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MIXTURE VISCOSITY INFLUENCE ON THE CONTAMINATED MIXING VOLUME IN BATCH TRANSFERS

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Abstract. *It is reported in this paper an investigation to assess the influence of the mixture viscosity on the mixing volumes which arise in batch transfers in multiproduct pipelines. To do so, two classical mixture viscosity correlations - Gambil and Arrhenius - were tested against viscosity measurements of gasoline-diesel mixtures of different concentrations. Afterwards, these correlations were implemented in a theoretical model to predict contaminated mixing volumes. Although these correlations do not present a good agreement when compared to viscosity experimental data, it is shown, by using a field test carried out in a Brazilian pipeline, that their impact on the mixing volumes are not significant. Moreover, no matter what correlation is used, the mixing volume predictions are shown to fit within the mixing volume uncertainties for the whole range of admissible concentrations investigated.*

Key-words: *Mixing volume, Batching transfer, Turbulent mass transfer*

1. INTRODUCTION

Multiproduct pipelines are large-diameter lines used to carry different petroleum products or different grades of a same product. Such a transport is performed by batching the products in continuous succession by either employing mechanical separators (usually

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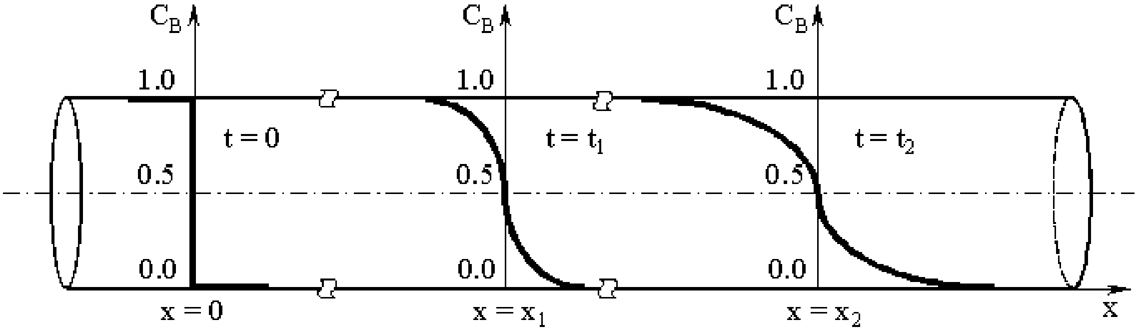

Figure 1: Evolution of the mixing region in a batching transfer.

pigs) between products or simply letting them to mix at batch interfaces. Since the first approach requires a somewhat complex operation, specially when intermediate pump-station manifolds are present, the second one is in general preferred.

On the other hand, the absence of a medium to separate the products gives rise to a mixing zone at the batches' boundary, which increases in length as it travels along the pipeline towards the receiving point. The prediction of the volume of the mixing zone, for a certain degree of admissible concentrations is of great concern in practice, in order to ensure an efficient operation.

From the operational viewpoint, the occurrence of mixing zones implies in additional costs associated to shipping the mixture back to refinery for later reprocessing. In other words, the minimization of the mixing volumes should always be pursued.

To better characterize the problem, consider a batching transfer of two distinct products labeled as "A" and "B", being "B" the following fluid and "A" the leading fluid. Let t and x represent, respectively, the time and the axial coordinate of the line which begins at the discharge of the pump station, $x = 0$, and ends at the receiving point labeled as $x = L$. Let $C_\imath(x,t) \in [0,1]$, with $\imath \in \{A, B\}$, be the time-averaged mean concentration of fluid $\imath$ within the mixture at the cross-section of the pipeline at x and t. Under certain circumstances, at beginning of a batching transfer, the concentration profile can be represented as a jump at $x = 0$. For instance, if we work with the concentration of fluid "B", then C_B jumps down as illustrated in Fig 1. During the transfer of the products through the pipeline, a mixing zone is formed at the boundary of the two adjacent products. This mixing region is constituted by a slug of contaminated material which increases in extension as the stream flows along the line. This phenomenon can be sketched in Fig 1 at subsequent time instants $t = t_1$ and $t = t_2$ at the spatial positions $x = x_1$ and $x = x_2$.

In order to estimate the mixing volume, most of the companies make use of theoretical semi-empiric models (Aunicky, 1970; Austin and Palfrey, 1964; Levenspiel, 1958; Ovádi and Török, 1977; Sjenitzer, 1958; Smith and Schulze, 1948a; Netchval et al., 1972), which assume steady-state and turbulent fluid flow. In the great majority of these models, the contamination is described by the process of dispersion of mass, being the dispersion coefficient the key factor responsible for accurate predictions of mixing volumes. Among other factors, the coefficient of turbulent dispersion is a function of the viscosity of the mixture of products "A" and "B". By inspecting the models, it is seen that different correlations have been used to evaluate the viscosity of the mixture of the petroleum products in terms of the concentrations. A simple comparison among these correlations reveals that, for a same pair of products, they lead to results of mixture viscosity quite different. So, a question arises as to the choice of the correlation which will render the

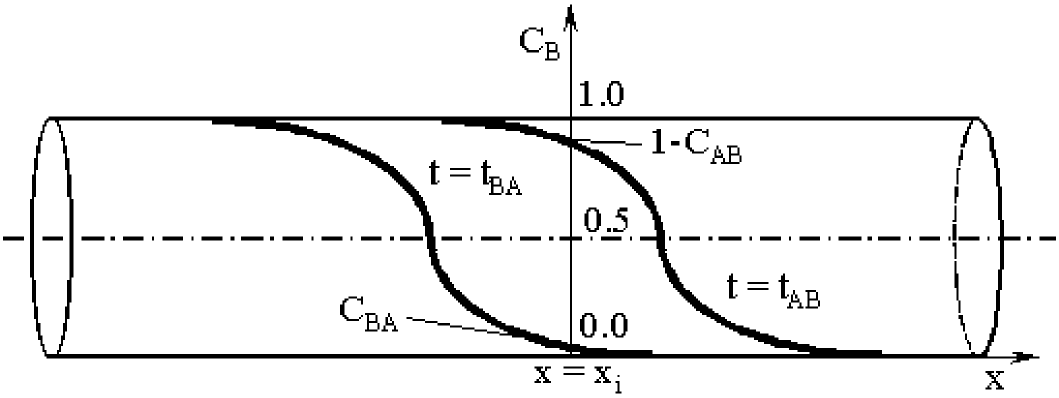

Figure 2: Characterization of the mixing volume at $x = x_i$ at different time instants, t_{BA} and t_{AB}.

most accurate mixing volume.

In this work, the influence of the correlation used to evaluate the mixture viscosity on the contaminated mixing volume is investigated. For this purpose, experimental measures of viscosities were carried out for a binary mixture of gasoline and diesel for different concentrations and the results compared with two available correlations used in the literature: one of Gambil and the other due to Arrhenius. In a subsequent stage, the three correlations (Gambil, Arrhenius and the experimental) were implemented in a model to forecast mixing volumes in steady-state batch transfers (Freitas Rachid et al, 2000) and the results were compared with the one of a field test driven in a Brazilian pipeline.

To allow a better understanding of the way mixing volumes are currently estimated and also to show in which part of the model the mixture viscosity is accounted for, we present in the next section the model formulation and the numerical technique used to compute the contaminated mixing volumes in batch transfers.

2. MATHEMATICAL FORMULATION

From both practical and operational viewpoints, the mixing volume at a position $x = x_i$ in the pipeline can only be evaluated once that the admissible concentration of product "B" in "A" at the beginning of the mixing zone, C_{BA}, and the admissible concentration of product "A" in "B" at the end of the mixing zone, C_{AB}, are specified as shown in Fig 2. Once these values (which are not necessarily equal) have been chosen in such a way that the products technical specifications are not altered, the mixing volume is formally defined as:

$$V_c = \int_{t_{BA}}^{t_{AB}} Q(x = x_i, t)\, dt \tag{1}$$

in which $Q(t)$ denotes the volumetric flow rate at $x = x_i$, t_{BA} is the time instant associated to the beginning of the contaminated zone with concentration C_{BA} at $x = x_i$ and t_{AB} is the time instant associated to the end of the contaminated zone with concentration C_{AB} at $x = x_i$ (see Fig 2). In other words, t_{BA} and t_{AB} are such that $C_B(x = x_i, t = t_{BA}) = C_{BA}$ and $C_A(x = x_i, t = t_{AB}) = C_{AB}$, respectively.

With the knowledge of flow rate as a function of the time and the field $C_\imath(x, t)$, the mixing volume can be promptly determined by (1) at any point $x = x_i$ sufficiently away from $x = 0$.

To establish the mathematical formulation of the problem we consider as basic assumptions the following:

1. the fluids are supposed to be Newtonian and incompressible;
2. the pipeline diameter is constant and there are no intermediate pump-station manifolds;
3. the mixture process as well as the fluid mixture can be disregarded in the balance of linear momentum;
4. the specific weights of the fluids do not significantly differ from each other;
5. minor losses can be neglected in the system.

Considering the above assumptions and designating by $x_m(t)$, with $x_m(t) \in [0, L]$, the position of the conventional half-length mixture (that is, the material coordinate in which $C_B(x = x_m(t), t) = C_A(x = x_m(t), t) = 0.5$) between the leading fluid "A" and the following fluid "B", the governing equations describing the mixing phenomenon can be written as:

$$\frac{f_B x_m + f_A(L - x_m)}{2Dg} u^2 = \frac{p_0}{\gamma_B} - \frac{p_L}{\gamma_A} + Z_0 - Z_L \tag{2}$$

$$\frac{dx_m}{dt} = u \tag{3}$$

$$\frac{\partial C_\imath}{\partial t} + u\frac{\partial C_\imath}{\partial x} = \frac{\partial}{\partial x}\Big[K^* \frac{\partial C_\imath}{\partial x}\Big] \tag{4}$$

for $(x, t) \in (0, +\infty) \times (0, T)$ in which $u = u(t)$ is the bulk time-average axial velocity in the tube, K^* is the effective dispersion coefficient, g is the gravitational acceleration, D stands for the pipeline diameter which is supposed to be constant along its length L. The specific weights of the products are denoted by $\gamma_\imath$, with $\imath \in \{A, B\}$, and the topographical heights at the beginning ($x = 0$) and at the receiving point ($x = L$) of the pipeline are designated by Z_0 and Z_L, respectively.

Equations (2) and (4) represent the balance of linear momentum for the fluids in the line and the concentration distribution which arises as a consequence of the dispersion phenomenon (Taylor, 1954). In equation (2) $f_\imath$, with $\imath \in \{A, B\}$, represent the Darcy-Weisbach friction factor related to the stretch of the pipeline where the flow of the fluid $\imath$ takes place and is given by

$$f_\imath = \left\{1.8 \log\left[\frac{6.9}{Re_\imath} + \left(\frac{\epsilon_r}{3.7}\right)^{1.11}\right]\right\}^{-2} \tag{5}$$

in which $Re_\imath = uD/\nu_\imath$ is the Reynolds number associated to the flow of fluid $\imath$ and ϵ_r is the pipeline relative roughness.

The terms p_0 and p_L in equation (2) are the pressures at $x = 0$ and at $x = L$, respectively. They describe different operational equipments and so are generically represented as functions of the time and fluid velocity. If during the transfer the pressure at the receiving point is held constant and the pump at $x = 0$ is supposed to be centrifugal and to be running at constant speed then:

$$p_0 = (a - bQ^m)\,\gamma_B \tag{6}$$

in which a, b and m are constant parameters of the pump curve and Q is the flow rate through the pump. If only a single pump is used in the transfer, $Q = u\pi D^2/4$.

By considering that the products "A" and "B" are pumped sequentially in such way that "A" is followed by "B" and that following relationship must always hold,

$$C_A + C_B = 1 \tag{7}$$

the initial conditions for (4) are given by, according the case $\imath = A$ or $\imath = B$:

$$\imath = B, \quad \begin{cases} C_B(x,t) = C_B(x \leq 0, 0) = 1 \\ C_B(x,t) = C_B(x > 0, 0) = 0 \end{cases} \tag{8}$$

or

$$\imath = A, \quad \begin{cases} C_A(x,t) = C_A(x \leq 0, 0) = 0 \\ C_A(x,t) = C_A(x > 0, 0) = 1. \end{cases} \tag{9}$$

As a consequence of the assumptions made so far, the bulk velocity $u(t)$ can be calculated by solving (2) along with (3) and (6) independently of (4). With the function $u(t)$ on one hand and the introduction of the following change of variables on the other:

$$y = \frac{x}{D} - \tau, \tag{10}$$

$$\tau = \frac{1}{D}\int_0^t u(t')dt', \tag{11}$$

the problem described by (4) along with (8) or (9) can be stated in a more convenient and peculiar form:

$$\frac{\partial C_\imath}{\partial \tau} = \frac{\partial}{\partial y}\left[K\frac{\partial C_\imath}{\partial y}\right] \quad \text{for } \imath \in \{A, B\} \tag{12}$$

subjected to

$$\imath = B, \quad \begin{cases} C_B(y,\tau) = C_B(y \leq 0, 0) = 1 \\ C_B(y,\tau) = C_B(y > 0, 0) = 0 \end{cases} \tag{13}$$

or

$$\imath = A, \quad \begin{cases} C_A(y,\tau) = C_A(y \leq 0, 0) = 0 \\ C_A(y,\tau) = C_A(y > 0, 0) = 1. \end{cases} \tag{14}$$

in which $K = K^*/Du$ stands for the dimensionless effective axial dispersion coefficient.

The problem of contamination in pipelines described by (12) with (13) or (14), which was formulated for the first time by (Taylor, 1954), is the basis for a number of models (Aunicky, 1970; Austin and Palfrey, 1964; Levenspiel, 1958; Ovádi and Török, 1977; Sjenitzer, 1958; Smith and Schulze, 1948a) currently used by several pipeline companies around the world. However, all of them consider, as simplifying hypotheses, that the axial velocity as well as the dispersion coefficient are constants. Moreover, they make use of poor correlations for the dispersion coefficient which, in turn, results in inaccurate values of mixing volumes, as it will be seen later.

Based on a bibliographical review, we have found that the correlation for the dispersion coefficient proposed by (Krantz and Wasan, 1974) is the most accurate among the usual ones. This assertion is based on a proper choice of the mean velocity and diffusivity distributions in the pipe wall region employed to compute K. Even though the work of (Krantz and Wasan, 1974) is devoted to non-Newtonian fluids, they have presented for Newtonian fluids a graphical correlation for K as a function of Re (for different Schmidt numbers), as a particular case. For $Sc = 1000$, the following fitted relationship between K and Re with $2 \times 10^3 \leq Re \leq 10^6$ is obtained (Krantz and Wasan, 1974)

$$\log K = -0.0641(\log Re)^4 + 1.1274(\log Re)^3 - 6.9173(\log Re)^2 + 16.379 \log Re \quad - 10.597. \tag{15}$$

To assign a more precise feature to the model (Freitas Rachid et al, 2002; Netchval et al., 1972), the Reynolds number Re in the above expression is computed as the Reynolds number of the mixture $Re = uD/\nu$, being ν the kinematic viscosity of the mixture. This viscosity depends on the concentration of the products in the mixture and can be evaluated by using different correlations. The most popular are the Gambil's correlation (Gambill, 1959)

$$\nu^{\frac{1}{3}} = C_A \nu_A^{\frac{1}{3}} + C_B \nu_B^{\frac{1}{3}}, \tag{16}$$

and the Arrhenius' correlation:

$$\nu = (\nu_A)^{C_A} (\nu_B)^{C_B}. \tag{17}$$

To numerically solve the non-linear problem described by (12) with (13) or (14) we have used the predictor-corrector method proposed by (Ames, 1977). Let $h > 0$ be the increment of y used to discretize the spatial computational domain of the problem in a set of points $\{y_i\}_{i=0}^n$ with $y_i = ih$, and $k > 0$ be the increment of τ used to discretize the time domain in a set of time instants $\{\tau_j\}_{j=0}^m$ with $\tau_j = jk$. By denoting the approximation of $C_\imath(y_i, \tau_j)$ by $C_{i,j}$, the expression for the predictor can be written as

$$\begin{aligned} C_{i-1,j+\frac{1}{2}} - \left[2 + \frac{4h^2}{kK(\tau_{j+\frac{1}{2}}, C_{i,j})}\right] C_{i,j+\frac{1}{2}} + C_{i+1,j+\frac{1}{2}} = \\ -C_{i-1,j} + \left[2 - \frac{4h^2}{kK(\tau_{j+\frac{1}{2}}, C_{i,j})}\right] C_{i,j} - C_{i+1,j} \\ -\frac{1}{8}\frac{\partial K}{\partial C}(\tau_{j+\frac{1}{2}}, C_{i,j}) \left[C_{i+1,j} - C_{i-1,j}\right]^2 \end{aligned} \tag{18}$$

and this expression for the corrector as

$$\begin{aligned} C_{i-1,j+1} - \left[2 + \frac{2h^2}{kK(\tau_{j+\frac{1}{2}}, C_{i,j+\frac{1}{2}})}\right] C_{i,j+1} + C_{i+1,j+1} = \\ -C_{i-1,j} + \left[2 - \frac{2h^2}{kK(\tau_{j+\frac{1}{2}}, C_{i,j+\frac{1}{2}})}\right] C_{i,j} - C_{i+1,j} \\ -\frac{1}{8}\frac{\partial K}{\partial C}(\tau_{j+\frac{1}{2}}, C_{i,j+\frac{1}{2}}) \left[C_{i+1,j+\frac{1}{2}} - C_{i-1,j+\frac{1}{2}}\right]^2 \end{aligned} \tag{19}$$

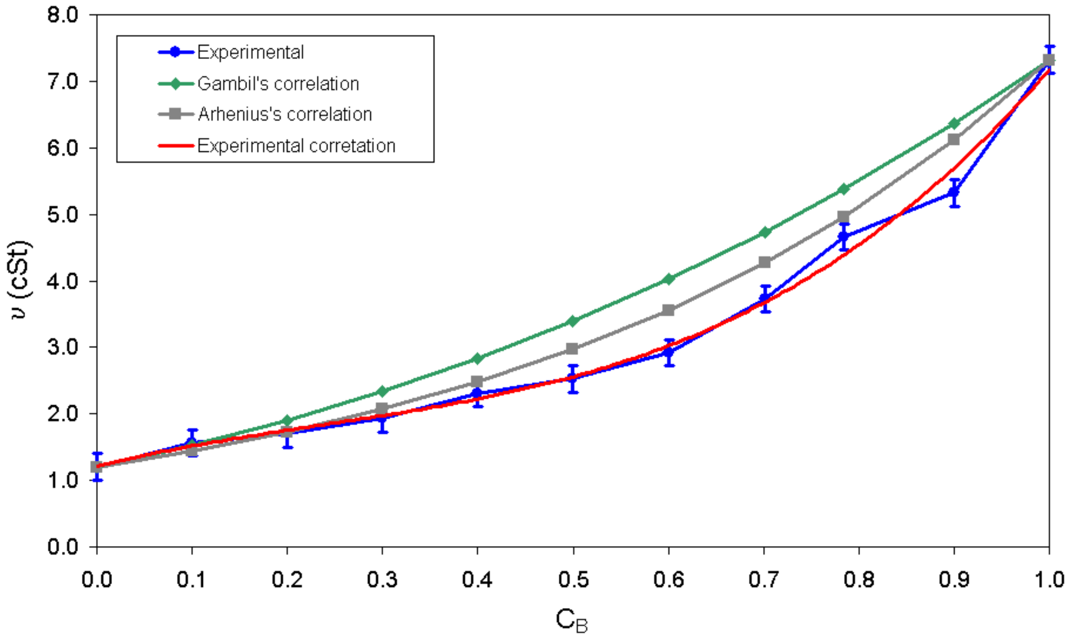

Figure 3: Comparison among mixture viscosity correlations and experimental data for different concentrations of product "B".

The solution of (19), $(C_{0,j+1}, C_{1,j+1}, \cdots, C_{n,j+1})$, at the time instant $\tau = (j+1)k$ is obtained through (18), $(C_{0,j+\frac{1}{2}}, C_{1,j+\frac{1}{2}}, \cdots, C_{n,j+\frac{1}{2}})$ with the values of C at time instant $\tau = jk$. This procedure is carried out by advancing in time a sufficient number of times to ensure that the end of the mixing zone has crossed the spatial point $x = x_i$. Finally, once a numerical approximation for $C_\imath(y, \tau)$ is available the mixing volume is readily calculated.

3. RESULTS AND DISCUSSION

To assess the influence of the mixture viscosity correlation on the mixing volumes arising in batch transfers, we first investigate the how significant the discrepancies in the mixture viscosity are when different correlations are employed. For this purpose, ten samples of a gasoline (product "A") and diesel (product "B") mixture, with volumetric concentrations equal to $0, 10, 20, \ldots, 90, 100\%$, were prepared and their viscosities experimentally measured in the Rheology Laboratory of the Universidade Federal Fluminense. To do so, a viscometer RS 50 Rheostress Haake was used at a controlled temperature of 20°C. The experimental results obtained along with the ones predicted by the Gambil's and Arrhenius' correlations are presented in Fig 3. Uncertainty levels of experimental measurements, which are of the order of 0.2 cSt, are also plotted in Fig 3 with a polynomial curve fitting, whose expression is

$$\nu = 8.2464 C_B^3 - 5.8274 C_B^2 + 3.5455 C_B + 1.2111, \tag{20}$$

having a correlation coefficient $r^2 = 0,992654$.

As one can see in Fig 3, the mixture viscosity given by the Gambil's and Arrhenius' correlations differ significantly from the experimental measures for concentrations C_B ranging from 0.4 to 0.9. Based on this observation, a question naturally arises as to what

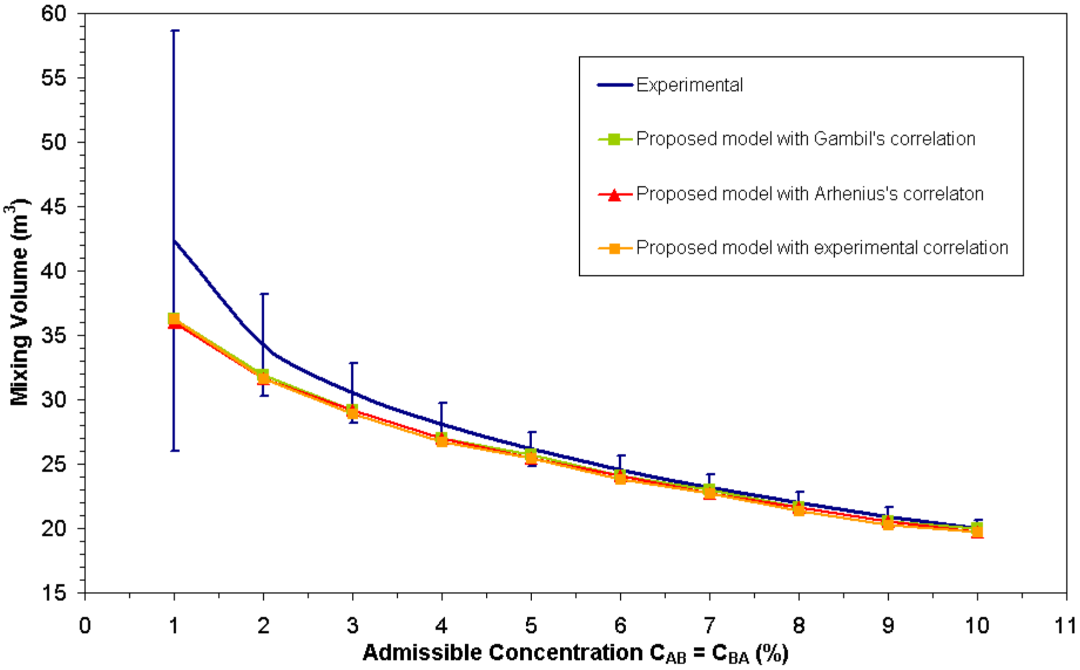

Figure 4: Predicted and measured mixing volumes as a function of admissible concentrations.

correlation one should use in Eq. (15) in order to have the best estimates in mixing volumes.

To answer this question, we compare the mixing volume predictions obtained by using the model presented herein with different correlations - Gambil, Arrhenius and the experimental correlation given by Eq. (15) - with experimental data of a batch transfer carried out in a Brazilian pipeline. The mixing volume experimental data are presented for different values of admissible concentrations $C_{AB} = C_{BA} = 1, 2, \ldots, 9, 10\%$ along with the aforementioned correlations in Fig 4. The experimental data refer to a transfer of gasoline/diesel, being the gasoline the leading fluid ($\gamma_A = 734 kgf/m^3$) and the diesel the following product ($\gamma_B = 833 kgf/m^3$) which took place in a 10" diameter pipeline operated by Petrobras. The pipeline is $200 km$ long and the topographic difference level between the receiving point and the pump station is $Z_L - Z_0 = -895 m$.

The experimental mixing volume was evaluated by continuous monitoring of the sonic velocity of the mixture at the position $x = 199.9 km$. To do so, it was used a clamp-on transit-time ultrasonic flow meter with an acquisition frequency of $0.2 Hz$ (Couto, 1998; Freitas Rachid et al, 1999a). The uncertainty analysis associated with this methodology is presented in (Baptista et al, 2000). The volumetric flow rate at the beginning of the transfer was $245 m^3/h$ and the pressure at the receiving point was $p_L = 9.21 kgf/cm^2$. The parameters a, b and m of equation (6) were determined from the pump curve and are equal to $a = 378.8 m$, $b = 5099.1 s^{1.75}/m^{4.25}$ and $m = 1.75$.

The mixing volume as a function of admissible concentrations $C_{AB} = C_{BA} = 1, 2, \ldots, 9, 10\%$ predicted by the model proposed in this paper with the Gambil, Arrhenius and the experimental correlations are depicted in Fig 4, along with the experimental mixing volume curve. The obtained results show that the mixture viscosity correlation has not a significant influence on the mixing volume since practically there is no distinction among the curves with the three different correlations. Even in the range of $0.4 < C_B < 0.9$, in which different values of mixture viscosity had been predicted by the three correlations (see Fig 3), one can not note any considerable distinction among the mixing volumes computed with these correlations. It should be noticed that no matter what correlation is used, the results obtained with the proposed model agree very well with the experimental data. It can be seen in Fig 4 that their predictions are within the experimental uncertainty bars over the whole range of concentration.

4. FINAL REMARKS

A model has been proposed in this paper to evaluate mixing volumes in pipeline batch transfers and then employed to assess the influence of the correlation used to compute the mixture viscosity on the mixing volume. The novel features of this model are the incorporation of the flow rate variation with time and the use of a more precise effective dispersion coefficient, which is considered to depend on the concentration. Although we have shown that neither the mixture viscosity correlation of Gambil nor the one due to Arrhenius give good results when compared to experimental measurements of gasoline-diesel mixtures, their impact on mixing volume is not significant. In other words, any correlation for mixture viscosity can be used with compromising the model prediction accuracy.

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