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MODAL IDENTIFICATION FROM IMPERFECT EXPERIMENTAL DATA USING A MATRIX FACTORIZATION BASED ON THE SINGULAR VALUE DECOMPOSITION

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Abstract. *The flexible structures vibration control problem requires accurate models to reach acceptable closed loop performance. It appears as a challenging task to the scientific community due to contrast between the infinity number of vibration modes of the plant and the finite number of sensors and actuators used for vibration control. As a consequence of that, the modeling problem appears as the fundamental step in dealing with active control of vibrations in flexible structures. This paper presents a modeling procedure for flexible structures based on the Eigensystem Realization Algorithm (ERA). The ERA algorithm, which includes an inherent quantitative criterion to optimize model reduction from experimental data, is extended to modal identification of flexible structures. Numerical results are presented to illustrate the modeling procedure. A flexible structure computational model with 22 vibration modes was used to generate clean and noisy experimental data. Mode shapes and modal frequencies from the resulting model are then compared with the ones from the original data and the results are analyzed. Finally, it is shown that the technique delivers accurate results even in the presence of noisy data.*

Keywords*: Modal Identification, Model Reduction, Eigensystem Realization Algorithm.*

1. INTRODUCTION

Control of dynamic systems is a model-accuracy dependent task. This is the case of vibration control of flexible structures in which the model accuracy plays a fundamental role in the control loop performance. In order to assure high performance control operation, the model must include as much vibration modes as possible. In general, distributed parameter systems (DPS) have dynamic realizations on infinity-dimension spaces and there are no means to ensure that a finite-dimension controller can produce closed loop stability.

Modal models for control design purposes are developed either from physical laws, from finite element methods (FEM's) or from experimental data. Due to the usual complexity of flexible structures the direct use of physical laws becomes a strenuous task. Thus, the most widely used methods for flexible structures modeling are the finite element method (FEM) and modal identification from experimental data. However these methods usually produce models with residual modes that do not accurate represent the actual flexible structure and may cause performance deterioration of the controller.

Besides that, the placement of the sensors and actuators in the flexible structure affects the observability and controllability properties of the system. Also, the usually non co-collocated sensors and actuators cause observation and control spillovers on the control loop. Besides that, experimental data for modal identification usually suffers from measurement noise that makes difficult to distinguish information from noise. In any case, the design of the control system must be performed based on some reduced order model (ROM) of the plant.

Residual dynamics (dynamics not included in the ROM) causes undesired performance deterioration and even instability of the control loop. The controller design problem in the presence of unmodeled dynamics has been addressed by several authors in the last decades among them one can refer the pioneer work from Balas (1978). Up to the present there is no clean design procedure that ensures closed loop stability of ROM based designed controllers, thus, the Large Scale Systems (LSS) control problem remains a challenging research field.

The modeling engineers have proposed several interesting algorithms for model reduction and modal identification in the last decades. They claim having produced the state of the art in modal identification. In view of that this paper's author believes that there is need for further experimentation to surely conclude for the best algorithm. On the other hand, the engineering community has also overlooked some very powerful algorithms due to their relatively high computational requirements. With the current development of computational resources, those modeling procedures should be addressed in more detail.

This paper tries to partially fulfill this lack of experimentation and with this goal in mind it presents an application of the Eigensystem Realization Algorithm (ERA) originally proposed by Juang and Pappa (1985,1986). The ERA algorithm is applied to the modal identification and model reduction of a 44-order flexible structure model.

2. PROBLEM FORMULATION

This section shows just the very fundamental ideas of the Eigensystem Realization Algorithm (ERA) as proposed in Juang and Pappa (1985). In the following, less important derivation steps and some results have been omitted in order to save printed space.

Consider a state space realization for a linear time-invariant discrete-time dynamic system given by

$$\begin{aligned} x(k+1) &= A(\boldsymbol{q})\,x(k) + B(\boldsymbol{q})\,u(k) \\ y(k) &= C(\boldsymbol{q})\,x(k) + v(k) \end{aligned} \tag{1}$$

where [***A*, *B*, *C***] defines a discrete-time state space realization, ***x*** is a n-dimensional state vector, ***u*** an m-dimensional control input, ***y*** a p-dimensional measurement vector and ***v*** represents measurement noise.

the system impulse response sequence is given by

$$h(k) = \{\, h(0) \quad h(1) \quad h(2) \quad h(3) \quad . \quad . \quad . \quad . \quad . \,\} \tag{2}$$

the impulse response compact form can be written as

$$h(k+1) = y(k+1) = C\,A^k\,B \tag{3}$$

a Hankel matrix can be constructed from the impulse response sequence as

$$H(k) = \begin{bmatrix} h(k+1) & h(k+2) & h(k+3) & . & . & . \\ h(k+2) & h(k+3) & h(k+4) & . & . & . \\ h(k+3) & h(k+4) & h(k+5) & . & . & . \\ h(k+4) & h(k+5) & h(k+6) & . & . & . \\ . & . & . & . & . & . \\ . & . & . & . & . & . \\ . & . & . & . & . & . \\ . & . & . & . & . & . \\ . & . & . & . & . & . \end{bmatrix} \quad (4)$$

or

$$H(k) = \begin{bmatrix} CA^{k}B & CA^{k+1}B & CA^{k+2}B & . & . & . \\ CA^{k+1}B & CA^{k+2}B & CA^{k+3}B & . & . & . \\ CA^{k+2}B & CA^{k+3}B & CA^{k+4}B & . & . & . \\ CA^{k+3}B & CA^{k+4}B & CA^{k+5}B & . & . & . \\ . & . & . & . & . & . \\ . & . & . & . & . & . \\ . & . & . & . & . & . \\ . & . & . & . & . & . \\ . & . & . & . & . & . \end{bmatrix} \quad (5)$$

also

$$H(0) = \begin{bmatrix} CB & CA^{1}B & CA^{2}B & . & . & . \\ CA^{1}B & CA^{2}B & CA^{3}B & . & . & . \\ CA^{2}B & CA^{3}B & CA^{4}B & . & . & . \\ CA^{3}B & CA^{4}B & CA^{5}B & . & . & . \\ . & . & . & . & . & . \\ . & . & . & . & . & . \\ . & . & . & . & . & . \\ . & . & . & . & . & . \\ . & . & . & . & . & . \end{bmatrix} = \begin{bmatrix} \overline{C} & \overline{B} \end{bmatrix} \quad (6)$$

where $\overline{C}$ is known as the system observability matrix and $\overline{B}$ as the system controllability matrix.

$$\overline{C} = \begin{bmatrix} C \\ CA \\ CA^2 \\ . \\ . \\ CA^{n-1} \end{bmatrix} \quad \text{and} \quad \overline{B} = \begin{bmatrix} B & AB & A^2B & . & . & A^{n-1}B \end{bmatrix} \tag{7}$$

where
n = system order
m = number of system outputs
p = number of system inputs

From the singular value decomposition (SVD)

$$H(0) = M \Sigma N^T \tag{8}$$

$$H(0) = M \left[\begin{array}{c|c} D & 0 \\ \hline 0 & 0 \end{array}\right] N^T \tag{9}$$

$$H(0) = M \begin{bmatrix} I_n \\ \hline 0 \end{bmatrix} D \begin{bmatrix} I_n & | & 0 \end{bmatrix} N^T \tag{10}$$

then

$$H(0) = P D Q^T = \overline{C}\,\overline{B} \tag{11}$$

and

$$H^+ = N \Sigma^+ M^T = Q D^{-1} P^T \tag{12}$$

where

M = orthogonal constant matrix
N = orthogonal constant matrix
D = constant (diagonal) matrix
In = nxn identity matrix
P = orthogonal constant matrix
Q = orthogonal constant matrix

Usually, $H(0)$ is not a square matrix and $dim\big(H(0)\big) = np \times nm$ with $rank\big(H(0)\big) \leq n$

We know that, there exist constant matrices *Ep* and *Em* such that

$$h(k+1) = E_p^T \, H(k) E_m \tag{13}$$

and that

$$H(k) = \overline{C} A^k \overline{B} \tag{14}$$

and also that

$$\overline{C}\overline{B} = P D Q^T = H(0) \tag{15}$$

then

$$h(k+1) = E_p^T H(k) E_m = E_p^T \overline{C} A^k \overline{B} E_m \tag{16}$$

$$h(k+1) = \left[E_p^T\right]\left[PD\right]\left[D^{-1}P^T\right]\left[H(k)\right]\left[QD^{-1}\right]\left[DQ^T\right]\left[E_m\right] \tag{17}$$

hence

$$h(k+1) = \left[E_p^T PD^{1/2}\right]\left[D^{-1/2}P^T H(1) QD^{-1/2}\right]^k \left[D^{1/2}Q^T E_m\right] \tag{18}$$

finally, a minimal order realization is given by

$$\begin{aligned} C &= \left[E_p^T PD^{1/2}\right] \\ A &= \left[D^{-1/2}P^T H(1) QD^{-1/2}\right] \\ B &= \left[D^{1/2}Q^T E_m\right] \end{aligned} \tag{19}$$

3. EXPERIMENTAL RESULTS

Impulse response tests were applied to a 44-order model that was assumed to be the exact model of the plant. The ERA algorithm originally proposed by Juang and Pappa (1985) was then applied to the experimental results obtained from the full model (the virtual plant). And finally, using the ERA algorithm, model identification and reduction were performed to finally obtain a 20-order model of the original plant.

The 22-vibration modes flexible structure model, used to generate experimental data and to assess the algorithm performance, includes modal frequencies from 3.18 rd/sec. to 457.71 rd/sec.

The experiment was set to extract the first 10 modes from experimental data and compare the resulting model with the original one. Thus, the sampling frequency was chosen to be 1000 rd/sec (T_s = 0.001 sec.).

To apply the ERA algorithm, two sets of data were generated. The first one was the noise-free pulse response of the flexible structure and the second one was the pulse response with 10% white noise added. Figures (1), (2), (3) and (4) show the simulation results for noise-free experimental data. Figures (5), (6), (7) and (8) show the simulation results for noisy experimental data.

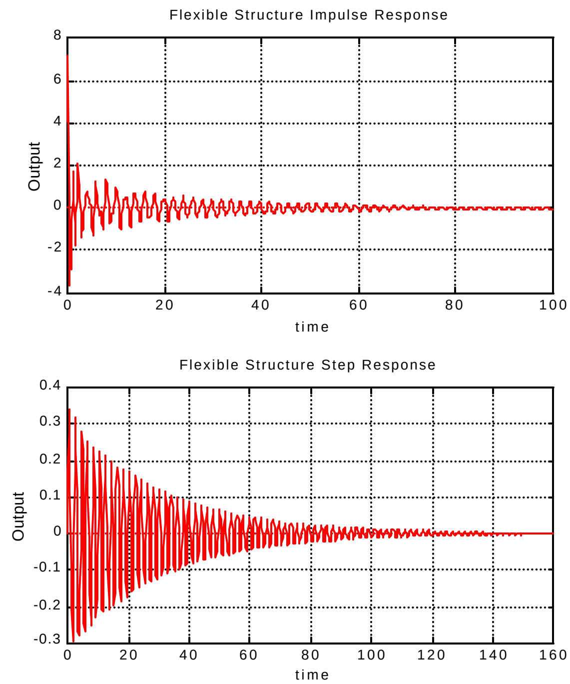

Figure 1. Plant Impulse and Step Responses (noise-free data).

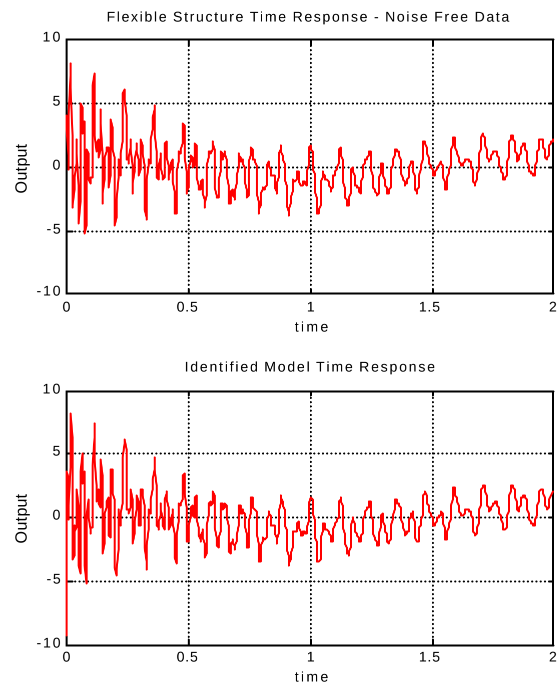

Figure 2. Time Responses of the Original and Identified Models (noise-free data).

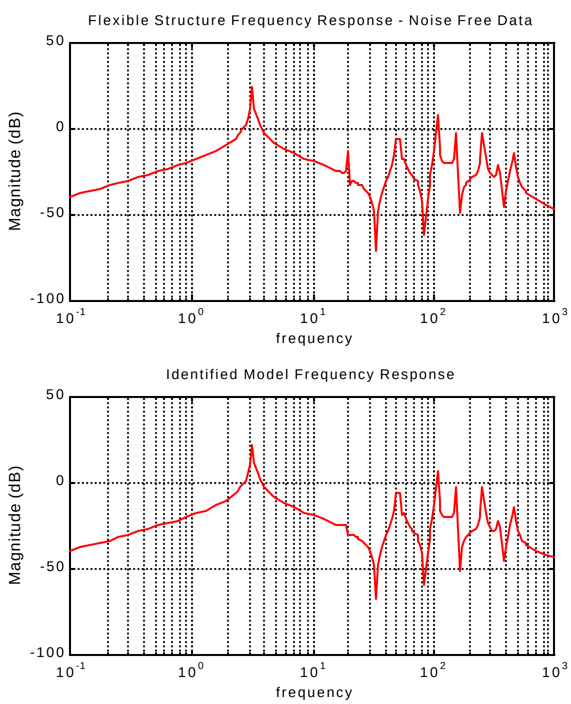

Figure 3. Frequency Response of the Original and Identified Models (noise-free data).

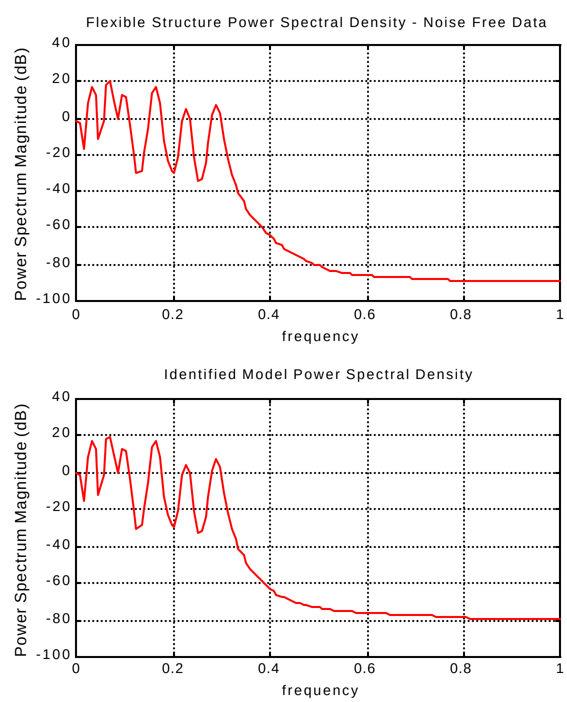

Figure 4. Power Spectral Density of the Original and Identified Models (noise-free data).

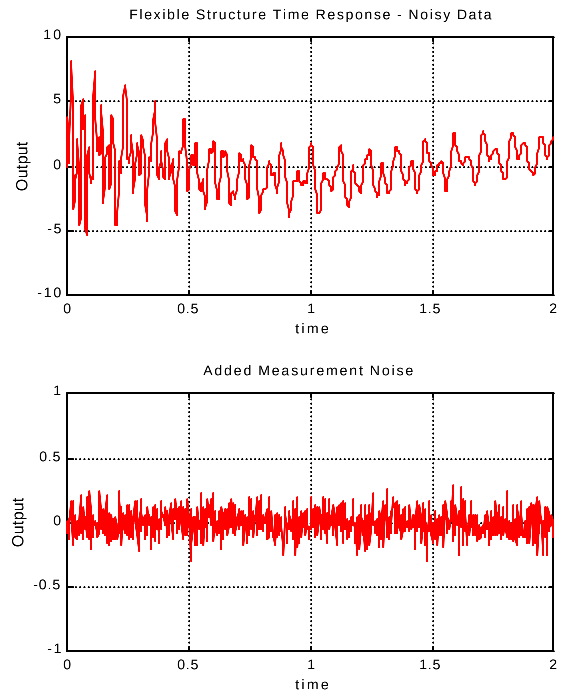

Figure 5. Plant Impulse Responses and Added Noise (noisy data).

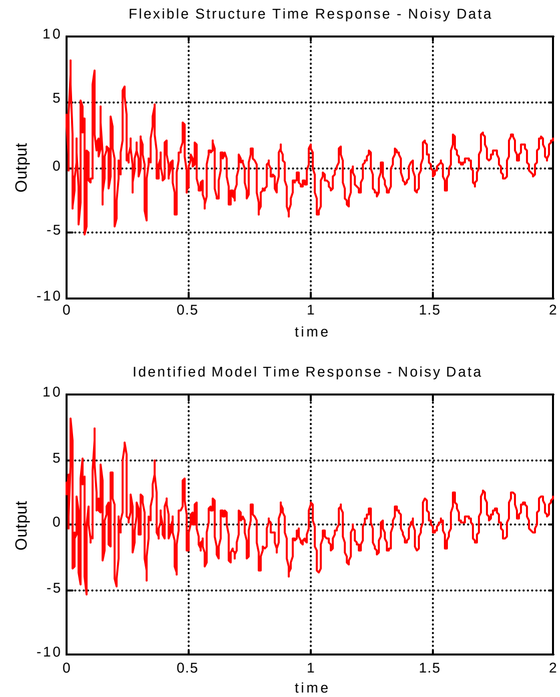

Figure 6. Time Responses of the Original and Identified Models (noisy data).

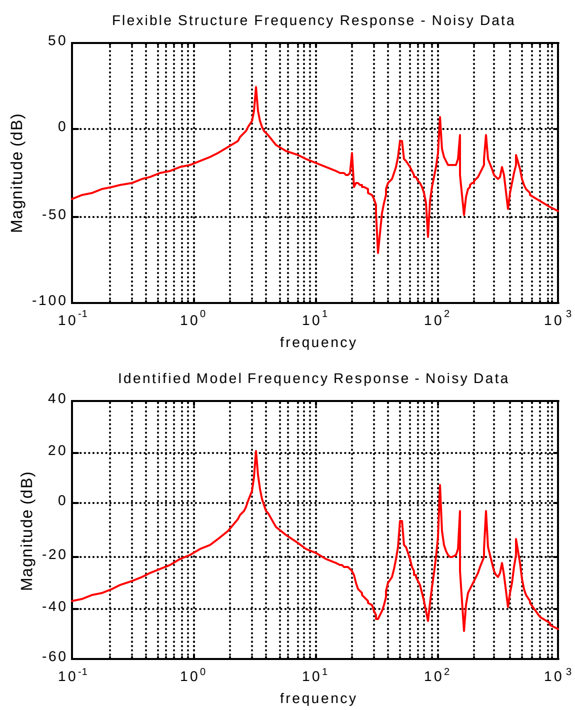

Figure 7. Frequency Response of the Original and Identified Models (noisy data).

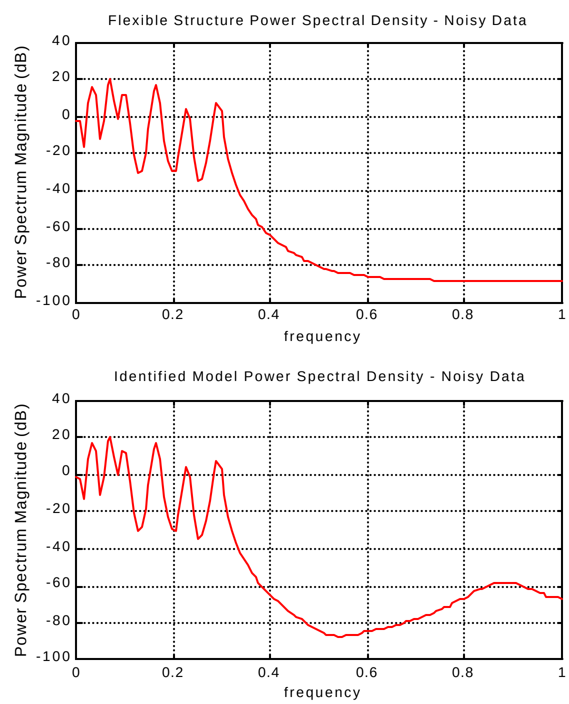

Figure 8. Power Spectral Density of the Original and Identified Models (noisy data).

4. FINAL COMMENTS

The simulation result shown that ERA algorithm performs quite well in the case of noise-free data and has a truly acceptable performance in the presence of noise. Besides that, Juang and Pappa (1986) have proposed two quantitative criteria to eliminate modal frequencies created by measurement noise. The Juang and Pappa criteria permitted the fine tune of the reduced model (results not shown in this work due to lack of space).

Due to the wide spread distribution of modes in frequency, high frequency modes are difficult to identify with accuracy. At high frequency, even with high resolution A/D converters, the contribution of high frequency modes are hidden by numerical round off and truncation (even in the hypothetical case of a noise-free environment). Also, the signal-noise relation of the experimental data at high frequency becomes too low for experimental purposes.

For accuracy of modal identification, the noise level plays an important role at high frequency. However, under regular noise presence, the sampling frequency appears to be more restrictive than the presence of noise in experimental data. This is because wide spread modal frequencies require a very fast sampling rate producing huge Hankel matrices. The difficulty of getting numerical accuracy with large dense matrices is well known by the technical community and there is no need for further comments.

Finally, model validation is a main difficulty in modal identification from experimental data that is frequently overlooked by inattentive experimentalists. Time sequences are usually exactly reproduced by the identification technique but the resulting model is not the one that produced the data. This can be easily verified in simulation but it can be overlooked in experiments since the plant model is actually unknown. The reason for that is the improper choice of sampling rates and the aliasing characteristic of sampled signals.

In any case and based on this author's experience the ERA algorithm is surely one of the most interesting tools for modal identification and model reduction of large scale systems such as flexible structures.

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