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MODAL IDENTIFICATION OF AN AERONAUTICAL STRUCTURE VIA THE EIGENSYSTEM REALIZATION ALGORITHM

Valdinei Sczibor
sczibor@sc.usp.br

Flávio Donizeti Marques
fmarques@sc.usp.br

Aeroelasticity, Flight Dynamics and Control Laboratory
School of Engineering of São Carlos – USP.
Av. Trabalhador Sancarlense, 400.
13566-590 – São Carlos, SP.

Abstract. *Traditionally, most aerospace structural system must be subjected to some form of modal verification prior to flight in order to ensure that the aircraft is free from any dangerous aeroelastic instability phenomena. The verification procedure often includes the experimental identification of structural characteristics such as the natural frequency and normal modes using modal testing. This paper presents a ground vibration testing (GVT) of a metallic wing of the Neiva Regente aircraft in order to assess the frequency response functions. The basic identification method used for this study is the Eigensystem Realization Algorithm (ERA). The essence of this method is based on a singular value decomposition of a matrix which contains information about the input/output relationship of the system. This matrix is related to the Markov parameters of the system. A minimal order realization of the system is determined as a function of the number significant singular values given by such decomposition. Because of nonlinearities and numerous local modes, modal identification of aircraft wing has proved to be surprisingly difficult.*

Keywords: *Aeroelasticity, GVT, ERA, Structural Dynamics.*

1. INTRODUCTION

Aircraft have been in constant development in order to become lighter, strengthener and more reliable. As consequence of such demand, aircraft structures have also become more flexible increasing the susceptibility to vibrations and aeroelastic instabilities. Aeroelastic instabilities can be disastrous and they are the result of coupling inertial, elastic and aerodynamic forces acting upon aircraft structures (Bisplinghoff *et al.*, 1955). Wing, panel and control surface flutter, buffeting, divergence and control reversal are typical aeroelastic phenomena in aeronautical engineering that must be avoided. Particularly for fixed- and rotary-wing aircraft, unwanted aeroelastic phenomena can raise from inappropriate structural dynamic features. Aircraft structures may also suffer from excessive vibration because of their structural flexibility, causing discomfort to the crew and fatigue problems. To avoid structural dynamic problems, aeronautical engineers must be able to attain the main dynamic characteristics, such as: natural/resonant frequencies and mode shapes. With those information it is possible to verify and re-design both structure and aerodynamics avoiding excessive vibration problems and suppressing unwanted aeroelastic instabilities.

Nonetheless, for complex structures, typical in aerospace industry, the attainment of dynamic characteristics is not an easy task. The finite element methods (FEM) have been the most successful tool for obtaining the dynamic features during preliminary design. Although the FEM has been consolidated in the aerospace industry, experimental verification of aircraft design is still necessary. Advances in acquisition system and in the field of sensors and/or actuators devices are also providing new possibilities to the aerospace structural dynamics. Kehoe and Freudinger (1993) and Abel (1997) present overviews on the most recent development in the main NASA research centers on aerospace structural dynamics. They have shown the main researches on this area, the modern techniques in use and highlight the vital role of modal parameters assessment, in particular, the so-called ground vibration testing, in assuring the prevention of aeroelastic instabilities.

The need for experiments to modal identification, i.e., the identification methods of modal parameters, is basically due to the complexity of typical aerospace structures. Accurate predictions of the structural modes is desired to understand the dynamic behaviour of the aircraft.

Modal identification methods may have several sources of errors. Even if errors from experimental procedures are minimized or eliminated, the errors related to both numerical methods and intrinsic identification algorithms limitations still are relevant. Another aspect that must be considered in the field of modal identification is the need for a complex experimental arrangements to achieve reliable data to proceed the identification. Novel methodologies that allow easy applicability at low cost are also desired (Tsunaki, 1999).

According to Juang (1994), experimental approaches for modal identification usually work with data in the form of free-decay vibration measurement, frequency response functions (FRFs), impulse response functions (IRFs), etc. Numerical algorithms have been also developed to calculate the modal parameters from the aforementioned experimental data.

The Eigensystem Realization Algorithm (ERA) is an identification method considered efficient and powerful, because it is capable to identify structures that present complex dynamic behaviour (Tsunaki, 1999). ERA has combined the algorithm of system realization of Ho-Kalman with singular value decomposition (SVD) to obtain the minimum system realization. Although the methods of minimum realization have been well known, the ERA was the first to propose the application of those methods to the identification of flexible structures. The utilization of SVD in ERA has allowed the assessment of system's order through the analysis of its singular values. Moreover, the mathematical formulation of ERA also allows the direct application of reliability coefficients to distinguish between computational and physical modes (Juang and Pappa, 1985).

This paper presents a modal identification process based on ERA. A brief mathematical description of the approach is given. The procedure proposed has been validated through data obtained from dynamic measurements accomplished in a steel beam. Moreover, experimental data from an aeronautical structure, the semi-span wing of the Neiva Regente aircraft has been used to illustrate the identification algorithm. Difficulties and assumptions adopted during the wing experimental analysis are also presented and discussed.

2. EIGENSYSTEM REALIZATION ALGORITHM APPROACH

A finite dimensional, discrete-time, linear, time-invariant dynamical system has the state-variable equations

$$x(k+1) = Ax(k) + Bu(k) \tag{1}$$

$$y(k) = Cx(k) \tag{2}$$

where x is a n-dimensional state vector, u a m-dimensional control input, and y a p-dimensional output or measurement vector. The integer k is the sample indicator. The transition matrix A characterizes the dynamics of the system. For flexible structures, it is a representation of mass, stiffness, and damping properties.

For the system Eqs. (1) and (2) with free pulse response, the time domain description is given by the function know, as the Markov parameter

$$Y(k) = C A^{k-1} B \tag{3}$$

or in the case of initial state response

$$Y(k) = C A^{k} \left[x_1(0), x_2(0), \ldots, x_m(0) \right] \tag{4}$$

where $x_i(0)$ represents the ith set of initial conditions and k is an integer. Note that B is an $n \times m$ matrix and C is a $p \times n$ matrix. The problem of system realization is: Given the functions $Y(k)$, construct constant matrices $\left[A, B, C \right]$ in terms of $Y(k)$ such that identities of Eq. (3) hold and the order of A is minimum.

The ERA approach, Juang and Pappa (1985), begins by forming the $r \times s$ block matrix (generalized Hankel matrix), that is:

$$H_{rs}(k-1) = \begin{bmatrix} Y(k) & Y(k+t_1) & \cdots & Y(k+t_{s-1}) \\ Y(j_1+k) & Y(j_1+k+t_1) & \cdots & Y(j_1+k+t_{s-1}) \\ \vdots & \vdots & \ddots & \vdots \\ Y(j_{r-1}+k) & Y(j_{r-1}+k+t_1) & \cdots & Y(j_{r-1}+k+t_{s-1}) \end{bmatrix} \tag{5}$$

where $j_i\,(i = 1,\ldots,r-1)$ and $t_i\,(i = 1,\ldots,s-1)$ are arbitrary integers. For the system with initial state response measurements, simply replace $H_{rs}(k-1)$ by $H_{rs}(k)$. Now observe that

$$H_{rs}(k) = V_r A^{K} W_s \tag{6}$$

in which

$$V_r = \begin{bmatrix} C \\ CA^{j_1} \\ \vdots \\ CA^{j_{r-1}} \end{bmatrix} \qquad W_s = \begin{bmatrix} B & A^{t_1} B & \cdots & A^{t_{s-1}} B \end{bmatrix} \tag{7}$$

where V_r and W_s are the observability and controllability matrices, respectively. Note that V_r and W_s are rectangular matrices with dimensions $rp \times n$ and $n \times ms$, respectively. Assume that there exists a matrix H^{+} satisfying the relation

$$W_s H^{+} V_r = I_n \tag{8}$$

where I_n is an identity matrix of order n. It will be shown that the matrix H^{+} plays a major role in deriving the ERA. Observe that, from Eqs. (7) and (8),

$$H_{rs}(0) H^{+} H_{rs}(0) = V_r W_s H^{+} V_r W_s = V_r W_s = H_{rs}(0) \tag{9}$$

The matrix H^{+} is thus the pseudoinverse of the matrix $H_{rs}(0)$ in a general sense.

The ERA process starts with the factorisation of the block data matrix Eq. (5), for k=1, using singular value decomposition:

$$H_{rs}(0) \;=\; R\,\Sigma\,S^{T} \tag{10}$$

where the columns of matrices R and S are orthonormal and Σ is a rectangular matrix, that is

$$\Sigma \;=\; \begin{bmatrix} \Sigma_n & 0 \\ 0 & 0 \end{bmatrix}$$

with $\Sigma_n = diag\left[\sigma_1, \sigma_2, \cdots, \sigma_i, \sigma_{i+1}, \cdots, \sigma_n\right]$ and monotonically non-increasing σ_i (i=1, 2, …, n), $\sigma_1 \geq \sigma_2 \geq \cdots \geq \sigma_i \geq \sigma_{i+1} \geq \cdots \sigma_n \geq 0$.

Next, let R_n and S_n be the matrices formed by the first n columns of R and S, respectively. Hence, the matrix $H(0)$ and its pseudoinverse become

$$H_{rs}(0) \;=\; R_n\,\Sigma_n\,S_n^{T}$$

where (11)

$$R_n^{T}\,R_n \;=\; I_n \;=\; S_n^{T}\,S_n$$

and

$$H^{+} \;=\; S_n\,\Sigma_n^{-1}\,R_n^{T} \tag{12}$$

Equation (12) can be readily proved by observing Eq. (9).

Defining 0_p as a null matrix of order p, I_p as an identity matrix of order p, $E_p^{T} = \left[I_p, 0_p, \cdots, 0_p \right]$, (where p is the number of outputs), and $E_m^{T} = \left[I_m, 0_m, \cdots, 0_m \right]$, (where m is the number of inputs). Using Eqs. (3), (6), (8), (11), and (12), a minimum order realization can be obtained as follows:

$$\begin{aligned}
Y(k+1) \;&=\; E_p^{T}\,H_{rs}(k)E_m \;=\; E_p^{T}\,V_r\,A^{k}\,W_s\,E_m \\
&=\; E_p^{T}\,V_r\left[W_s\,H^{+}\,V_r\right]A^{k}\left[W_s\,H^{+}\,V_r\right]W_s\,E_m \\
&=\; E_p^{T}\,H_{rs}(0)\left[S_n\,\Sigma_n^{-1}\,R_n^{T}\right]V_r\,A^{k}\,W_s\left[S_n\,\Sigma_n^{-1}\,R_n^{T}\right]H_{rs}(0)E_m \\
&=\; E_p^{T}\,H_{rs}(0)S_n\,\Sigma_n^{-1/2}\left[\,\Sigma_n^{-1/2}\,R_n^{T}\,H(1)S_n\,\Sigma_n^{-1/2}\,\right]^{k}\Sigma_n^{-1/2}\,R_n^{T}\,H(0)E_m \\
&=\; E_p^{T}\,R_n\,\Sigma_n^{1/2}\left[\Sigma_n^{-1/2}\,R_n^{T}\,H_{rs}(1)S_n\,\Sigma_n^{-1/2}\right]^{k}\Sigma_n^{1/2}\,S_n^{T}\,E_m
\end{aligned} \tag{13}$$

This is the basic formulation of realization for the ERA. The triplet

$$\begin{aligned}
\hat{A} \;&=\; \Sigma_n^{-1/2}\,R_n^{T}\,H(1)S_n\,\Sigma_n^{-1/2} \\
\hat{B} \;&=\; \Sigma_n^{1/2}\,S_n^{T}\,E_m \\
\hat{C} \;&=\; E_p^{T}\,R_n\,\Sigma_n^{1/2}
\end{aligned} \tag{14}$$

is a minimum realization. Here the quantities denoted with ^ mean estimated quantities to distinguish from the true quantities. The order of the matrix $\hat{A}$ is n which is the order of the system for sufficiently low-noise data. Due to measurement noise, nonlinearity, and computer roundoff, the block matrix $H_{rs}(k)$ will usually be of full rank which does not, in general, equal the true order of the system under test. It should not be aim to obtain a system realization which exactly reproduces the noisy sequence of data. A realization which produces a smoothed version of the sequence, and which closely represent the underlying linear dynamics of the system, is more desirable (Juang, 1994). In this context, several accuracy indicators have been investigated for quantitatively partitioning the realized model into pure (principal) and noise (perturbational) portions so that the noise portion can be disregarded (Juang and Pappa, 1986).

The realized discrete-time model represented by the matrices $\hat{A}$, $\hat{B}$, $\hat{C}$ and $\hat{D}$ can be transformed to the continuous-time model. The system frequencies and dampings may then be computed from the eigenvalues of the estimated continuous-time state matrix. The eigenvectors allow a transformation of the realization to modal space and hence the determination of the complex (or damped) mode shapes and the initial modal amplitudes (or modal participation factors) (Juang, 1994).

3. ALGORITHM VALIDATION WITH A BEAM STRUCTURE

The experimental test used to validate the proposed method has been proceeded with a steel beam. The experimental set-up used to the beam modal characterization is shown in Fig. (1). The procedure has been based on the measured FRFs assuming a single excitation point and one acceleration measurement point (single-input/single-output approach). The structure has been excited with a force-instrumented hammer, and frequency response functions has been calculated between the applied force and each of the 11 accelerometers positions. The FRFs measured in Fig. (2) have been obtained using impulsive excitation signal, exponential windows in the 0-750 Hz frequency range.

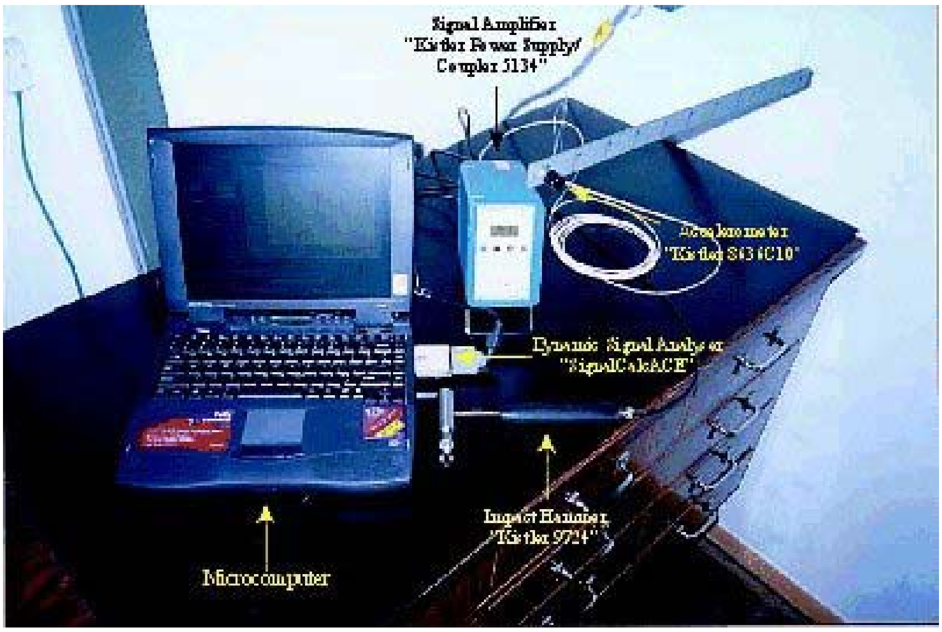


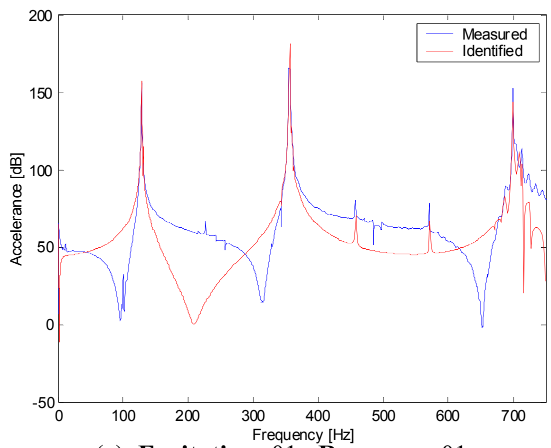
Figure 1. Experimental assembly

The identification procedure includes a search for the realization in a given interval, determined by the designer. The selected interval is used to span the number of significant singular values. Thus, a set for realization is estimated as a function of these singular values. Then, based on the sum of the absolute error between the measured and predicted impulsive responses, the designer selects the realization order of the system.

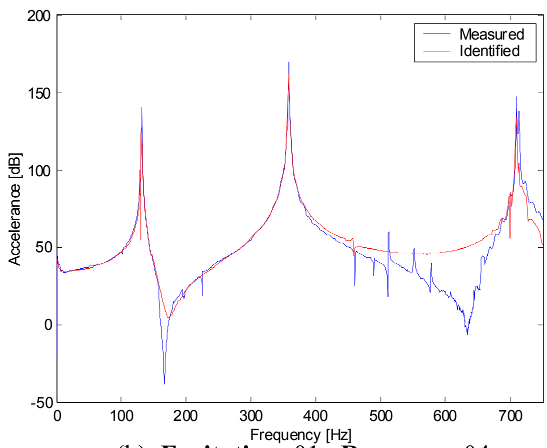
The identification algorithm consists of:

- compute the Hankel for r and s selected;
- performed the singular value decomposition;
- scan in a given model order interval, to evaluate the sum of the absolute error;
- select the order and identify the final model; and
- compare the frequency responses functions.

Figure (2) shows some the identification results for the ERA approach. The results clearly show the good agreement between experimental and identified model FRFs, despite of possible minor problems during measurements (accelerometer fixation and positioning, e.g.).



(a) **Excitation:** 01, **Response:** 01



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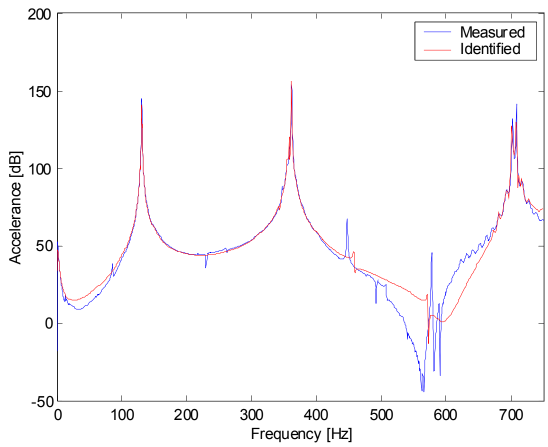
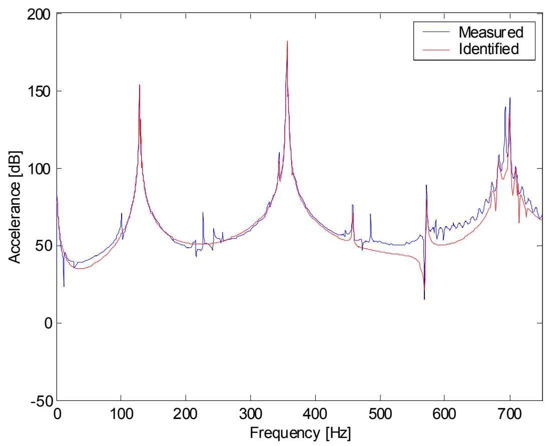


(c) **Excitation:** 01, **Response:** 07



(d) **Excitation:** 01, **Response:** 11

Figure 2. Comparison of ERA identification frequency response with measured data

4. IDENTIFICATION APPROACH APPLIED TO AN AERONAUTICAL STRUCTURE

Ground vibration testing (GVT) is performed on an aeronautical structure to identify its structural modes and their associated natural frequencies and dampings. This type of testing is an important part of the final aircraft design flight activity. The reasons for its important role in flight test include correlating and verifying the test modal data with dynamic finite-element models used to predict potential structural instabilities (such as flutter), assessing the significance of modifications to the aircraft structure by comparing the modal data before and after the modification, and helping to resolve in-flight anomalies. In this section are presented the necessary steps in a GVT, structure under testing, equipment setup, data acquisition, and frequency response analysis.

4.1. Structure Under Testing

Figure (3) shows the Neiva Regente aircraft. The airplane was manufactured in the 60's by the Brazilian Aeronautical Industry Neiva Ltda., Botucatu, SP. The construction is almost totally metallic comprising a semi-monocoque fuselage and semi-wings with simple mounts. Only a semi-wing (of conventional construction) has been taken to the experiments. The wing has been suspended through cables and springs in order to achieve a free-in-space boundary condition. The system to be identified and testing set-up, is shown in Fig. (4).

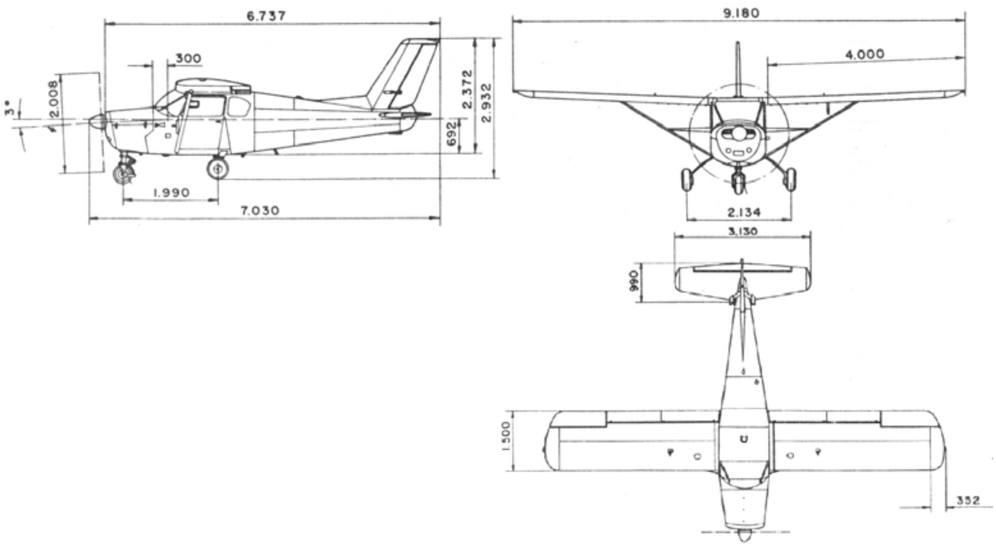


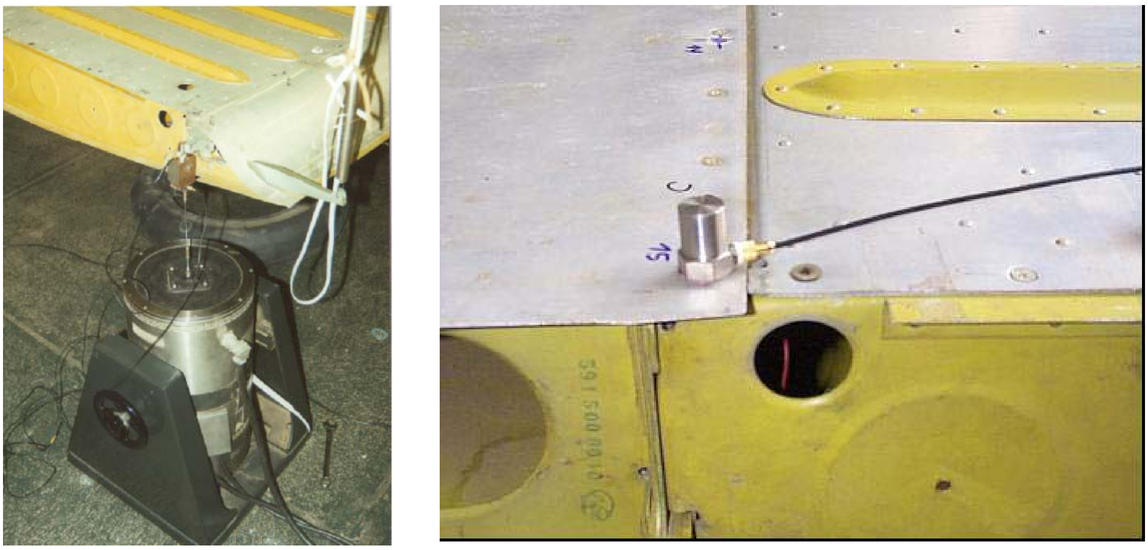
Figure 3. Neiva Regente aircraft (dimensions in meters)

Figure 4. Test environment

4.2. Experimental Set-up and Analysis

The experimental set-up used to the wing modal characterization is presented in this section. The GVT has been based on measured FRFs assuming a single excitation point and one acceleration measurement point (single-input/single-output approach).

Figure (5) shows the excitation and measure devices used in the modal testing. The structure under test has been driven by an electrodynamic shaker attached to the suspended wing by a flexible stinger. The structure has been excited with random and chirp excitation and FRFs have been calculated between the applied forces and the accelerometer. For each excitation position a FRF has been measured for each point scattered along the wing spars external surface. Figure (6) illustrates all the 30 measurement points used. The two excitations points used are: front and rear spar mountings. As shown in the Figure (6), the experimental tests used to the wing modal characterization do not present driving point. The input and output signals have been gathered by standard piezoelectric sensors. The force and acceleration measurements are in the perpendicular direction to the wing surface (y), which emphasizes the primary out-of-plane bending and torsional modes of the structure (Pappa *et al.*, 1997). The electrodynamic exciter used has been a B&K 4812 associated with a B&K 2707 power amplifier. The signals have been measured by a four channel 2630 Tektronix Spectrum Analyser. Input and output signals have been measured respectively by a Kistler 912 (13,3 *pC/N*) force transducer and a B&K 4383 (30,5 *pC/g*) accelerometer.



(a) Excitation point (shaker) (b) Measure point (accelerometer)

Figure 5. Excitation and measurement devices

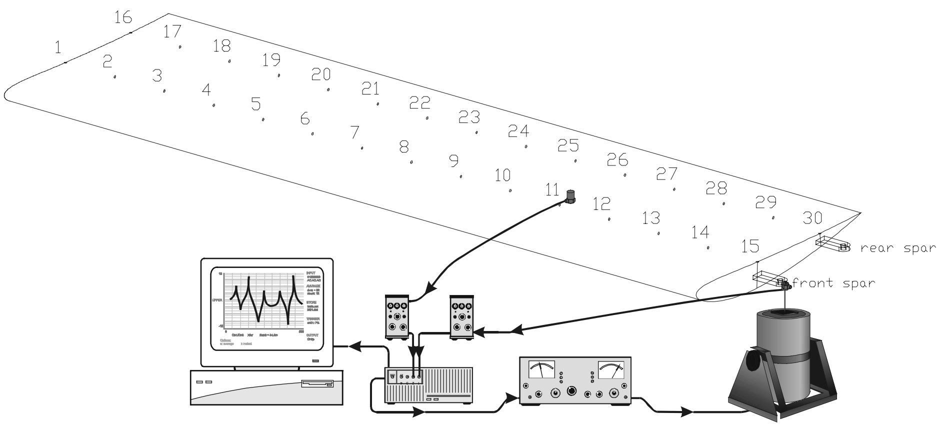
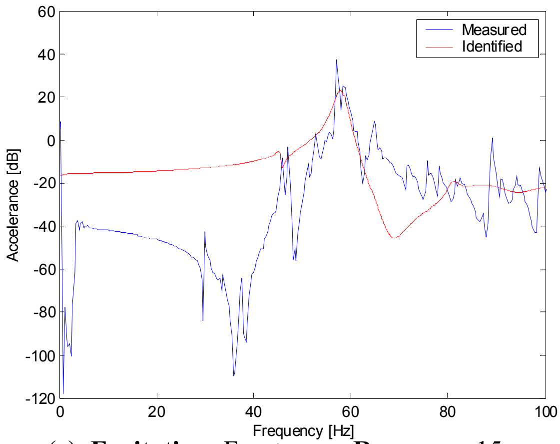


Figure 6. Test setup wing excitation/response locations

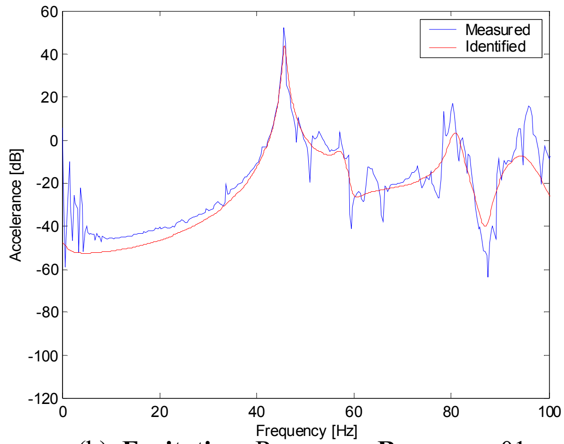
5. RESULTS AND DISCUSSION

The final aspect of the ERA method available for assessing identification accuracy is the process of data reconstruction. This procedure consists of comparing the original frequency response with those calculated using the ERA-reduced model. If the ERA modal decomposition process is performed accurately, the reconstruction results will closely match the original data. Figure (7) shows a typical comparison from the wing aircraft data analysis. The relatively noisy measured data in Fig. (7) is representative of most measurements obtained using *y*-direction excitation at the front and rear spars of the structure. Although have been used other excitation and windows forms, the FRFs measured in Fig. (7) have been obtained using chirp excitation signal, box-car windows in the 0-500 Hz frequency range.

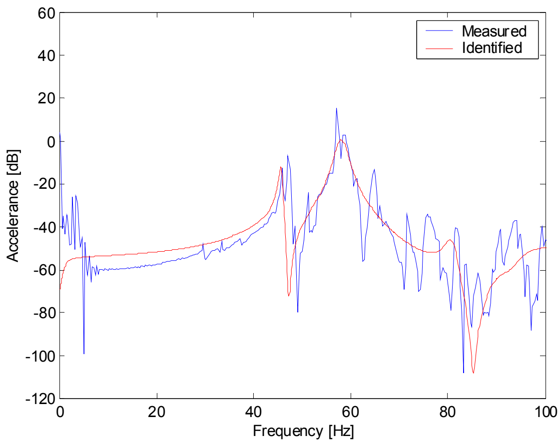
In modal-survey tests, identification difficulties arise primarily from high modal density, nonlinearity, weakly excited modes, local modes, nonstationarities, rattling, etc., not from instrumentation noise. The simultaneous effects of these conditions are in general impossible to include explicitly in reliable calculations. It has been observed a large amount of noise in the measured FRFs. This feature has been assigned by the influence of wing structure construction method (riveted plates and stringers). This problem is common in aeronautical structures and some time must be expent in searching for unreliably fixations.



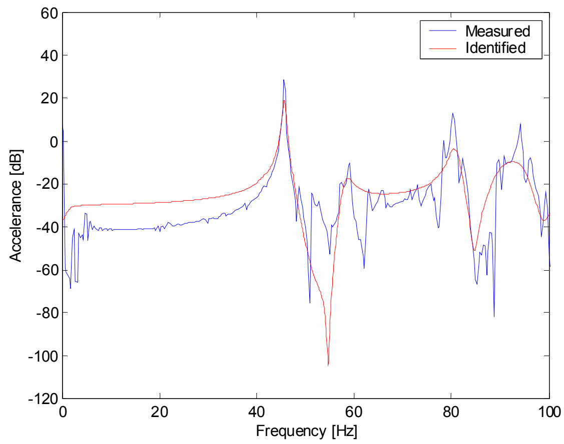
(a) **Excitation:** Front spar, **Response:** 15y



(b) **Excitation:** Rear spar, **Response:** 01y



(c) **Excitation:** Front spar, **Response:** 07y



(d) **Excitation:** Rear spar, **Response:** 25y

Figure 7. Comparison of ERA identification frequency response with measured data

6. CONCLUSIONS AND FUTURE WORK

This paper has presented the eigensystem realization algorithm for modal parameter identification and model reduction applied to a typical aeronautical structure. Firstly, the algorithm has been validated through data obtained from dynamic measurements accomplished in a steel beam. Secondly, an aeronautical structure, the semi-span wing of the Neiva Regente aircraft, has been used to illustrate the identification algorithm for a complex structure. The wing has been driven by an electrodynamic vibration exciter using random input signals and one accelerometer to collect the structural response at 30 points on the wing external surface. The FRFs have been measured on the wing span using standard piezoelectric sensors.

The results obtained in the algorithm validation through the modal test in the steel beam, show to be coherent in comparison with the original frequency response of those calculated using the ERA-reduced model despite of the noisy levels in the measured FRFs. The results indicate that the method is adequate to structures that present complex behaviour.

The modal identification of an aircraft wing has proved to be surprisingly difficult. Due to nonlinearities and numerous local modes achieved in the ground vibration test, the results of the comparison the original frequency response with those calculated using the ERA-reduced are not completely satisfactory.

For the future, it seems clear that most sophisticated methodologies must be used in order to overcome noisy measurements the undesired influence of localized modes to the whole structure dynamic characteristics. In this way, the use of new ground vibration test techniques and ERA variations, like accuracy indicators to quantitatively identify the system and noise modes, seems to be an adequate approach for further investigations to the wing structure used in this work.

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