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SOLUTION OF A TWO-DIMENSIONAL DRYING PROBLEM IN CYLINDRICAL COORDINATES

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Abstract*. This work deals with the solution of the heat and mass transfer problem during drying of capillary porous media. The physical problem considered here is described by the linear Luikov's equations in cylindrical coordinates. The two-dimensional problem is solved with the Generalized Integral Transform Technique (GITT). This is a powerful hybrid numerical-analytical approach, which has been successfully used for the solution of different classes of problems. The convergence of the series-solutions for the problem is addressed in the paper, for different radial Biot numbers.*

Keywords: *Luikov's equations, Heat and mass transfer, Drying, Generalized Integral Transform Technique.*

1. INTRODUCTION

The phenomena of heat and mass transfer in capillary porous media has practical applications in several different areas including, among others, drying and the study of moisture migration in soils and construction materials. For the mathematical modeling of such phenomena, Luikov (1966) has proposed his widely known formulation, based on a system of coupled partial differential equations, which takes into account the effects of the temperature gradient on the moisture migration.

Different approaches have been used for the solution of Luikov's equations in one-dimensional and multi-dimensional problems (Comini and Lewis, 1976, Mikhailov and Ozisik, 1984, Lobo et al., 1987, Lobo et al., 1995, Guigon et al., 1999, Ribeiro et al., 1993, Cotta, 1993, Ribeiro and Cotta, 1995, Ribeiro and Lobo, 1998, Duarte and Ribeiro, 1998). The use of the *Generalized Integral Transform Technique* with simple eigenvalue problems involving analytical eigenfunctions, can avoid the calculation of complex eigenvalues for the drying problem based on Luikov's formulation. For more details on the use of such hybrid numerical-analytical technique, the reader is referred to the works of Ribeiro et al. (1993), Cotta (1993), Ribeiro and Cotta (1995) and Ribeiro and Lobo (1998).

In this paper we examine the solution of a two-dimensional drying problem in cylindrical coordinates. The coupled heat and mass transfer in the capillary-porous body is formulated with Luikov's equations. The resulting two-dimensional problem is solved with the Generalized Integral Transform Technique (GITT) and the convergence of the series-solutions for the problem is examined, as described next.

2. PHYSICAL PROBLEM AND MATHEMATICAL FORMULATION

The physical problem under picture in this work involves a cylindrical capillary porous medium of radius R_0 and length l, initially at uniform temperature and uniform moisture content. One of the boundaries, which is impervious to moisture transfer, is put in contact with a heater. The other boundary is put in contact with the dry surrounding air, thus resulting in a convective boundary condition for both the temperature and the moisture content. The lateral surface of the cylinder is also supposed to be impervious to mass transfer, but heat losses at this boundary are taken into account through a convective boundary condition. The linear system of equations proposed by Luikov (1966), for the modeling of such physical problem involving the drying of a capillary porous media, can be written in dimensionless form as (Luikov, 1966, Mikhailov and Özisik, 1984, Cotta, 1993, Ribeiro, 1993, Ribeiro and Lobo, 1998):

$$\frac{\partial\theta(R,Z,\tau)}{\partial\tau}=\alpha\frac{\partial^2\theta(R,Z,\tau)}{\partial Z^2}-\beta\frac{\partial^2\phi(R,Z,\tau)}{\partial Z^2}+\frac{r_a^2\alpha}{R}\frac{\partial}{\partial R}\left[R\frac{\partial\theta(R,Z,\tau)}{\partial R}\right]-\frac{r_a^2\beta}{R}\frac{\partial}{\partial R}\left[R\frac{\partial\phi(R,Z,\tau)}{\partial R}\right]$$

$$\frac{\partial\phi(R,Z,\tau)}{\partial\tau}=Lu\frac{\partial^2\phi(R,Z,\tau)}{\partial Z^2}-LuPn\frac{\partial^2\theta(R,Z,\tau)}{\partial Z^2}+$$
$$+\frac{r_a^2 Lu}{R}\frac{\partial}{\partial R}\left[R\frac{\partial\phi(R,Z,\tau)}{\partial R}\right]-\frac{r_a^2 LuPn}{R}\frac{\partial}{\partial R}\left[R\frac{\partial\theta(R,Z,\tau)}{\partial R}\right]$$

in $0<R<1$ and $0<Z<1$, for $\tau>0$ (1.a,b)

$$\theta(R,Z,0)=0,\quad \phi(R,Z,0)=0,$$ for $\tau=0$, in $0<R<1$ and $0<Z<1$ (1.c,d)

$$\frac{\partial\theta(0,Z,\tau)}{\partial R}=0,\quad \frac{\partial\phi(0,Z,\tau)}{\partial R}=0,$$ at $R=0$ and $Z=0$ for $\tau>0$ (1.e,f)

$$\frac{\partial\theta(1,Z,\tau)}{\partial R}-Bi_{qr}\,[1-\theta(1,Z,\tau)]=0,$$ at $R=1$ and $0<Z<1$ for $\tau>0$ (1.g)

$$\frac{\partial\phi(1,Z,\tau)}{\partial R}=Pn\frac{\partial\theta(1,Z,\tau)}{\partial R},$$ at $R=1$ and $0<Z<1$ for $\tau>0$ (1.h)

$$\frac{\partial\theta(R,1,\tau)}{\partial Z}-Bi_q\,[1-\theta(R,1,\tau)]+(1-\varepsilon)KoLuBi_m\,[1-\phi(R,1,\tau)]=0,$$

$$\frac{\partial\phi(R,1,\tau)}{\partial Z}+Bi_m^*\phi(R,1,\tau)=Bi_m^*-PnBi_q\left[\theta(R,1,\tau)-1\right],$$ at $Z=1$ and $0<R<1$, for $\tau>0$ (1.i,j)

The various dimensionless groups appearing above are defined as

$$\theta(R,Z,\tau)=\frac{T(r,z,t)-T_0}{T_s-T_0},\quad \phi(R,Z,\tau)=\frac{u_0-u(r,z,t)}{u_0-u_s},\quad Q=\frac{ql}{k(T_s-T_0)},\quad \tau=\frac{at}{l^2},\qquad (2.a\text{-}d)$$

$$Lu=\frac{a_m}{a},\quad Pn=\delta\frac{T_s-T_0}{u_0-u_s},\quad Bi_q=\frac{hl}{k},\quad Bi_m=\frac{h_m l}{k_m},\quad Ko=\frac{\lambda}{c}\frac{u_0-u_s}{T_s-T_0},\quad Bi_{qr}=\frac{h_r R_0}{k},\qquad (2.e\text{-}j)$$

$$r_a = \frac{l}{r}, \qquad R = \frac{r}{R_0}, \qquad Z = \frac{z}{l}, \qquad Bi_m^* = Bi_m\,[1-(1-\varepsilon)\,Pn\,Ko\,Lu] \tag{2.k-n}$$

$$\alpha = 1 + \varepsilon KoLuPn, \qquad \beta = \varepsilon KoLu \tag{2.o,p}$$

where a is the thermal diffusivity of the porous medium, a_m is the moisture diffusivity in the porous medium, c is the specific heat of porous medium, h *and* h_r are the heat transfer coefficients at the top and lateral surfaces, respectively, h_m is the mass transfer coefficient, k is the thermal conductivity, k_m is the moisture conductivity, q is the prescribed heat flux, λ is the latent heat of evaporation of water, T_s is the temperature of the surrounding air, T_o is the uniform initial temperature in the medium, u_s is the moisture content of the surrounding air, u_o is the uniform initial moisture content in the medium, δ is the thermogradient coefficient and ε is the phase conversion factor. *Lu*, *Pn* and *Ko* denote the Luikov, Posnov and Kossovitch numbers, respectively.

3. SOLUTION OF THE PROBLEM

We use in this work the GITT for the solution of the two-dimensional problem (1). In order to reduce the effects on the convergence of the series solution of the non-homogeneities along the axial direction and assuming that the heat losses through the lateral surface at R=1 are small, we filter problem (1) by writing its solution as

$$\theta(R,Z,\tau) = \theta_s(Z) + \theta_h(R,Z,\tau) \qquad \phi(R,Z,\tau) = \phi_s(Z) + \phi_h(R,Z,\tau) \tag{3.a,b}$$

where the filtering solutions are obtained from the following steady-state problem

$$\alpha \frac{d^2\theta_s(Z)}{dZ^2} = \beta \frac{d^2\phi_s(Z)}{dZ^2} \qquad \text{in } 0<Z<1, \tag{4.a}$$

$$\frac{d^2\phi_s(Z)}{dZ^2} = Pn \frac{d^2\theta_s(Z)}{dZ^2} \qquad \text{in } 0<Z<1, \tag{4.b}$$

$$\frac{d\theta_s(0)}{dZ} = -Q, \quad \frac{d\phi_s(0)}{dZ} = -PnQ, \qquad \text{at } 0<Z<1, \tag{4.c,d}$$

$$\frac{d\theta_s(1)}{dZ} + Bi_q\,\theta_s(1) = Bi_q - (1-\varepsilon)KoLuBi_m[1-\phi_s(1)], \qquad \text{at } Z=1, \tag{4.e}$$

$$\frac{d\phi_s(1)}{dZ} + Bi_m^*\phi_s(1) = Bi_m^* - PnBi_q[\theta_s(1)-1], \qquad \text{at } Z=1, \tag{4.f}$$

in the form

$$\theta_s(Z) = 1 + Q\left(1 + \frac{1}{Bi_q} - Z\right) \qquad \phi_s(Z) = 1 + PnQ(1-Z) \tag{5.a,b}$$

By substituting equations (3.a,b) into equations (1) and using equations (4), we obtain the filetered problem as:

$$\frac{\partial\theta_h(R,Z,\tau)}{\partial\tau} = \alpha\frac{\partial^2\theta_h(R,Z,\tau)}{\partial Z^2} - \beta\frac{\partial^2\phi_h(R,Z,\tau)}{\partial Z^2} + \frac{r_a^2\alpha}{R}\frac{\partial}{\partial R}\left[R\frac{\partial\theta_h(R,Z,\tau)}{\partial R}\right] - \frac{r_a^2\beta}{R}\frac{\partial}{\partial R}\left[\frac{\partial\phi_h(R,Z,\tau)}{\partial R}\right]$$

$$\text{in } 0<R<1 \text{ and } 0<Z<1, \text{ for } \tau>0 \tag{6.a}$$

$$\frac{\partial \phi_h(R,Z,\tau)}{\partial \tau} = Lu\frac{\partial^2 \phi_h(R,Z,\tau)}{\partial Z^2} - LuPn\frac{\partial^2 \theta_h(R,Z,\tau)}{\partial Z^2} + \frac{r_a^2 Lu}{R}\frac{\partial}{\partial R}\left[\frac{\partial \phi_h(R,Z,\tau)}{\partial R}\right] - \frac{r_a^2 LuPn}{R}\frac{\partial}{\partial R}\left[\frac{\partial \theta_h(R,Z,\tau)}{\partial R}\right]$$

in $0<R<1$ and $0<Z<1$, for $\tau>0$ (6.b)

$$\theta_h(R,Z,0) = -\theta_s(Z), \quad \phi_h(R,Z,0) = -\phi_s(Z),$$ at $\tau=0$, in $0<R<1$ and $0<Z<1$ (6.c,d)

$$\frac{\partial \theta_h(0,Z,\tau)}{\partial R} = 0, \quad \frac{\partial \phi_h(0,Z,\tau)}{\partial R} = 0,$$ at $R=0$, $0<Z<1$ and $\tau>0$ (6.e,f)

$$\frac{\partial \theta_h(R,0,\tau)}{\partial Z} = 0, \quad \frac{\partial \phi_h(R,0,\tau)}{\partial Z} = 0,$$ at $Z=0$, $0<R<1$ and $\tau>0$ (6.g,h)

$$\frac{\partial \theta_h(1,Z,\tau)}{\partial R} + Bi_{qr}\,\theta_h(1,Z,\tau) = Bi_{qr}[1-\theta_s(Z)],$$ at $R=1$, $0<Z<1$ and $\tau>0$ (6.i)

$$\frac{\partial \phi_h(1,Z,\tau)}{\partial R} = Pn\frac{\partial \theta_h(1,R,\tau)}{\partial R} = PnBi_{qr}[1-\theta_h(1,Z,\tau)-\theta_s(Z)],$$ at $R=1$, $0<Z<1$ and $\tau>0$ (6.j)

$$\frac{\partial \theta_h(R,1,\tau)}{\partial Z} + Bi_q\,\theta_h(R,1,\tau) = (1-\varepsilon)KoLuBi_m\phi_h(R,1,\tau),$$ at $Z=1$, $0<R<1$ and $\tau>0$ (6.k)

$$\frac{\partial \phi_h(R,1,\tau)}{\partial Z} + Bi_m^*\phi_h(R,1,\tau) = -PnBi_q\theta_h(R,1,\tau),$$ at $Z=1$, $0<R<1$ and $\tau>0$ (6.l)

The following eigenvalue problems are used in order to define the integral transform/inverse formula pairs for temperature:

$$\frac{d^2\varphi_j(\gamma_j Z)}{dZ^2} + \xi_j^2\varphi_j(\gamma_j Z) = 0$$ in $0<Z<1$ (7.a)

$$\frac{d\varphi_j(0)}{dZ} = 0; \quad \frac{d\varphi_j(\gamma_j 1)}{dZ} + Bi_q\varphi_j(\gamma_j 1) = 0$$ (7.b,c)

$$\frac{d^2\Omega_i(\eta_i R)}{dR^2} + \frac{1}{R}\frac{d\Omega_i(\eta_i R)}{dR} + \eta_i^2\Omega_i(\eta_i R) = 0$$ in $0<R<1$ (8.a)

$$\frac{d\Omega_i(0)}{dR} = 0; \quad \frac{d\Omega_i(\eta_i 1)}{dR} + Bi_{qr}\Omega_i(\eta_i 1) = 0$$ (8.b,c)

Similarly, the following eigenvalue problems are used for the moisture content:

$$\frac{d^2\Gamma_j(\xi_j Z)}{dZ^2} + \xi_j^2\Gamma_j(\xi_j Z) = 0$$ in $0<Z<1$ (9.a)

$$\frac{d\Gamma_j(0)}{dZ} = 0; \quad \frac{d\Gamma_j(\xi_j 1)}{dZ} + Bi_m^*\Gamma_j(\xi_j 1) = 0$$ (9.b,c)

and

$$\frac{d^2\Pi_i(\sigma_i R)}{dR^2} + \frac{1}{R}\frac{d\Pi_i(\sigma_i R)}{dR} + \sigma_i^2\Pi_i(\sigma_i R) = 0$$ in $0<R<1$ (10.a)

$$\frac{d\Pi_i(0)}{dR}=0\,; \qquad \frac{d\Pi_i(\sigma_i 1)}{dR}=0 \tag{10.b,c}$$

The eigenfunctions, normalization integrals and transcendental equations for the determination of the eigenvalues are given by:

$$\varphi_j(Z)=cos(\gamma_j Z) \qquad N_1(\gamma_j)=\frac{1}{2}\left[1+\frac{Bi_q}{\gamma_j^2+Bi_q^2}\right] \qquad (\gamma_j)tan(\gamma_j)=Bi_q \tag{11.a-c}$$

$$\Omega_i(\eta_i R)=J_0(\eta_i R) \qquad N_3(\eta_i)=\left[\frac{J_0^2(\eta_i 1)}{2}\frac{Bi_{qr}^2+\eta_i^2}{\eta_i^2}\right] \qquad \eta_i J_0'(\eta_i)+Bi_{qr}J_0(\eta_i)=0 \tag{12.a-c}$$

$$\Gamma_j(Z)=cos(\xi_j Z) \qquad N_2(\xi_j)=\frac{1}{2}\left[1+\frac{Bi_m^*}{\xi_j^2+Bi_m^{*2}}\right] \qquad (\xi_j)tan(\xi_j)=Bi_m^* \tag{13.a-c}$$

$$\Pi_i(\sigma_i R)=\begin{cases} J_0(\sigma_i R) & , \text{ for } i\neq 0 \\ 1 & , \text{ for } i=0 \end{cases} \qquad N_4(\sigma_i)=\begin{cases} \dfrac{J_0^2(\sigma_i)}{2} & , \text{ for } i\neq 0 \\ \dfrac{1}{2} & , \text{ for } i=0 \end{cases} \tag{14.a,b}$$

$$\begin{cases} J_0'(\sigma_i)=0 & , \text{ for } i\neq 0 \\ \sigma_0=0 & , \text{ for } i=0 \end{cases} \tag{14.c}$$

The integral transform / inverse formula pairs for temperature and moisture content are defined, respectively, as

$$\theta_h(R,Z,\tau)=\sum_{i=1}^{\infty}\sum_{j=1}^{\infty}\overline{\Omega}_i(\eta_i R)\overline{\varphi}_j(\gamma_j Z)\tilde{\overline{\theta}}_{ij}(\tau) \qquad \text{(inverse)} \tag{15.a}$$

$$\tilde{\overline{\theta}}_{ij}(\tau)=\int_{R=0}^{1}\int_{Z=0}^{1}R\overline{\Omega}_i(\eta_i R)\overline{\varphi}_j(\gamma_j Z)\,\theta_h(R,Z,\tau)\,dZ\,dR \qquad \text{(transform)} \tag{15.b}$$

and,

$$\phi_h(R,Z,\tau)=\sum_{i=0}^{\infty}\sum_{j=1}^{\infty}\overline{\Pi}_i(\sigma_i R)\overline{\Gamma}_j(\xi_j Z)\tilde{\overline{\phi}}_{ij}(\tau) \qquad \text{(inverse)} \tag{16.a}$$

$$\tilde{\overline{\phi}}_{ij}(\tau)=\int_{R=0}^{1}\int_{Z=0}^{1}R\overline{\Pi}_i(\sigma_i R)\overline{\Gamma}_j(\xi_j Z)\phi_h(R,Z,\tau)\,dZ\,dR \qquad \text{(transform)} \tag{16.b}$$

The integral transformation of problem (6) results on the following system of coupled ordinary differential equations:

$$\sum_{i=1}^{\infty}\sum_{j=1}^{\infty}\sum_{l=1}^{\infty}\sum_{m=1}^{\infty}\delta_{ijlm}\frac{d\tilde{\bar{\theta}}_{ij}(\tau)}{d\tau}+\sum_{i=1}^{\infty}\sum_{j=1}^{\infty}\sum_{1=l}^{\infty}\sum_{m=1}^{\infty}C_{ijlm}\tilde{\bar{\theta}}_{ij}(\tau)+\sum_{i=0}^{\infty}\sum_{j=1}^{\infty}\sum_{l=0}^{\infty}\sum_{m=1}^{\infty}D_{ijlm}\tilde{\bar{\phi}}_{lm}(\tau)=\sum_{i=1}^{\infty}\sum_{j=1}^{\infty}G_{ij} \tag{17.a}$$

$$\sum_{i=0}^{\infty}\sum_{j=1}^{\infty}\sum_{l=0}^{\infty}\sum_{m=1}^{\infty}\delta_{ijlm}\frac{d\tilde{\bar{\phi}}_{ij}(\tau)}{d\tau}=\sum_{i=1}^{\infty}\sum_{j=1}^{\infty}\sum_{l=1}^{\infty}\sum_{m=1}^{\infty}E_{ijlm}\tilde{\bar{\theta}}_{lm}(\tau)+\sum_{i=0}^{\infty}\sum_{j=1}^{\infty}\sum_{l=0}^{\infty}\sum_{m=1}^{\infty}F_{ijlm}\tilde{\bar{\phi}}_{ij}(\tau) \tag{17.b}$$

$$\tilde{\bar{\theta}}_{ij}(0)=-\int_{R=0}^{1}\int_{Z=0}^{1}R\,\bar{\Omega}_i(\eta_i R)\bar{\varphi}_j(\gamma_j Z)\theta_s(Z)\,dZ\,dR \tag{17.c}$$

$$\tilde{\bar{\phi}}_{ij}(0)=-\int_{R=0}^{1}\int_{Z=0}^{1}R\bar{\Pi}_i(\sigma_i R)\bar{\Gamma}_j(\xi_j R)\phi_s(Z)\,dZ\,dR \tag{17.d}$$

where

$$A_{ijlm}=\int_{R=0}^{1}\int_{Z=0}^{1}R\,\bar{\Omega}_i(\eta_i R)\bar{\varphi}_j(\xi_j Z)\bar{\Pi}_l(\sigma_l R)\bar{\Gamma}_m(\xi_m Z)\,dZ\,dR \tag{18.a}$$

$$B_{ijlm}=\int_{R=0}^{1}\int_{Z=0}^{1}R\,\bar{\Pi}_i(\sigma_i R)\bar{\Gamma}_j(\xi_j Z)\bar{\Omega}_l(\eta_l R)\bar{\varphi}_m(\gamma_m Z)\,dZ\,dR \tag{18.b}$$

$$C_{ijlm}=-\delta_{\mathrm{ijlm}}\alpha(\mathrm{r}_a^2\eta_i^2+\gamma_j^2)+\delta_{il}\beta Pn\,\mathrm{Bi}_q\bar{\varphi}_j(\gamma_j)\bar{\varphi}_m(\gamma_m)+\delta_{jm}r_a^2\beta PnBi_{qr}\bar{\Omega}_i(\eta_i)\bar{\Omega}_l(\eta_l) \tag{18.c}$$

$$D_{ijlm}=\beta(r_a^2\eta_i^2+\gamma_j^2)A_{ijlm}+\left[KoLuBi_m-\beta Bi_q\right]I_{1,il}\bar{\varphi}_j(\gamma_j)\bar{\Gamma}_m(\xi_m)-r_a^2\beta Bi_{qr}\bar{\Omega}_i(\eta_i)\bar{\Pi}_l(\sigma_l)I_{2,jm} \tag{18.d}$$

$$E_{ijlm}=LuPn(r_a^2\sigma_i^2+\xi_j^2)B_{ijlm}-LuPnBi_m^{*}I_{3,il}\bar{\Gamma}_j(\xi_j)\bar{\varphi}_m(\gamma_m) \tag{18.e}$$

$$F_{ijlm}=-\delta_{ijlm}Lu(r_a^2\sigma_i^2+\xi_j^2)-\delta_{il}Lu^2Pn(1-\varepsilon)Ko\,Bi_m\,\bar{\Gamma}_j(\xi_j)\bar{\Gamma}_m(\xi_m) \tag{18.f}$$

$$G_{ij}=r_a^2Bi_{qr}\bar{\Omega}_i(\eta_i)I_{4,j} \tag{18.g}$$

$$I_{1,il}=\int_{R=0}^{1}R\,\bar{\Omega}_i(\eta_i R)\bar{\Pi}_l(\sigma_l R)\,dR=\frac{\eta_i J_0(\sigma_l)J_1(\eta_i)-\sigma_l J_0(\eta_i)J_1(\sigma_l)}{\left(\eta_i^2-\sigma_l^2\right)\sqrt{N_3(\eta_i)}\sqrt{N_4(\sigma_l)}} \tag{18.h}$$

$$I_{2,jm}=\int_{Z=0}^{1}\bar{\varphi}_j(\gamma_j Z)\bar{\Gamma}_m(\xi_m Z)\,dZ=\frac{\gamma_j\cos(\xi_m)sen(\gamma_j)-\xi_m sen(\xi_m)\cos(\gamma_j)}{\sqrt{N_1(\gamma_j)}\sqrt{N_2(\xi_m)}(\gamma_j^2-\xi_m^2)} \tag{18.i}$$

$$I_{3,jm}=\int_{Z=0}^{1}\bar{\Gamma}_j(\xi_j Z)\bar{\varphi}_m(\gamma_m Z)\,dZ=\frac{\gamma_m\cos(\xi_j)sen(\gamma_m)-\xi_j sen(\xi_j)\cos(\gamma_m)}{\sqrt{N_1(\gamma_m)}\sqrt{N_2(\xi_j)}(\gamma_m^2-\xi_j^2)} \tag{18.j}$$

$$I_{4,j}=\int_{Z=0}^{1}\bar{\varphi}_j(Z)\left[1-\theta_s(Z)\right]dZ \tag{18.k}$$

The solution for the system (17), truncated to a sufficiently large order to reach convergence, is obtained with the subroutine DIVPAG of the IMSL (1987). Then, the temperature and moisture content along the axial direction can be computed by using the following expressions, derived with an integral balance approach (Cotta, 1993):

$$\phi_h(R,Z,\tau) = \sum_{l=0}^{\infty}\sum_{m=1}^{\infty}\left[\overline{\Gamma}_m(\xi_m) - r_a^2\sigma_l^2 \mathrm{I}_{6,\mathrm{m}}\right]\overline{\Pi}_l(\sigma_l R)\,\tilde{\bar{\phi}}_{lm}(\tau) - Pn\sum_{l=1}^{\infty}\sum_{m=1}^{\infty}\overline{\Omega}_l(\eta_l R)\,\mathrm{I}_{5,\mathrm{m}}\frac{d\tilde{\bar{\theta}}_{lm}(\tau)}{d\tau} -$$
$$-\frac{\alpha}{Lu}\sum_{l=0}^{\infty}\sum_{m=1}^{\infty}\overline{\Pi}_l(\sigma_l R)\,\mathrm{I}_{6,\mathrm{m}}\frac{d\tilde{\bar{\phi}}_{lm}(\tau)}{d\tau} \qquad (19.a)$$

$$\theta_h(R,Z,\tau) = \frac{\beta}{\alpha}\phi_h(R,Z,\tau) + \sum_{l=1}^{\infty}\sum_{m=1}^{\infty}\left[\overline{\varphi}_m(\gamma_m) - \frac{\beta}{\alpha}\overline{\varphi}_m(\gamma_m) - r_a^2\eta_l^2 I_{5,m}\right]\overline{\Omega}_l(\eta_l R)\,\tilde{\bar{\theta}}_{lm}(\tau) +$$
$$+\frac{r_a^2\beta}{\alpha}\sum_{l=0}^{\infty}\sum_{m=1}^{\infty}\sigma_l^2\overline{\Pi}_l(\sigma_l R) I_{6,m}\tilde{\bar{\phi}}_{lm}(\tau) - \frac{1}{\alpha}\sum_{l=1}^{\infty}\sum_{m=1}^{\infty}\overline{\Omega}_l(\eta_l R) I_{5,m}\frac{d\tilde{\bar{\theta}}_{lm}(\tau)}{d\tau} \qquad (19.b)$$

where $I_{5,m} = \int_Z^1\int_0^{Z'}\overline{\varphi}_m(\gamma_m Z')\,dZ''dZ'$ $\qquad I_{6,m} = \int_Z^1\int_0^{Z'}\overline{\Gamma}_m(\xi_m Z')\,dZ''dZ'$ (19.c,d)

4. RESULTS AND DISCUSSION

The solution *via* GITT of the two-dimensional problem given by Eqs. (1) is now examined. Tables 1 and 2 show the results for Bi_{qr} = 0, as well as the converged one-dimensional solution *via* GITT (Guigon et al, 1999), for the temperature and moisture content at the axial positions Z=0.1, 0.5 and 0.9, for τ = 0.2 and τ = 0.5, respectively. Other parameters of importance for the analysis were taken as: Lu=0.4, Pn=0.6, Ko=5.0, Bi_q=Bi_m=2.5, ε =0.2, Q=0.9 and r_a=1. The radial position was taken as R=0.5. The 2D solution was obtained by using 500 terms in each summation appearing in equations (19.a,b) in the axial direction and NR=2 in the radial direction. We note in Tables 1 and 2 that the two-dimensional problem is in very good agreement with the 1D solution for Bi_{qr}=0.

Table 1 – Comparison of 1D and 2D solutions for Bi_{qr} = 0.0, R=0.5 and τ = 0.2.

	$\theta(R,Z,\tau)$		$\phi(R,Z,\tau)$	
Solution	2D	1D	2D	1D
Z=0.1	0.2694	0.2690	0.0770	0.0772
Z=0.5	0.0279	0.0275	0.0501	0.0506
Z=0.9	-0.0228	-0.0232	0.3324	0.3328

Table 2 – Comparison of 1D and 2D solutions for Bi_{qr} = 0.0, R=0.5 and τ = 0.5

	$\theta(R,Z,\tau)$		$\phi(R,Z,\tau)$	
Solution	2D	1D	2D	1D
Z=0.1	0.4755	0.4752	0.1963	0.1967
Z=0.5	0.2562	0.2560	0.1924	0.1928
Z=0.9	0.2267	0.2264	0.4780	0.4781

Tables 3 and 4 illustrate the convergence of the solution with respect to the number of terms retained in the summations appearing in equations (19), at the radial position R=0.9, for different axial positions and for τ=0.2 and τ = 0.5, respectively. Similar results are shown in tables 5, 6 and 7, 8, for Bi_{qr} = 1 and 10, respectively. We can notice in Tables 3-8 that, for the different radial heat transfer coefficients examined, the solutions are converged to at least 3 significant digits, even for a small dimensionless time such as τ=0.2. However, for a very low number of terms such as N=30, the solution is converged to 2 decimal places. The CPU time for the solution obtained with N=30 was around 40 seconds, while the solution with N=150 took about 24 hours. Therefore, quite accurate solution for engineering purposes can be obtained with N=30 and with a small CPU time. The accuracy of the solution can be greatly improved by increasing the number of terms to N=150, but increasing the CPU time too. For the cases shown in Tables 3-8 we have used NR=10.

Table 3 – Convergence behavior of moisture content and temperature Bi_{qr}=0.1, τ=0.2 and R=0.9.

	$\theta(R,Z,\tau)$					$\phi(R,Z,\tau)$				
N	30	60	90	120	150	30	60	90	120	150
Z=0.1	0.3473	0.3408	0.3387	0.3377	0.3370	0.0579	0.0645	0.0667	0.0678	0.0685
Z=0.5	0.1002	0.0942	0.0922	0.0913	0.0910	0.0361	0.0424	0.0445	0.0455	0.0461
Z=0.9	0.0263	0.0209	0.0191	0.0183	0.0180	0.3249	0.3295	0.3310	0.3318	0.3322

Table 4 – Convergence behavior of moisture content and temperature Bi_{qr} = 0.1, τ = 0.5 and R=0.9.

	$\theta(R,Z,\tau)$					$\phi(R,Z,\tau)$				
N	30	60	90	120	150	30	60	90	120	150
Z=0.1	0.5736	0.5696	0.5683	0.5677	0.5673	0.1882	0.1930	0.1946	0.1954	0.1959
Z=0.5	0.3447	0.3406	0.3393	0.3386	0.3382	0.1882	0.1924	0.1938	0.1945	0.1949
Z=0.9	0.2838	0.2791	0.2775	0.2768	0.2763	0.4739	0.4764	0.4773	0.4777	0.4779

Table 5 – Convergence behavior of moisture content and temperature $Bi_{qr} = 1$, $\tau = 0.2$ and R=0.5.

	$\theta(R,Z,\tau)$					$\phi(R,Z,\tau)$				
N	30	60	90	120	150	30	60	90	120	150
Z=0.1	0.8334	0.8262	0.8239	0.8227	0.8220	-0.0735	-0.0669	-0.0648	-0.0637	-0.0630
Z=0.5	0.4789	0.4722	0.4700	0.4689	0.4683	-0.0733	-0.0671	-0.0650	-0.0640	-0.0634
Z=0.9	0.1621	0.1559	0.1538	0.1528	0.1522	0.2693	0.2738	0.2753	0.2760	0.2765

Table 6 – Convergence behavior of moisture content and temperature $Bi_{qr} = 1$, $\tau = 0.5$ and R=0.5.

	$\theta(R,Z,\tau)$					$\phi(R,Z,\tau)$				
N	30	60	90	120	150	30	60	90	120	150
Z=0.1	1.1797	1.1744	1.1726	1.1718	1.1712	0.1292	0.1339	0.1354	0.1362	0.1366
Z=0.5	0.8214	0.8159	0.8141	0.8132	0.8130	0.1340	0.1381	0.1394	0.1401	0.1405
Z=0.9	0.4684	0.4625	0.4605	0.4596	0.4590	0.4218	0.4243	0.4251	0.4255	0.4257

Table 7 – Convergence behavior of moisture content and temperature $Bi_{qr} = 10$, $\tau = 0.2$ and R=0.5.

	$\theta(R,Z,\tau)$					$\phi(R,Z,\tau)$				
N	30	60	90	120	150	30	60	90	120	150
Z=0.1	5.0219	5.0134	5.0106	5.0092	5.0089	-0.2335	-0.2276	-0.2256	-0.2247	-0.2244
Z=0.5	3.5789	3.5707	3.5679	3.5666	3.5663	-0.2074	-0.2018	-0.1999	-0.1991	-0.1987
Z=0.9	1.0185	1.0105	1.0079	1.0066	1.0063	0.1843	0.1883	0.1896	0.1903	0.1904

Table 8 – Convergence behavior of moisture content and temperature $Bi_{qr} = 10$, $\tau = 0.5$ and R=0.5.

	$\theta(R,Z,\tau)$					$\phi(R,Z,\tau)$				
N	30	60	90	120	150	30	60	90	120	150
Z=0.1	5.3283	5.3215	5.3192	5.3182	5.3179	0.0781	0.0822	0.0836	0.0842	0.0844
Z=0.5	3.8772	3.8704	3.8681	3.8670	3.8668	0.0909	0.0945	0.0957	0.0963	0.0964
Z=0.9	1.2879	1.2808	1.2784	1.2772	1.2770	0.3762	0.3785	0.3792	0.3796	0.3797

5. CONCLUSIONS

In this paper we presented the solution of the two-dimensional linear Luikov's equations in cylindrical coordinates. The drying cylindrical body is heated from the bottom boundary, while the top boundary is open to the surrounding air, resulting on a convective boundary condition for both heat and mass transfer. The lateral surface of the body is impervious to mass transfer, but heat can be lost by convection to the surroundings.

The solution for such a problem was obtained with the Generalized Integral Transform Technique. The solution for the case involving an adiabatic lateral surface was in very good agreement with one-dimensional results previously obtained with the same technique. The convergence of the series-solutions for the problem was examined for different radial Biot numbers. For the cases examined, involving Bi_{qr} = 0.1, 1 and 10, the solution was converged within an accuracy of 4 significant digits, with N=150 terms in each summation in the axial direction and NR=10 terms in each summation in the radial direction. We are now examining physical aspects of the solution, as well as the effects of other parameters of importance for the physical processes involved, such as the Luikov, Kossovich and Posnov's numbers, as well as the cylinder aspect ratio.

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