II CONGRESSO NACIONAL DE ENGENHARIA MECÂNICA
II NATIONAL CONGRESS OF MECHANICAL ENGINEERING
12 a 16 de Agosto de 2002 - João Pessoa – PB

COMPARING ELECTRONIC AND NUMERICAL METHODS FOR THE INTEGRATION OF RANDOM SIGNALS IN ACCELEROMETRY

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Summary: *The main objective of this work is to review the Fourier Transform based numerical integration method applied to acceleration signals to obtain velocity signals used in vibration analysis of mechanical equipment. The acceleration signals are acquired from piezoelectric sensors positioned at the equipment. These signals are random continuous functions that are digitized by a data acquisition system generating a vector of discrete values. Several well established numerical integration schemes (Trapezoidal, Runge-Kutta, Open and Closed integration formulas) were tested and they failed since initial condition is not always available when random signals is being integrated . The Fourier Transform method proved to be the most adequate to integrate random signals since due its batch processing characteristic, it does not require initial conditions. The testing were conducted using inputs obtained from a high precision electronic function generator as well as real acceleration and velocity signals obtained experimentally in a rotating machinery fault simulator. The integration was carried out using both a high quality integrator amplifier as well as numerically.*

Keywords: :*methods of integration, acceleration signals, velocity signals, vibration*

1. Introduction

In a previous article, (Ting and Padovese, 2001) the authors presented a comparison among several numerical integration schemes applied to signals in accelerometry. The objective of that work was to assess a not commonly used method to obtain the velocity signal from the integration of acceleration signals based on the Fourier Transform and compare it against classical schemes, namely: Trapezoidal, Runge Kutta, Open and Closed Integration Formulas (Carnahan et all, 1969 and Pearson, 1974). The comparisons were carried out using analytically generated deterministic waveforms with well defined initial conditions (sine, cosine, squares, triangles and composition of them) and their analytical integrated forms. Noise was also analytically added.

The main conclusions both in the time domain as well as in the frequency domain are that all integration methods performed well provided the initial condition is known but the one based on the Fourier Transform required the lowest sampling frequency (near Nyquist frequency) to reach a given precision while the others required much larger sampling frequencies. Also, the Fourier Transform integration method proved to be more robust against random noise addition.

The monitoring and diagnosis of industrial equipment and systems using vibration analysis requires generally the integration of acceleration signals sampled from accelerometers to generate velocity signals. The industrial standards and common practice indicate the velocity as the most appropriate variable to be used in vibration analysis for a wide range of applications. Sensors which generate directly signals that are proportional to the velocity are rarely used since they are difficult to fabricate and to use. The most commonly used sensors industry wise are based on piezoelectric elements actuated by inertial mass generating acceleration signals of the measured point.

Obtaining velocity from acceleration signals can be done either by electronic integration or by numerical digital integration. Electronic integration is performed during the data acquisition before the signal conversion to digital format, by electronic RC type circuitry contained in the signal conditioning equipment. This technique is well consolidated and present adequate results requiring however conditioners which time response are compatible with the sample rate and the desired frequency range.

The objective of this present work is to expand the validation of the Fourier Transform method for integrating acceleration signals by comparing it against integration performed by well established industrial electronic analog integrator amplifiers. Two classes of signals were employed:

a) Analog waveforms inputs originated in a high precision function generator
b) Real acceleration signals acquired from a rotating machinery fault simulator

This work discuss the general problem of integrating signals and particularly, it address the numerical integration problem of both deterministic as well as of random signals. The influence of some parameters like the number of samples and the sampling frequency upon the velocity signal amplitudes and frequencies are also studied. The results shows that in the case of random signals when the initial condition for velocity is not known, the Fourier Integration method is a good alternative for electronic integration amplifiers.

2. Numerical Integration Formulas

Well established integration formulas are presented below without derivation, being the reader advised to reference numerical methods text books such as the one by Carnahan (1969) for details. Another method based on a Fourier transform property which can be used to integrate time dependent functions is also presented. The general problem statement is described as follows.

Given a time dependent continuous function *a(t)* we wish to calculate its integral *v(t*) point wise for $v(0)=v_0$ for:

$$v(t) = \int_0^t a(t)dt \tag{1}$$

For digitized signals, the vector a_i (i=1,2...N) obtained from accelerometers, which is the case being studied, the function is random and continuous, representing the oscillatory vibration at the measured point. This peculiar characteristic of the function being integrated presents some special challenge to the problem.

It is presented below the integration rules for the four numerical methods. In these formulas, i=1,2....N, where N is the total number of sampled points and Δt is the time interval between two sampled points and equal to the inverse of the sampling frequency.

a) The Trapezoidal Rule (or the Euler Method) is calculated by:

$$v_{i+1} = v_i + 0.5*\Delta t\,(a_{i+1} + a_i) \tag{2}$$

b) The Runge-Kutta fourth order method with Kutta’s coefficients is:

$$v_{i+1} = v_i + \Delta t(a_{i+1} + 4*a_i + a_{i-1})/3 \tag{3}$$

One should notice that in this case where the function to be integrated is time dependent only, this formula reduces to Simpson's rule. Although this formula is included, its results are not presented since it presents similar results compared to Trapezoidal rule.

c) The Open Integration Formula with order of the interpolating polynomial equal to 3:

$$v_{i+1} = v_i + \Delta t(55*a_i - 59*a_{i-1} + 37*a_{i-2} - 9*a_{i-3})/24 \quad (4)$$

d) The Closed Integration Formula with order of the interpolating polynomial equal to 3:

$$v_{i+1} = v_i + \Delta t(9*a_{i+1} + 19*a_i - 5*a_{i-1} + a_{i-2})/24 \quad (5)$$

It is important to notice that the methods presented above require the specification of the velocity initial condition v_0 which in the case of integrating deterministic functions it can be well defined. However, in the case of random functions, the initial condition is not well defined making the integration ill posed.

3. Integration using a Fourier Transform property.

Given the definition of the Fourier Transform of a time dependent function *a(t)* as :

$$A(w) = \int_{-\infty}^{+\infty} a(t)e^{-jwt}\,dt \quad (6)$$

where w is the frequency domain and j is the imaginary unit number, we wish to calculate *v(t)* from *A(w)*. One can demonstrate by integrating Eq. (6) by parts, after substituting *a(t)=dv/dt* that the following equation is true:

$$A(w) = jwV(w) = \int_{-\infty}^{+\infty} \frac{dv}{dt}e^{-jwt}\,dt \quad (7)$$

In other words, to calculate *v(t)* from the Fourier transform *A(w)* of *a(t)*, the following steps will produce the desired result:

a) Calculate the Fourier transform of *a(t)*, generating *A(w)*.
b) Calculate the velocity in the frequency domain *V(w)* using the first part of Eq. (7).
c) Calculate the inverse Fourier transform of *V(w)* to obtain *v(t).*

Two additional conditions must be satisfied in order to use this scheme which are:

d) The signals should be zero mean or centered. To accomplish this one need to subtract the mean value of the signal.
e) When applying Equation 7, a singular condition is reached when dividing *A(w)* by *w* at *w*=0. This can be avoided by using the L'Hopital theorem, since *A(0)* and *w* are both null.

4. Integrating random functions.

In order to compare the performance of the different integration methods presented above, we studied the integration of two classes of signals :

a) Analog waveforms inputs originated in a high precision function generator
b) Real acceleration signals acquired from a rotating machinery fault simulator

In both cases, the signals were integrated using both a electronic integrator amplifier and the numerical integration methods and the results compared.

In the first case, in order to get analog waveforms, a HP 33120A high precision, calibrated function generator was used. The generated function was then input to two B&K Nexus 2692 A 0I1 charge amplifier with integration function included to generate in one of them the acceleration signal already filtered and amplified and in the second one the same signal which is integrated. The two outputs, one representing the acceleration and the other the velocity signals are then fed to a National Instruments DAQ AI-16E-4 card which digitalize them and send to a computer with LABVIEW data acquisition software. MATLAB was used afterwards to implement and run the algorithms with the numerical methods present above. The Figure 1 shows the block diagram of this setup. The Figure 2 is the photo of the setup.

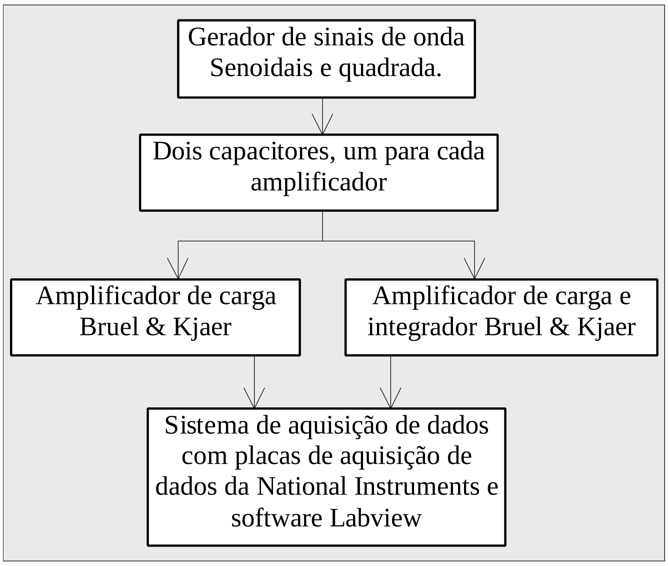

Figure 1. Block Diagram of the Experimental Setup

Figure 2. Picture of the Experimental Setup

Two types of waveforms were generated: sinusoidal and square waveforms with different amplitudes and frequencies. The amplitude range was defined in such a manner to have the best signal/noise ratio. To simulate a real condition, the initial time for sampling was not automatically triggered being therefore the velocity initial condition unknown introducing a randomness in a originally deterministic function.

A important result and perhaps the most important conclusion of this paper is that it was impossible to adjust a initial condition to be used in the classical stepwise type numerical integration schemes. The use of improper initial condition introduces offset values which generates unacceptable errors. In view of this, Trapezoidal, Runge-Kutta, Open and closed integration formulas were discarded. The Fourier Transform method being a batch type method since it processes the entire data set independently does not require a initial condition. This method proved to be applicable in this condition.

To illustrate this part of the study, two figures were included. Figure 3 shows the integrated velocity signals for both the analog electronic technique as well as for the Fourier Transform method. The simulated acceleration signal is sinusoidal with 10Hz frequency sampled at 200Hz. The comparison is shown in the time domain as well as in the frequency domain. From figure 3, one can notice that both the waveform and the spectrum agree well with a deviation of about 10% in amplitudes.

The several cases studied revealed that the sampling rate is a important parameter to set the deviation between the two methods. In figure 4, the same 10Hz sinusoidal signal and a 10Hz square waveform are analyzed for different sampling rates and the percentage deviation of the spectrum amplitude at the 10Hz peak is presented. Starting from sampling frequencies near twice the Nyquist frequency where the deviation is large (90%) one can notice that the deviation drops exponentially with increasing sampling frequency to values around 10% at around 200Hz. The results of figure 4 lead us to the following:

a) The obtain acceptable precision in the integrated function amplitude in both the time as well as the frequency domain, a sampling frequency of about 10 times de Nyquist frequency is needed.
b) To obtain a acceptable precision in the frequency values the same criteria used for avoiding aliasing is sufficient, i.e. the sampling frequency should be greater than 2 times the Nyquist frequency (Shannon theorem).

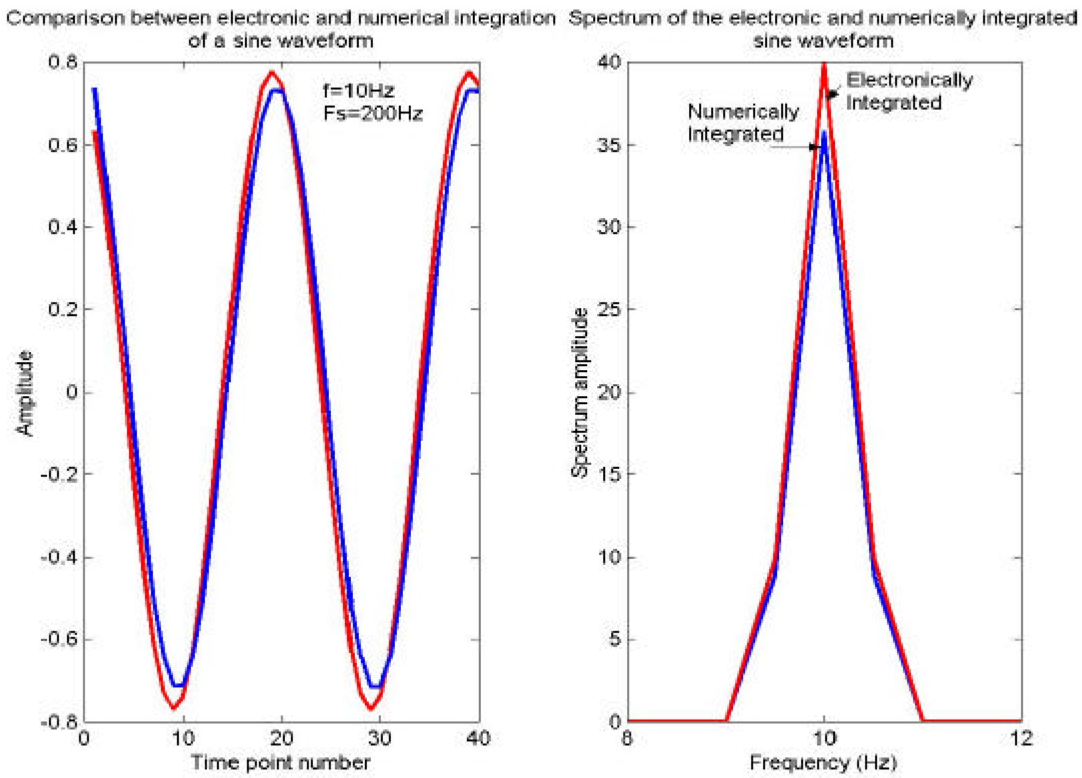

Figure 3. Comparison between electronic integration and numerical integration.

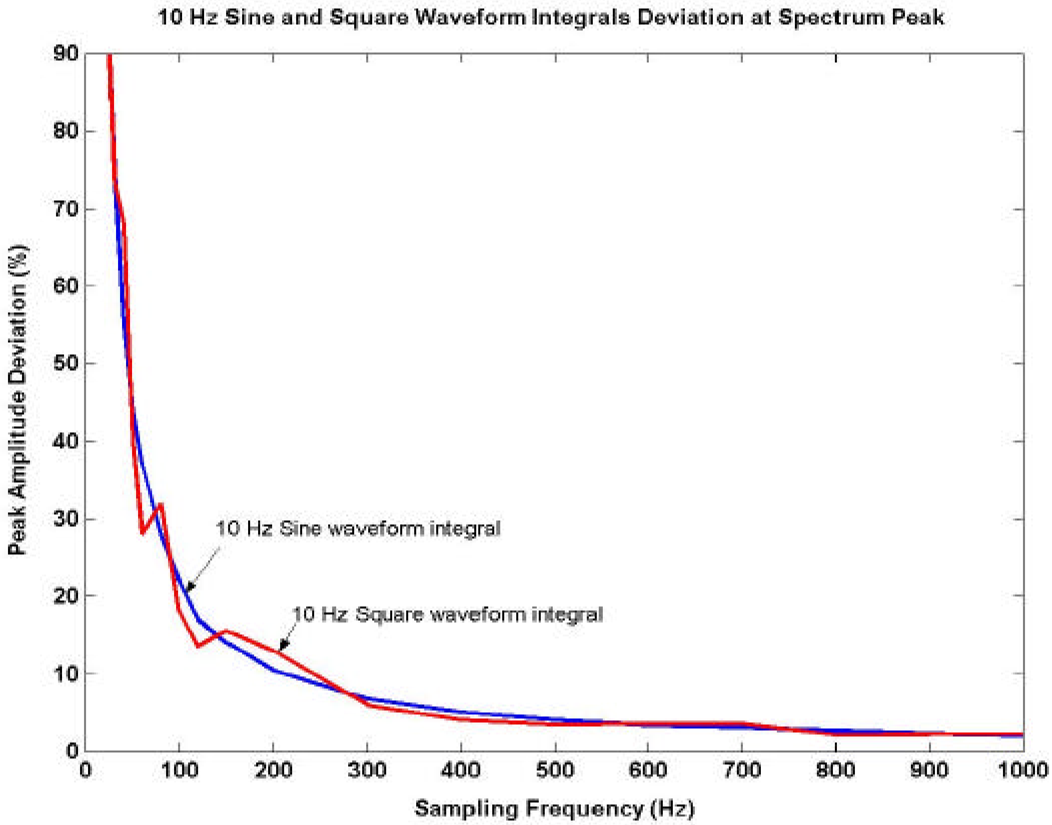

Figure 4. Deviation at the peak spectrum amplitude between numerical and electronic integration as function of sampling frequency.

5. Integration of a actual vibration signal.

To further extend the validation on the use of the Fourier Transform integration method for random signals, we acquired acceleration signals from a rotating machinery fault simulator (Spectra Quest MFSv2.0). This simulator has a electric motor which speed is controlled by a frequency inverter. The rotation is transferred to a shaft through a flexible coupling. The shaft is assembled on two rolling bearing closed case with a movable rotor in between. This experimental setup allows for the controlled simulations of unbalanced rotors, misalignments, defective bearings, soft base and resonance conditions. The simulator is instrumented with eight ICP type accelerometers allowing for additional instrumentation as necessary. A range of different speed up to 10000 rpm is possible.

The results for a particular case of a unbalanced shaft is shown in this paper. Two B&K 7701A-100 charge type accelerometer were installed on the bearings casing to measure the vibration. The motor is set to run at 600 rpm (10Hz). The signals from the accelerometers are conditioned by two B&K Nexus charge amplifiers being one of them set to integrate the acceleration signal to velocity. The data is acquired using LABVIEW and the processing is done afterwards with MATLAB. The experimental setup can be seen in the picture of the Figure 2 and in block diagram of the Figure 1.

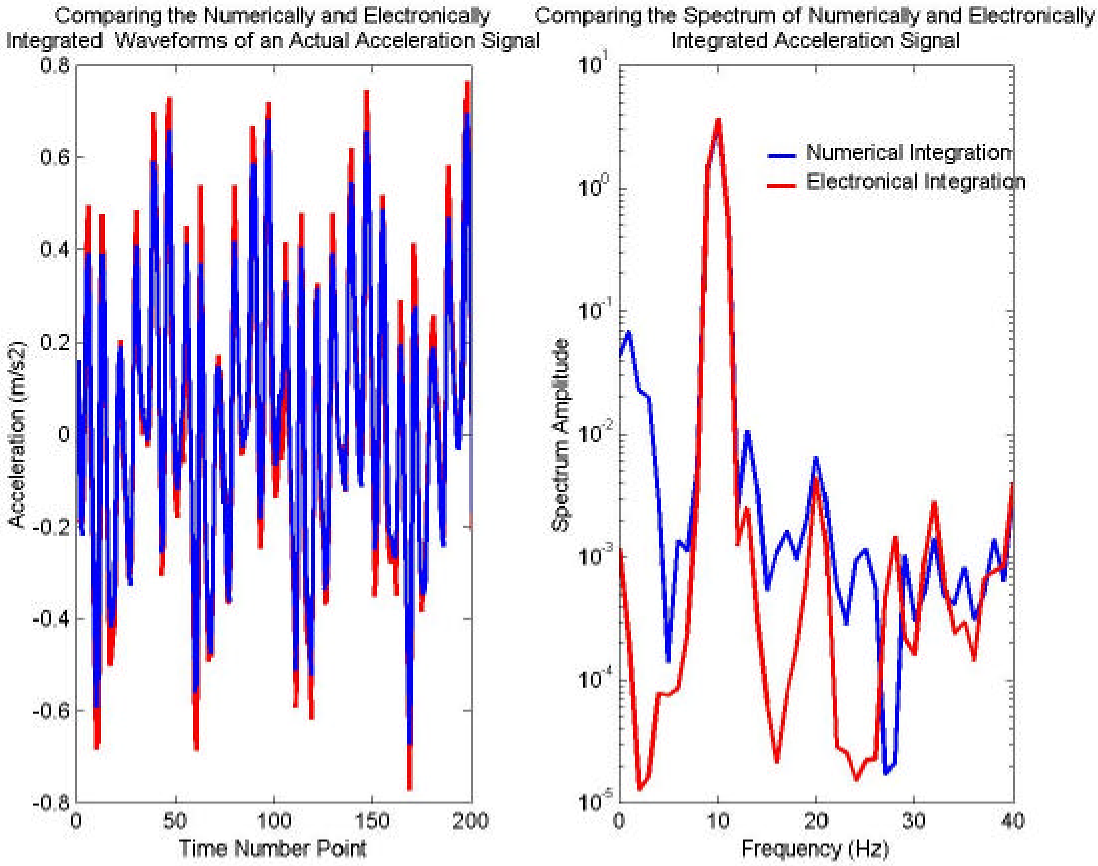

Figure 5. Comparison between electronic and numerical integration of a actual acceleration signal

In the right hand side of Figure 5 it is shown the velocity waveforms obtained both by electronic integration (red line) as well as by numerical integration (blue line) of the acceleration produced by a unbalanced rotor. It is well known that in this type of fault the acceleration and the velocity show a sinusoidal behavior with the same frequency as the rotation speed, i.e. 10 Hz in this particular case. Both wave forms agreed well as far as shape goes with a deviation of about 10% at the extreme points of the sine. At the right hand side of Figure 5 the power spectrum density of both signals are plotted. Again, at the 10 Hz frequency the agreement is quite good with around 10% deviation. The only noticeable difference shows up at very low frequencies near DC components. The low frequency gain in integration problems is well known because of the division by the frequency which for small values greatly amplifies any noise which is present. In this case, the B&K amplifier has already filtered out this noise with a high pass filter at around 1 Hz.

6. Conclusions

a) The most important conclusion obtained in this paper is that classical stepwise integration formulas like the Trapezoidal, Runge Kutta and other schemes fail to integrate random signals where the initial condition is not known and that the Fourier Transform integration methods produced correct integration in this situation provided the random signal is centered.

b) In using the Fourier Transform method for integrating zero mean random signals, the sampling rate of the original signal play the foremost important part. The results obtained indicate that the sampling rate to reach acceptable precision in frequency determination is the same as the Shannon theorem indicates, i.e. greater than 2 times the Nyquist frequency. However, to reach good precision in the amplitude calculation, a sampling frequency of about 10 times the Nyquist frequency is required.

c) The low frequency gain present in all signal integration problems is unavoidable and a external pos processing filter is necessary to remove high amplitudes near the DC components.
d) The Fourier Transform Integration Method proved to be a robust and efficient integration scheme and our future works will be dedicated to improve the range of the validations as well as to implement in a computer routine and in firmware processed via hardware.

7. References.

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