noticed that a reasonable agreement occurs between the two approaches, and significant variations occurs in the T-block approach for frequencies above 200 Hz.

The next measurement location was the beam's mid point. This is an interesting location since it represents the node point for all of the beam's even mode shapes, i.e., even though the excitation mechanism is placed at the end point, only the odd natural frequencies will appear, since the T-block and the angular accelerometer are both connected at the node point for even natural frequencies. The results obtained are shown in Fig. (9). Again, a reasonable agreement is found between the T-block approach and the direct measured angular FRF.

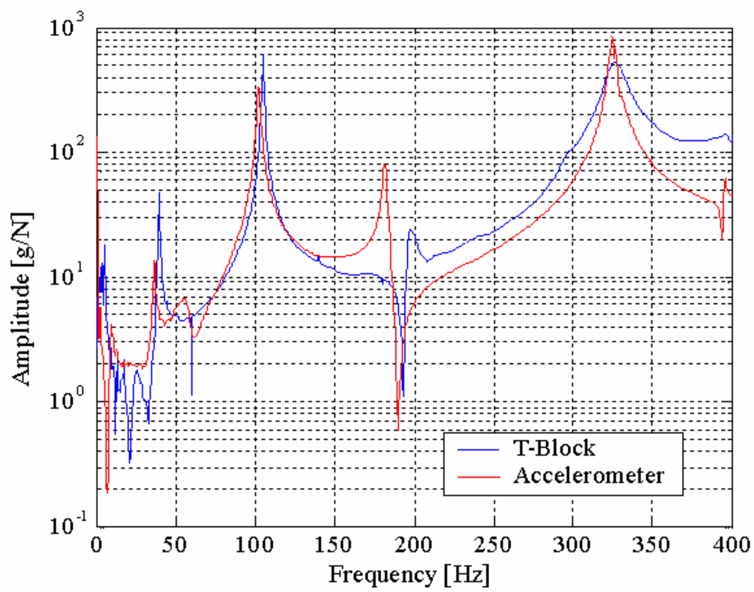

Figure 9. Angular FRF for beam's mid point

Finally the last measurement location was the opposite beam's end. The results for this angular transfer FRF are shown in Fig. (10).

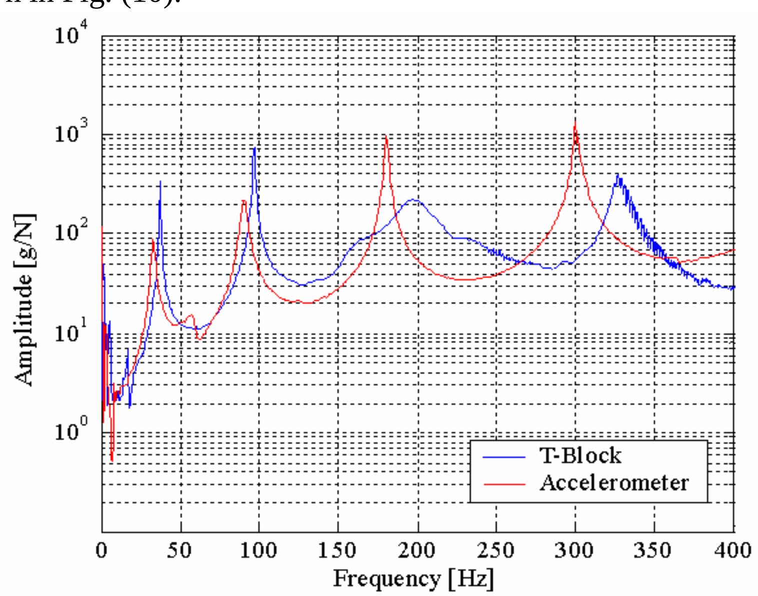

Figure 10. – Transfer angular FRF for beam's end point

5. CONCLUSION

This paper addresses the problem of using the T-block approach in order to measure angular FRF on a free free beam. The results obtained with this procedure are compared to the actual angular/linear FRF measured by an angular piezoelectric accelerometer recently available. The results shown here reveal that the T-block must be carefully attached to the structure under test otherwise undesired results can be obtained in terms of spurious natural frequencies present in the working frequency range. The results for the angular/linear driving point FRF showed good agreement with the results obtained through the angular sensor. Larger deviations were found in the results obtained for the transfer angular FRF. These deviations are due to a number of reasons, specially the calculation procedure adopted to manipulate the signals measured with the T-block. Further results will be communicated by the authors attempting to improve the results as well as addressing the issue of applying pure moments in order to get angular/angular FRF.

6. ACKNOWLEDGMENTS

The authors kindly acknowledge the financial supported received from CAPES and FAPESP for the execution of this project.

7. REFERENCES

Maia, N.M.M and Silva, J.M.M, 1997, “Theoretical and Experimental Modal Analysis”, Research Studies Press LTD, John Wiley & Sons INC, 468 p.

McConnell, K.G.,1995, “Vibration Testing – Theory an Practice”, Research Studies Press LTD, John Wiley & Sons INC, 606 p.

Yoshimura, T and Hosoyo, N, 1999 “FRF Estimation on Rotational DOFs by Rigid Block Attachment Approach”, Proceedings of the ISMA, 2000.

Yoshimura, T and Hosoyo, N, 2000 “FRF Estimation on Rotational DOFs of Estructures”, Proceedings of 18 IMAC - San Antonio – USA, pp. 1667-1671