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EVALUATION AND OPTIMIZATION OF THE INSTABILITY REGIONS ON ROTORS IN WOUNDING SHAFT

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Abstract. *In this work we purpose to analyse the behavior of rotors in wounding shaft considering the instability. The finite element method is used and the mechanical properties of the shaft are obtained using an equivalent modulus. The internal damping due to composite materials is introduced in the analysis by using a model proposed by Adams. The disks are considered rigids, the bearings are flexible supposed be isotropics or anisotropics and the external excitation is synchronous. Firstly, we analyse the effect of the internal damping on the instability in rotors when coupling terms and external damping are included. After that, an optimization technique is used in order to avoid instability regions by maximizing the logarithmic decrement on the first critical speed, considering as design variables the wounding angle, the stiffness of the bearings and the position of the disk.*

Keywords: *rotors, composite materials, optimization, instability.*

1. INTRODUCTION

The great majority of rotordynamic problems encountered involves synchronous whirl, i.e., response to an unbalance mass. The remaining minority of problems involves nonsynchronous whirl that can be subdivided into three classes, Vance (1988): supersynchronous vibrations due to shaft misalignment; subsynchronous and supersynchronous vibrations due to cyclic variations of parameters, mainly caused by loose bearing housings or shaft rubs; nonsynchronous rotor whirling that becomes unstable, typically when a certain speed called the threshold speed of instability is reached. Problems in the first and second classes have obvious solutions such as align the shafts, tighten the bearings housings, or eliminate the rub. Problems in the third class, although relatively uncommon, have a history of causing expensive failures in rotors, with elusive causes and cures.

The instability in rotating machine is usually produced by destabilizing forces which are tangential to the rotor whirl orbit, acting in the same direction as the instantaneous motions. Most of the known destabilizing forces are represented by cross-coupled stiffness, K_{xz} and K_{zx}. Inside of destabilizing forces, it can be included the forces produced by internal damping in a shaft of rotor.

For a rotor with the shaft in conventional material, the influence of internal damping can be usually omitted. However, for rotor with the shaft in composite materials, the internal damping can be two times major, Wettergren (1996), because of high capacity of damping due to the matrix of the composite material. Wettergren (1996), Gupta et al. (1998) and Silveira (2001) have introduced the internal damping due to composite materials in rotodynamics analysis considering a damping model similar to the one proposed by Adams et al. (1973).

Experimental measurements have shown that the internal damping is not viscous, but model it as hysteretic damping is an usefull approach for stability analysis, Vance (1988). Considering that, the hysteretic damping due to composite materials can be treated as a viscous damping by using an equivalence between the energy dissipated by both mechanisms, Singh et al. (1994). Zorzi et al. (1977), Özgüven et al. (1984), Taylor et al. (1995), Melanson et al. (1998) and Ku (1998) have incorporated the hysteretic and the viscous damping in rotor problems in order to investigate the instability regions on rotors in isotropic materials. Silveira (2001) has also included the hysteretic damping on analysing the instability and the response on frequency on rotors in wounding shaft..

In this work the finite element method is used to analyse the instability zones of composite rotors. The shaft is obtained by winding several layers of embbebed fibers over a mandrel. The disk is supposed to be rigid and the assembly is supported by flexible bearings. An equivalent modulus approach is used in order to represent the orthotropic properties of the composite shaft. The effect of the damping in composite materials is made by introducting a model proposed by Adams et al. (1973). The strain stress relation that includes the internal damping on the strain energy in bending developed in Silveira (2001) is used. The purpose of this work is to analyse the effect of the internal damping of the composite materials on the stability of rotors. Optimization techniques are used in order to avoid instability in which the wounding angle, the stiffness of the bearings and the position of the disk are used as design variables.

2. THE FINITE ELEMENT MODEL

The finite element model of a rotor is composed by beam elements and rigid elements to represent the shaft and the disks respectively. The rotor is supposed to be simple-supported and the wounding angle of each layer of the shaft is j. As shown by Lalanne et al. (1998), the kinetic energy of a disk can be expressed by:

$$T_D = \frac{1}{2} M_D \left(\dot{u}^2 + \dot{w}^2\right) + \frac{1}{2} I_{Dx} \left(\dot{q}^2 + \dot{y}^2\right) + I_{Dy}\Omega \dot{y} q + \frac{1}{2} I_{Dy}\Omega^2 \qquad (1)$$

where M_D is the mass of the disk, u and w are the coordinates of the center of inertia of the disk on the inertial axes, $\dot{u}$ and $\dot{w}$ are yours time derivatives, I_{Dx} and I_{Dy} are the moments on the principal directions of inertia. The rotation speed of rotor is W and $\dot{y}$ and $\dot{q}$ are instantaneous velocities.

For an element of the shaft, the kinetic energy can be expressed by:

$$T_S = \frac{r S}{2} \int_0^L \left(\dot{u}^2 + \dot{w}^2\right) dy + \frac{r I_{xx}}{2} \int_0^L \left(\dot{q}^2 + \dot{y}^2\right) dy + r I_{xx} L \Omega^2 + 2 r I_{xx} \Omega \int_0^L \dot{y} q \, dy \qquad (2)$$

where r is the volumetric mass, S is the area of the cross section, I_{xx} is the inertia moment of the cross section and L is the lenght of the element.

The general expression for the strain energy of the shaft in bending is:

$$U = \frac{1}{2} \int_V \{e\}^t [s] \, dV \qquad (3)$$

As proposed by Silveira (2001), the stress-strain relation for a composite beam, including the effect of hysteretic damping can be given as:

$$s = E_{eq} e + E_{eq}^{y} \dot{e} \tag{4}$$

where E_{eq} is the equivalent Young's modulus and E_{eq}^{y} is the equivalent damped Young's modulus. Considering small deformations, the longitudinal strain and the longitudinal strain rate can be expressed as:

$$\begin{aligned} e &= -x \frac{\partial^2 u^*}{\partial y^2} - z \frac{\partial^2 w^*}{\partial y^2} \\ \dot{e} &= -x \frac{\partial^2 \dot{u}^*}{\partial y^2} - z \frac{\partial^2 \dot{w}^*}{\partial y^2} \end{aligned} \tag{5}$$

where u^* and w^* are displacements of the geometric center measured on the rotating axes of the shaft, Lalanne et al. (1998). Considering the relation between the displacements u^* and w^* and the displacements u and w measured on the inertial axes, Lalanne et al. (1998), and using Eqs. (3)-(4), we obtain the strain energy in bending:

$$\begin{aligned} U &= \frac{1}{2} E_{eq} I_{xx} \int_0^L \left[\left(\frac{\partial^2 w}{\partial y^2} \right)^2 + \left(\frac{\partial^2 u}{\partial y^2} \right)^2 \right] dy + \frac{1}{2} E_{eq}^{y} I_{xx} \int_0^L \left[\left(\frac{\partial^2 u}{\partial y^2} \frac{\partial^2 \dot{u}}{\partial y^2} \right) + \left(\frac{\partial^2 w}{\partial y^2} \frac{\partial^2 \dot{w}}{\partial y^2} \right) \right] dy \\ &+ \frac{1}{2} E_{eq}^{y} I_{xx} \int_0^L \left[-\Omega \left(\frac{\partial^2 u}{\partial y^2} \frac{\partial^2 w}{\partial y^2} \right) + \Omega \left(\frac{\partial^2 w}{\partial y^2} \frac{\partial^2 u}{\partial y^2} \right) \right] dy \end{aligned} \tag{6}$$

The second term and the third term of Eq. (6) are related to the hysteretic damping named $[H_b]$ and $[H_c]$. The equivalence between the hysteretic damping and the viscous damping is made by:

$$\begin{aligned} [K_b] &= \frac{[H_b]}{p\,[w]} \\ [K_c] &= \frac{[H_c]}{p\,[w]} \end{aligned} \tag{7}$$

where $[w]$ is the diagonal frequencies matrix.

The equation of motion of the rotor is obtained by applying Lagrange's equations, on the kinetic energy and on the strain energy of the elements, and can be written as:

$$[M]\{\ddot{u}\} + [K_b + \Omega G]\{\dot{u}\} + [K + \Omega K_c]\{u\} = \{F_d\,(t)\} \tag{8}$$

where $[M]$, $[G]$ and $[K]$ are global mass, global Coriolis and global stiffness matrices. $[K_b]$ and $[K_c]$ are global dissipation matrix and global circulation matrix. Vectors $\{\ddot{u}\}$, $\{\dot{u}\}$, and $\{u\}$ are nodal acceleration, nodal velocity and nodal displacement respectively and $\{F_d(t)\}$ is the generalised force vector due to the unbalanced mass. The elementaries matrices are obtained according to the Euler-Bernoulli equation for beams and are presented in Zorzi et al. (1977).

3. THE EQUIVALENT MODULUS AND THE INTERNAL DAMPING MODEL

Considering that the shaft is thin walled and slender, and the laminate is symmetric and balanced, the equivalent Young's modulus is found as, Singh et al. (1994):

$$E_{eq} = \frac{\left[4(U_1 - U_5)(U_5 + U_3 g) - b^2 U_2^2 \right]}{U_1 - bU_2 + gU_3} \tag{9}$$

where $U_{1\text{-}5}$ are the laminate invariants and, Tsai et al. (1980):

$$g = \sum_{k=1}^{N} \frac{h_k}{h} \cos(4j_k) \qquad b = \sum_{k=1}^{N} \frac{h_k}{h} \cos(2j_k) \tag{10}$$

where h_k is the thickness of layer k, h is the laminate thickness and j_k the wounding angle.

On the prediction of damping on multi-layer shell structure the model proposed by Adams is used and the specific damping capacity is defined as:

$$y = \frac{\Delta U}{U} \tag{11}$$

From Eq. (3) and Eq. (11) and assuming plane stress state, the dissipative energy for a single layer of unidirectionally fiber in the orthotropic axis is:

$$\Delta U = \frac{1}{2}\int_V \{e\}^t [y]\{s\} dV \tag{12}$$

where $[y]$ is the specific damping capacity matrix in the form:

$$[y] = \begin{bmatrix} y_{11} & 0 & 0 \\ 0 & y_{22} & 0 \\ 0 & 0 & y_{12} \end{bmatrix} \tag{13}$$

and y_{11}, y_{22}, y_{12} are the specific damping capacities of a layer on longitudinal, transverse and shear direction. From Eq. (12) and considering the constitutive relation, we obtain:

$$\Delta U = \frac{1}{2}\int_V \{e\}^t [y][Q]\{e\} dV \tag{14}$$

Using the same procedure to derive E_{eq}, the equivalent damped Young's modulus E_{eq}^{y} as a function of the specific damping is derived from Eq. (14).

4. INSTABILITY IN ROTORDYNAMICS

The natural frequencies and the zones of instability can be determined from the solution of the eigenvalue problem as a result of the homogeneous equation:

$$[m]\{\ddot{q}\} + [c]\{\dot{q}\} + [k]\{q\} = 0 \tag{15}$$

where $[m]$, $[c]$ and $[k]$ are modal matrices obtained from Eq. (8) by using the pseudo-modal method, Lalanne et al. (1998). Solutions for this problem are sought as:

$$\{q\} = [P]\{e^{rt}\} \tag{16}$$

Substituting Eq. (16) in the Eq. (15) we obtain:

$$\left[\{r\}^2 [m] + \{r\}[c] + [k]\right][P] = 0 \tag{17}$$

Equation (17) can be rearranged as follow:

$$\begin{bmatrix} [0] & [I] \\ -[m]^{-1}[k] & -[m]^{-1}[c] \end{bmatrix} \begin{Bmatrix} [P] \\ \{r\}[P] \end{Bmatrix} = \{r\} \begin{Bmatrix} [P] \\ \{r\}[P] \end{Bmatrix} \tag{18}$$

where $[I]$ is the identity matrix. The eigenvectors of Eq. (18) are obtained in the complex form:

$$\{r\} = \{l\} \pm i\{w\} \tag{19}$$

where $\{w\}$ is the natural frequencies vector and $\{l\}$ is the vector that determine the stability of the system.

5. APPLICATION

In order to emphasize the influence of the internal damping due to composite materials in rotordynamics analysis, the zones of instability are determined for a rotor with differents configurations of bearings. In this case, the rotor is composed by a wounding shaft, two disks equidistants from the ends and the assembly is supported by flexible bearings, Fig. (1).

The wounding shaft has length of 1.2m, inner radius of 0.04m, outer radius of 0.048m, with eight layer of 0.001m thickness in a balanced and symmetric configuration such as $[\pm j]_s$. The disks have inner radius of 0.048m, outer radius of 0.15m and thickness of 0.05m. The material data are given in Tab.(1) and the stiffness of the bearings are given in Tab. (2).

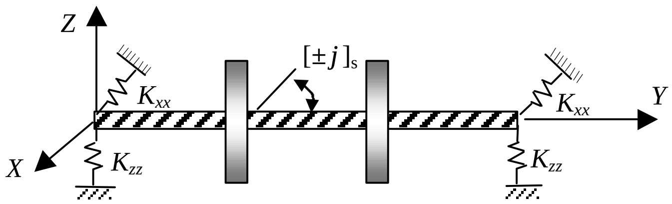

Figure 1. Rotor in wounding shaft with two disks.

Table 1. Material data of the shaft and the disks.

	E_1 (GPa)	E_2 (GPa)	G_{12} (GPa)	ρ (kg/m^3)	ψ_{11} (%)	ψ_{22} (%)	ψ_{12} (%)	ν_{12}
Shaft (carbon/epoxy)	172.7	7.20	3.76	1446.2	0.45	4.22	7.05	0.3
Shaft (glass/epoxy)	37.78	10.90	4.91	1813.9	0.87	5.05	6.91	0.3
Disk (steel)	-	-	-	7800	-	-	-	-

Table 2. Stiffness and damping data of the bearings.

	K_{xx} (N/m)	K_{zz} (N/m)	K_{xz} (N/m)	K_{zx} (N/m)	A_{xx} (N/m/s)	A_{zz} (N/m/s)	A_{xz} (N/m/s)	A_{zx} (N/m/s)
Anisotropic bearing	1.10^7	1.10^8	0	0	0	0	0	0
Isotropic bearing with external damping	1.10^7	1.10^7	0	0	1.10^3	1.10^3	0	0
Isotropic bearing with coupled terms	1.10^7	1.10^7	-1.10^6	1.10^6	0	0	0	0

Figures (2), (3) and (4) show the influence of the wounding angle on the position of the natural frequencies, as well as the influence of the internal damping on the instability zones for rotors in carbon/epoxy supported by isotropic bearings. In all figures, the legend is as follow: ---

synchronous exicitation; —— stable natural frequency; —●— unstable natural frequency. As was observed by Silveira (2001), as higher is the wounding angle, higher is the internal damping introduced by the matrix of the composite material. Hence, it can be observed that for small wounding angle, the internal damping has a less influence on the instability. With these conditions, it can be seen that the external damping increase quite the threshold speed of instability and the coupling terms destabilized all forward modes.

On the other hand, for large wounding angle, the internal damping has a strong influence on the instability, and consequently, the external damping introduced by the bearings are not large enough to increase the threshold speed of instability. And it can also be observed that the internal damping trends to stabilize forward modes until the critical speed.

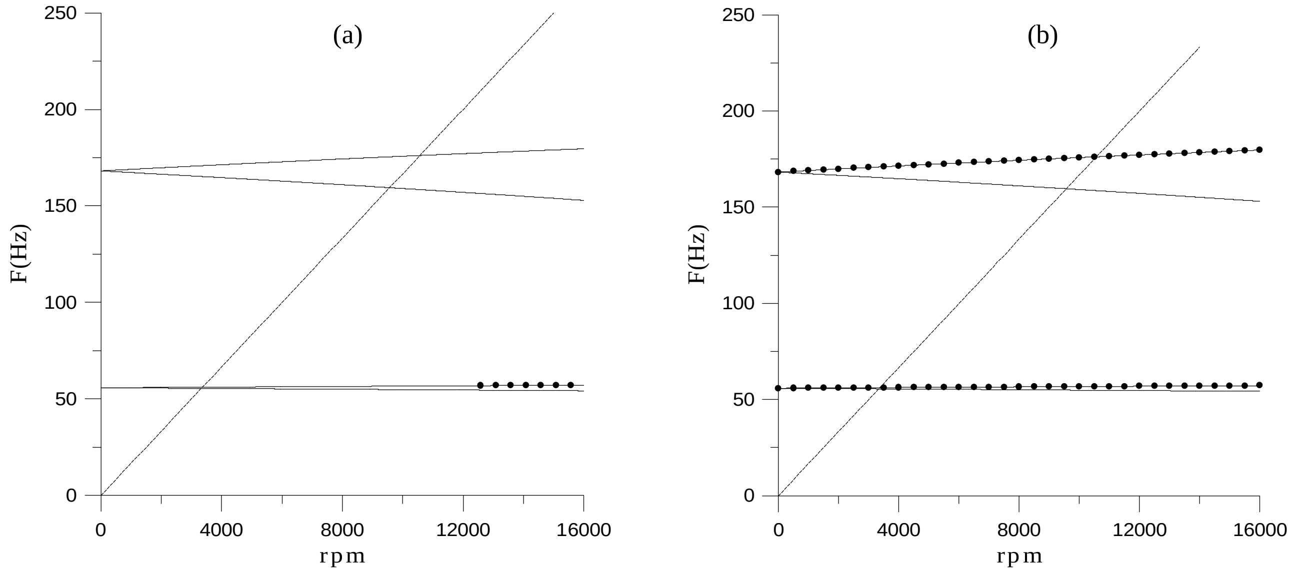

Figure 2. Campbell diagram and instability regions for j = 15º, (a) with external damping; (b) with coupling terms.

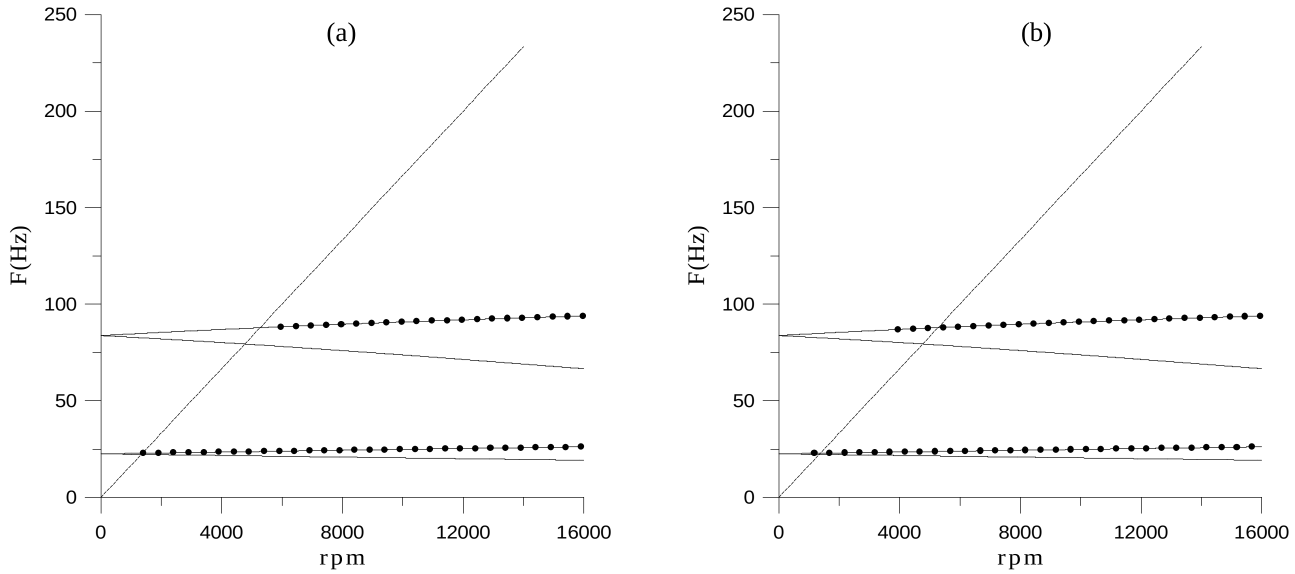

Figure 3. Campbell diagram and instability regions for j = 45º, (a) with external damping; (b) with coupling terms.

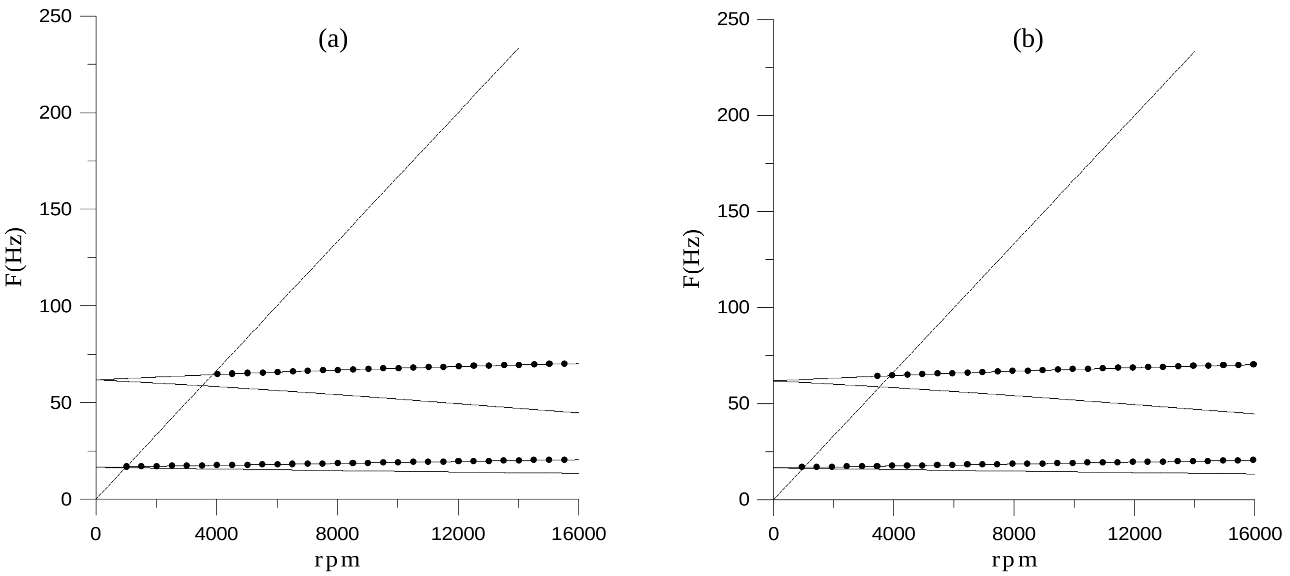

Figure 4. Campbell diagram and instability regions for j = 75º, (a) with external damping; (b) with coupling terms.

Figures (5a), (5b) and (5c) show the influence of the anisotropic bearings on the stability of the rotor. It can be seen that as higher is the wounding angle, higher is the internal damping introduced by the composite material, and consequently, higher is the instability regions. It can be added, the fact of as higher is the wounding angle, lower is the effect of the anisotropic bearings, because of the decrease of the equivalente stiffness of the shaft.

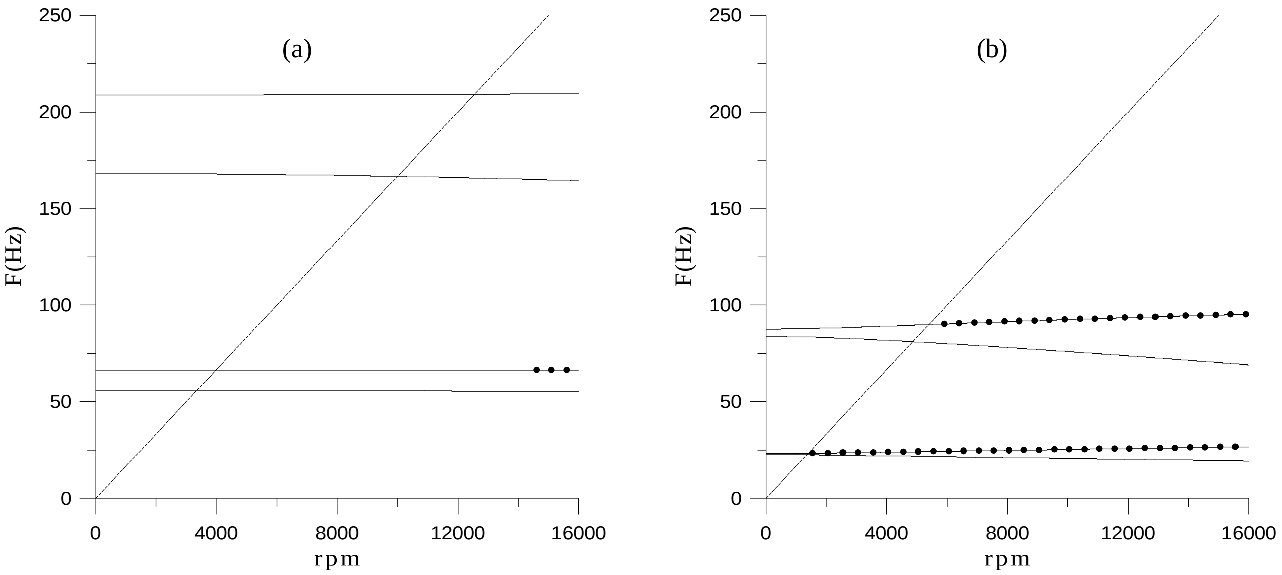

Figure 5. Campbell diagram and instability regions for rotor with anisotropic bearings (a) j = 15º ; (b) j = 45º.

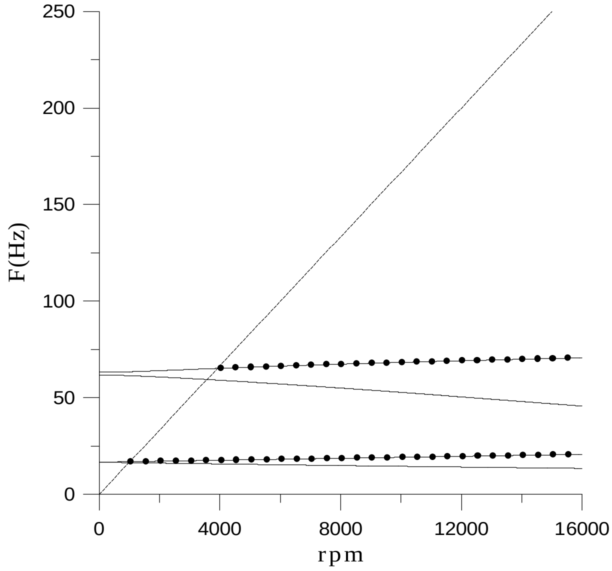

Figure 5c. Campbell diagram and instability regions for rotor with anisotropic bearings and j = 75º.

6. OPTIMIZATION OF THE INSTABILITY REGIONS

A non-linear unconstrained optimization technique was used in order to avoid instability regions by maximization of the logarithmic decrement on the first critical speed. It was used the Quasi-Newton method, in which the approach of the Hessian is made by the BFGS, Arora (1989), and the gradient of the objective function was determined by the forward finite difference.

The problem of optimization was formulated as follow:

Max $\boldsymbol{d}_i$

$15^{\circ} \leq j_1 \leq 75^{\circ}$

$15^{\circ} \leq j_2 \leq 75^{\circ}$

$1.10^7 \leq K_{xx} \leq 1.10^8$

$5.10^7 \leq K_{zz} \leq 8.10^8$

$0.2 \leq y_c \leq 0.4$

where y_c is the position of the disk on the shaft and $\boldsymbol{d}$ is the logarithmic decrement that is evaluated only on the first critical speed.

The rotor is composed now with only one disk, located at 0.33m of the left end. The wounding shaft has lenght of 1m, inner radius of 0.031m, outer radius of 0.039m, with eight layer of 0.001m thickness in a balanced and symmetric configuration, such as $[\pm j_1, \pm j_2]_s$. The disk has inner radius of 0.039m, outer radius of 0.15m and thickness of 0.03m. In order to introduce instability on the system, coupled terms was taken as K_{xz} = -1.10^7 N/m and K_{zx} = 1.10^{7}N/m.

The initial and optimal configurations for rotors in carbon/epoxy and in glass/epoxy are shown in Tab. (3) and the results plotted on Fig. (6) and Fig. (7). For both carbon/epoxy and glass/epoxy shafts, it can be observed that on the initial configuration, the rotor with a synchronous excitation and operating on a super-critical speed, it will be unstable. However, for the rotor on the optimal configuration and on the same conditions of excitation and rotation, it will not be unstable.

Table 3. Optimal and initial configurations of the rotor.

	j_1	j_2	K_{xx} (MPa)	K_{zz} (MPa)	y_c (m)
Initial configuration (carbon/epoxy and glass/epoxy)	60.0º	60.0º	1.10^7	1.10^7	0.333
Optimal configuration (carbon/epoxy)	43.7º	43.7º	3.10^6	8.10^7	0.4
Optimal configuration (glass/epoxy)	44.07º	44.07º	3.10^6	8.10^7	0.4

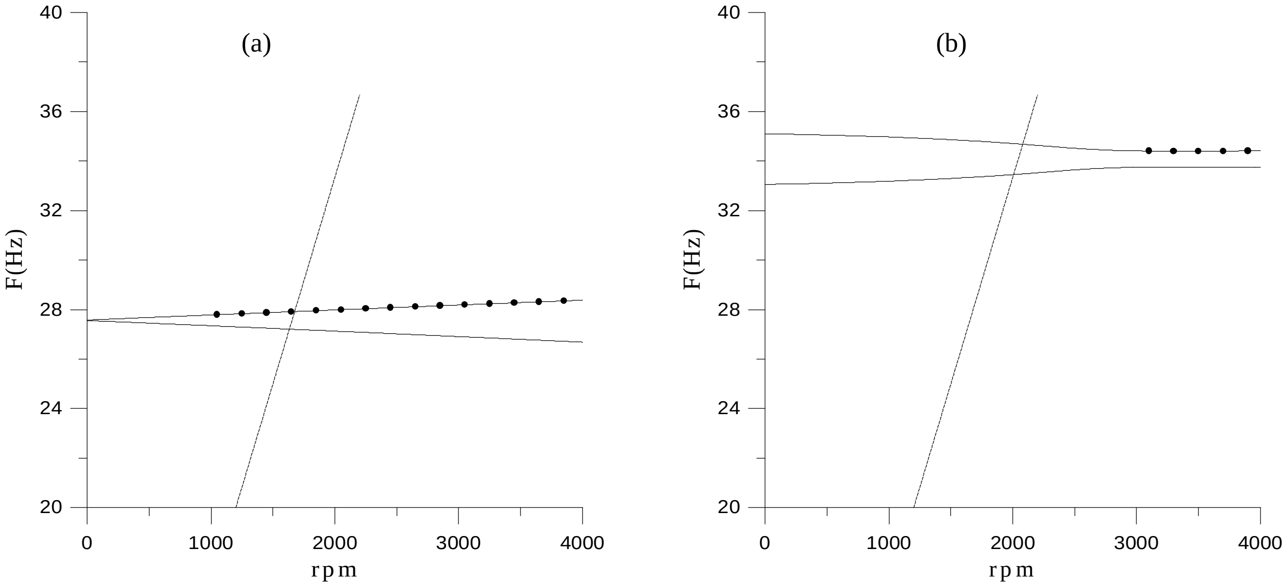

Figure 6. Instability regions for a rotor in carbon/epoxy, (a) initial configuration; (b) optimal configuration.

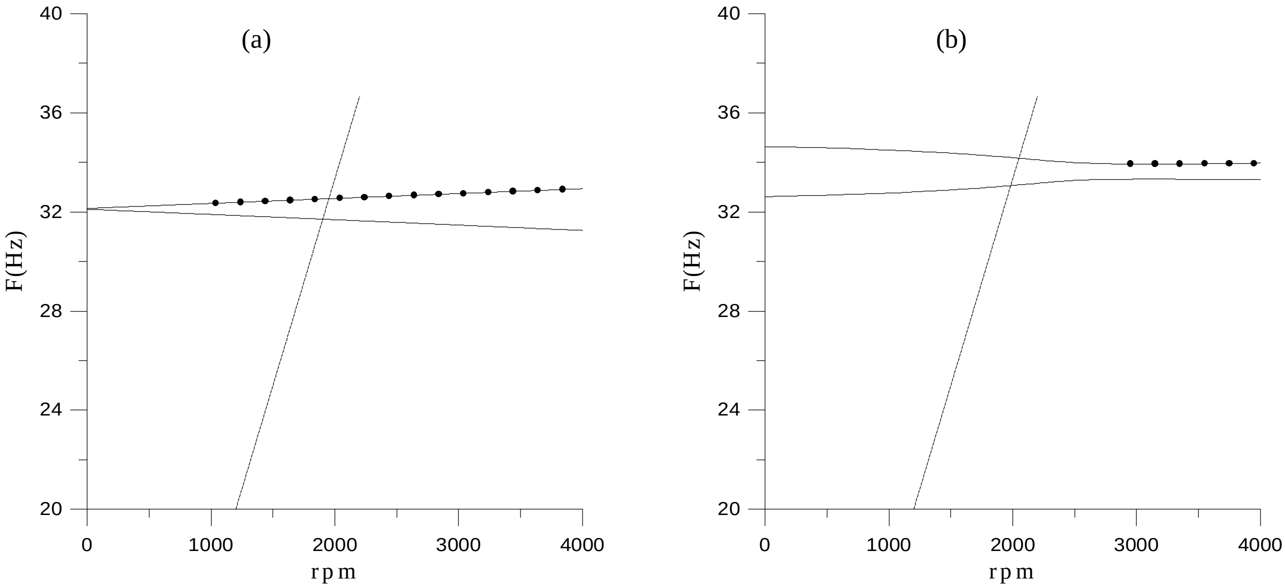

Figure 7. Instability regions for a rotor in glass/epoxy, (a) initial configuration; (b) optimal configuration.

7. CONCLUSION

In this work it was investigated the behavior of carbon/epoxy and glass/epoxy rotors considering the stability of the system. It can be observed that as higher is the wounding angle, higher is the internal damping introduced by composite material, and consequently, higher is the instability regions. Thus, the quantity of the external damping necessary to avoid the instability on the cirtical speeds can be changed according to the wounding angle, i.e., according to the quantity of internal damping introduced by wounding shaft. In the same manner, the presence of anisotropic bearings trends to reduce the instability regions, however it will depend of the woulding angle. In the case of the presence of bearings with coupled terms, the internal damping trends to stabilize until the first critical speeds. We can conclude that the internal damping offered by the composite materials to the rotors should be used with care if we consider the stability, nevertheless, the internal damping associated with others parameters can be explored in rotordynamics analysis in order to search the optimal design.

8. REFERENCES

Adams, R. D., and Bacon, D. G. C., 1973, "Effect of Fibre Orientation and Laminated Geometry on the Dynamic Properties of CFRP," Journal of Composite Materials, Oct., Vol. 7, pp. 402-428.

Arora, J. S., 1989, Introduction to Optimum Design, McGraw-Hill International Editions.

Gupta, K. and Singh, S. P., 1998, "Damping Measurements in Fiber Reinforced Composite Rotor." Journal of Sound and Vibration, 211(3), pp. 513-520.

Ku, D. -M., 1998, "Finite Element Analysis of Whirl Speeds for Rotor-Bearing Systems with Internal Damping." Mechanical Systems and Signal Processing, Vol.12, No. 5, pp. 599-610.

Lalanne, M. and Ferrraris, G., 1998, Rotordynamics Prediction in Engineering, 2nd Edition, John Wiley and Sons.

Melanson, J. and Zu, J. W., 1998, "Free Vibration and Stability Analysis of Internally Damped Rotating Shafts with General Boundary Conditions." Journal of Vibrations and Acoustics, Vol.120, Jully, pp. 776-783.

Özgüven, H. N., Özkan Z. L., 1984, "Whril Speeds and Unbalance Response of Multibearing Rotors Using Finite Elements," Journal of Vibration, Acoustics, Stress, and Reliability in Design, Vol. 106, January.

Silveira, M. E., 2001, Análise do Comportamento Dinâmico de Rotores em Eixos Bobinados, Universidade Federal de Santa Catarina, Departamento de Engenharia Mecânica, Dissertação de Mestrado, Florianópolis.

Singh, S. P. and Gupta, K., 1994, "Free Damped Flexural Vibration Analysis of Composite Cylindrical Tubes Using Beam and Shell Theories," Journal of Sound and Vibration, 172(2), pp. 171-190.

Taylor, A. G. and Craggs, A., 1995, "The Effect of Damping on the Stability of a Finite Element Model of a Flexible Non-axisymmetric Rotor on Tilting Pad Bearings." Journal of Sound and Vibration, Vol.185, No. 5, pp. 809-825.

Tsai, S. W. and Hahn, H. T., 1980, Indroduction to Composite Materials, Technomic Publishing Company, Inc.

Vance, J. M., 1988, Rotordynamics of Turbomachinery. John Wiley and Sons.

Wettergren, H., 1996, Rotordynamics Analysis with Special Reference to Composite Rotors and Internal Damping, Linköping University, Dept. of Mechanical Engineering, doctor’s degree thesis, Sweden.

Zorzi, E. S. and Nelson, H. D., McVaugh, J. M., 1977, "Finite Element Simulation of Rotor-Bearing Systems with Internal Damping," Journal of Engineering for Power, January, pp 71 - 76.