II CONGRESSO NACIONAL DE ENGENHARIA MECÂNICA
II NATIONAL CONGRESS OF MECHANICAL ENGINEERING
12 a 16 de Agosto de 2002 - João Pessoa – PB

LAMINAR MIXED CONVECTION IN VERTICAL TUBES WITH TEMPERATURE-DEPENDENT THERMAL PROPERTIES

Cláudia Regina de Andrade - claudia@mec.ita.br
Edson Luiz Zaparoli - zaparoli@mec.ita.br
Instituto Tecnológico de Aeronáutica – Departamento de Energia – IEME
Pça Marechal Eduardo Gomes, 50 - 12228-900 - São José dos Campos, SP, Brasil

Abstract. *This work reports a numerical study of the laminar mixed convection in vertical tubes considering temperature-dependent properties (density, conductivity, specific heat and viscosity. The buoyancy-assisted flow is considered fully developed and a constant axial temperature gradient is assumed with a peripherally uniform wall temperature condition. The mass conservation, momentum and energy equations are solved by the Galerkin finite element method. An unstructured mesh with triangular elements of six nodes and second-degree interpolation polynomials is employed with an adaptive mesh refinement in the more intense gradient regions. The resultant algebraic equations system is solved by an iterative procedure in a coupled way combining the Conjugated Gradient and Newton-Raphson methods. The refinement can be controlled by a default or a user specified control parameter. Thus, as each iteration is completed the code calculates the error limit each patch and subdivides only those patches where the error exceeds the specified value. In the variable properties case the heat transfer rate depends on both Rayleigh number and a parameter involving the tube wall and the bulk mean temperatures. Heat transfer rate results using constant properties (density varies under the Boussinesq approach) presented a good agreement with the previous literature data in the Rayleigh number range studied ($1{\cdot}10^3 < Ra < 1{\cdot}10^6$). Nusselt number results obtained for the variable properties case showed that when the temperature dependence is taking account the Nusselt number values increase when compared with the constant properties ones for a same Ra number. Besides, the velocity and temperature profiles along the tube transversal section are also affected by the thermal properties variation.*
Keywords: *variable properties, vertical tube, buoyancy-assisted flow*

1. INTRODUCTION

Heat transfer problems in which free convection and forced convection mechanisms interact are termed “mixed or combined convection”. The buoyancy force acts vertically, but the forced flow may be upward or downward directions. Downflow heating is called “opposing flow” when the natural convection currents effects are opposite. Buoyancy-assisted flow occurs when those currents are in the same direction of the forced flow, resulting in “aiding flow”.

These phenomenon have received attention due to several applications, including shell-and-tube heat exchangers, nuclear reactors and some aspects of electronic cooling. Many works has focused on the mixed convection considering both aiding and opposing flow conditions and different geometries.

Aung and Worku (1987) presented a numerical study dealing with combined free and forced laminar convection in a parallel plate vertical channel with opposing flow. These authors considered asymmetric wall heating at uniform heat fluxes and no flow reversal occurrence was reported.

Joye (1996) studied experimentally the upflow and downflow heating in vertical circular tubes, varying the Reynolds number range ($7.0{\cdot}10^2 < Re < 2.5{\cdot}10^4$). Those results showed that the way in which the Nusselt number values were reduced was not identical for aiding and opposing flows.

Laplante and Bernier (1997) presented a numerical analysis of the laminar mixed convection in vertical pipes. They studied downward water-flow and also the effects of wall conduction on the heat transfer rate.

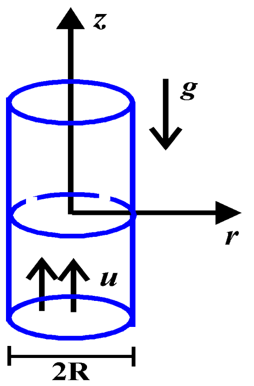
Cheng and Yu (2000) analyzed the buoyancy-assisted flow in vertical circular channels within an asymmetrically-heated slab and also assuming all thermophysical properties to be constant. The heat transfer results allowed establishing criteria to determine the flow reversal occurrence.

Zamora and Hernandez (1997) studied the effect of temperature-dependent properties on natural convection in rectangular vertical channels. These authors concluded that the temperature-dependent properties case shows an increase in the mass flow rate induced in the channel in comparison with the Boussinesq approximation one, for high Rayleigh numbers and a specific temperature difference range.

At the present work the effect of temperature-varying thermophysical properties on the laminar mixed convection in vertical tubes is studied employing the Galerkin finite element method. Heat transfer rate results using constant properties (density varies under the Boussinesq approach only in the buoyancy-term) are compared with temperature-dependent properties simulations. These results showed that the velocity and temperature profiles along the tube transversal section are affected by the thermal properties variation for the Rayleigh number range analyzed.

2. MATHEMATICAL FORMULATION

Steady-state laminar incompressible buoyancy-assisted air-flow in a vertical tube as schematized in Fig.(1) is numerically studied. The flow is both hydrodynamically and thermally fully developed, with negligible viscous dissipation and axial conduction.



with:

R= tube radius
z = axial coordinate
y = radial coordinate
u = z-axis velocity component
v = r-axis velocity component
g = gravity acceleration vector

Figure 1. Vertical tube: buoyancy-assisted flow

The fully developed flow (v = 0) and the constant axial temperature gradient assumptions result in the following conditions for velocity and temperature profiles:

$$\rho \frac{\partial u}{\partial z} = -u \frac{\partial \rho}{\partial z} \quad \text{and} \quad \frac{\partial T}{\partial z} = \frac{dT_w}{dz} = \frac{dT_b}{dz} = \text{constant} \tag{1}$$

where ρ is the fluid density; T_w is the wall temperature and T_b is the bulk mean temperature.

Density varies linearly as a function of the temperature as follow:

$$\rho = \rho_m [1 - \beta (T - T_m)] \tag{2}$$

where $\beta = -\frac{1}{\rho}\frac{\partial \rho}{\partial T}$ is the coefficient of volumetric expansion and ρ_m is the density evaluated at the mean temperature (T_m) calculated as:

$$T_m = \frac{T_w + T_b}{2} \tag{3}$$

Using Eqs. (1) and (2) the z-momentum and energy equations are represented by:

$$\nabla \cdot (\mu \nabla u) + u^2 \frac{\partial \rho}{\partial T} \frac{dT_b}{dz} = \frac{dP^*}{dz} + (\rho_m - \rho)\, g \tag{4}$$

$$\nabla \cdot (k \nabla T) = \rho\, C_p u \frac{dT_b}{dz} \tag{5}$$

Where:

Cp = fluid constant pressure specific heat
k = fluid thermal conductivity
μ = fluid viscosity
∇ = divergence operator

With the following boundary conditions (see Fig.1): at $r = R \Rightarrow u = 0$ and $T = T_w$

For the Boussinesq approximation all properties are assumed constant, except the density in the body force term of the z-momentum. Otherwise, for the variable properties numerical simulations, density depends on the temperature as described in Eq. (2) in all terms of Eqs. (4) and (5). The remaining thermophysical properties temperature-dependent are calculated by the following equations:

$\mu (T) = a + bT + cT^2$; a_μ =1.58598e-6, b_μ = 6.44859e-8, c_μ = -2.72873e-11 (6)

$k (T) = a + bT + cT^2$; a_k = 6.98435e-5, b_k = 9.73789e-5, c_k = -3.35029e-8 (7)

$Cp (T) = a + bT + cT^2 + dT^3$; a_{cp} = 1032.91, b_{cp} = 0.201, c_{cp} = 0.00035922, d_{cp} = 6.26095e-8 (8)

The above equations were obtained by a polynomial fit to the air thermophysical properties data presented in Incropera and DeWitt (1981). Viscosity, specific heat and thermal conductivity variations in the temperature range studied in this work are shown in Figures 2 to 4, respectively.

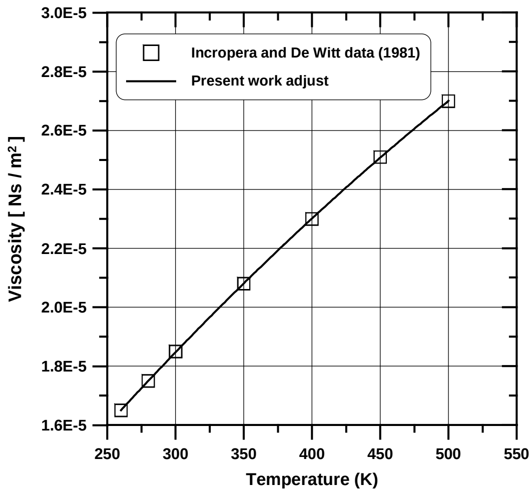


Figure 2. The temperature-varying viscosity for air

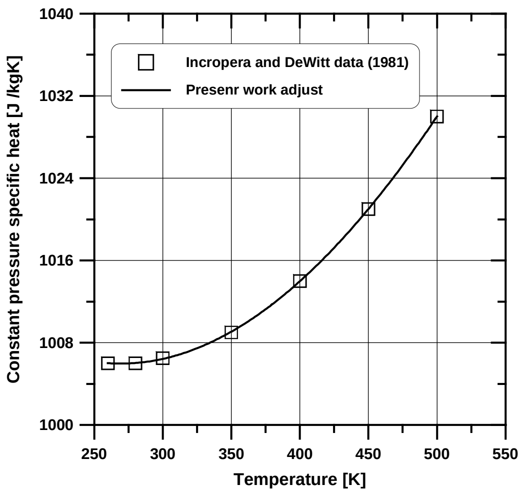


Figure 3. The temperature-varying specific heat for air

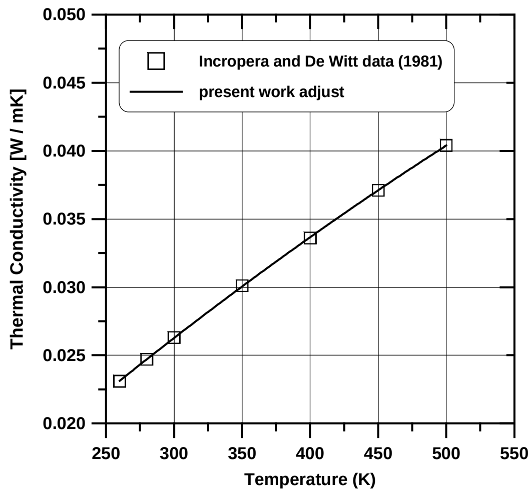


Figure 4. The temperature-varying thermal conductivity for air

The mixed convection problem with variable properties can be analyzed as a function of the dimensionless parameters below:

Prandtl number: $$\Pr = \frac{\mu_m \, Cp_m}{k_m} \tag{9}$$

Rayleigh number: $$Ra = \frac{g\,\beta\, R^4 (dT_b / dz) \Pr}{(\mu_m / \rho_m)^2} \tag{10}$$

Nusselt number: $$Nu = \frac{q_w}{k_m \, (T_w - T_b)}\ , \tag{11}$$

Temperature ratio: $$\theta = \frac{T_w - T_b}{T_m} \tag{12}$$

where q_w is the heat flux at the tube wall and Tm is determined by Eq. (3).

In this work, all simulations are carried on the range $1{\cdot}10^3 \le Ra \le 1{\cdot}10^6$, with Prandtl number (Eq. 11) equal to 0.7 (air-flow) and $0.01 \le \theta \le 0.1$.

Dimensionless results presented for velocity ($\bar{u}$) and temperature (ϕ) fields are calculated as:

$$\phi = \frac{T - T_w}{T_w - T_b} \quad \text{and} \quad \bar{u} = \frac{u}{u_m}, \quad \text{with } u_m = \frac{\int_0^R (u)\, 2\pi r dr}{\pi R^2} \text{ and } T_b = \frac{\int_0^R (\rho u Cp T)\, 2\pi r dr}{\int_0^R (\rho u Cp)\, 2\pi r dr} \tag{13}$$

3. SOLUTION METHODOLOGY

Finite element modeling has been used for converting the spatial components of complex sets of continuous partial differential equations (with well defined boundary values) to a set of discrete nodal equations for numerical solving. In this method the spatial area of interest is gridded into small patches called finite elements over which the variables are represented by simple polynomials. If a sufficiently fine grid is used and a sufficiently high order polynomial is used, a solution can be found to within any preset error limit. At the present work the Galerkin Finite Element Method of weighted residuals with quadratic basis to convert continuous partial differential equations into discrete nodal equations is used. Fig. (5) shows the computational domain represented by the vertical tube transversal section.

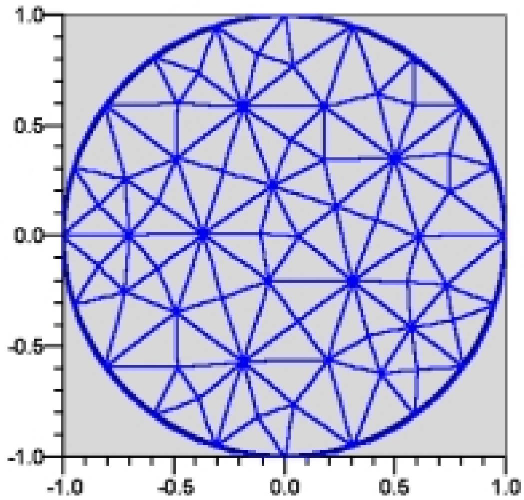


Figure 5. Computational domain

One difficult decision to make, when accuracy is required, in implementing the finite element method, is how to divide the area of interest into patches. Small patches require excessive computer time and memory. At the present work the PDEase2D code was employed by starting with a coarse grid of triangular patches (the most universal patch geometry that can be selected) and then using an iterative process to refine the grid to suit the problem. As each iteration is completed, the program determines the error in each patch and subdivides only those patches where the error exceeds a default or user specified error limit. After subdividing, recalculation is fast because of the good starting estimate provided by the previous iteration. In this way the code uses fine initial gridding only in those areas where sharp curvatures and tight geometries exist, providing near optimum speed and memory utilization. This adaptive mesh refinement is then applied in the more intense gradient regions. The resultant algebraic equations system is solved by an iterative procedure in a coupled way combining the Conjugated Gradient and Newton-Raphson methods using the PDEase code.

4. RESULTS

To validate the numerical code, the Nusselt number results (Eq. (11)) obtained using the Boussinesq approximation were compared with experimental data provided by Hallman (1961). Other correlations presented in the literature for the mixed convection in vertical tubes (with uniform heat flux as boundary condition) are also presented in Tab. (1).

Table 1: Correlations for the Nusselt number using the Boussinesq approach

Hallman experimental data correlation (1961)	$Nu\_a = 1.4\ Ra^{0.28}$
Petukhov correlation (1988)	$Nu\_b = 4.364\left(1+\frac{Ra}{60}\right)^{0.28}$
Churchill correlation (1992)	$Nu\_c = \{(4.364)^6 + [0.846\ (16Ra)^{0.25}]^6\}^{1/6}$

Figure 6 illustrates the Nusselt number as a function of the Rayleigh applying the correlations exhibited in the Tab. (1). The present work results and the experimental data show good agreement.

It is verified that for lower Ra values, the forced convection mechanism dominates the problem and the Nusselt number is near the pure forced convection value (Nu = 4.364). As the Ra increases, the natural convection effect is stronger and the hear transfer rate also enhances. For example, when Ra = $1{\cdot}10^5$, the Nu number is about four times the obtained for the pure forced convection.

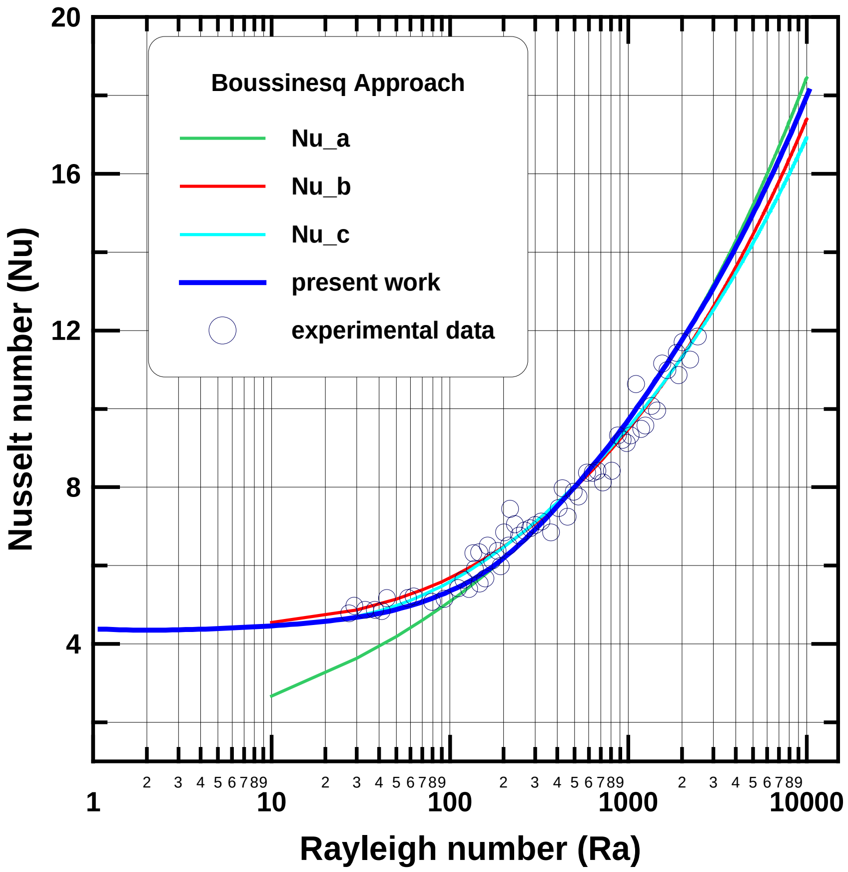


Figure 6. Nu number as a function of Ra number for the mixed convection in vertical tube.

Figures (7) and (8) show the dimensionless velocity calculated by Eq. (13) for the mixed convection in vertical tube with constant and variable thermal properties. Both simulations density varies with temperature according to Eq. (2) and for the variable case the temperature dependence for other properties is obtained applying Eq. (6) to (8).

For Rayleigh number equal to 10^3 the differences between constant and variable properties obtained for the velocity distribution along the vertical tube transversal section are more significant. As the Ra values increase (Fig. 8)) the velocity gradients become more intense and concentrate near the tube wall.

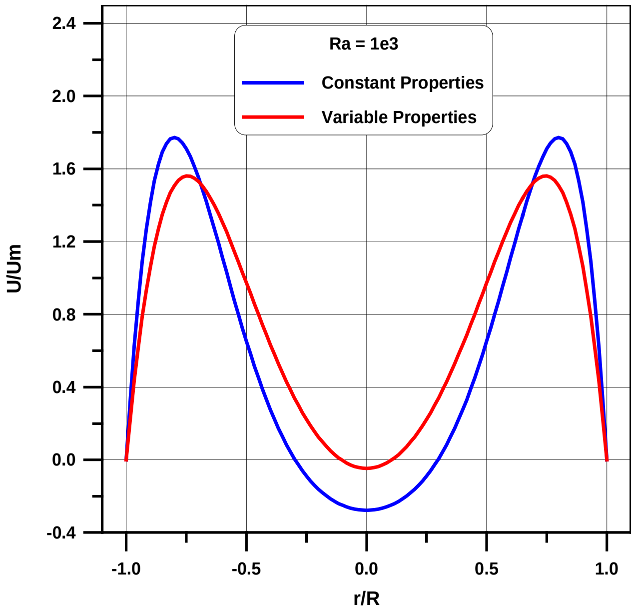


Figure 7. Dimensionless velocity profile at Ra = 10^3 and θ =0.02 along the tube transversal section

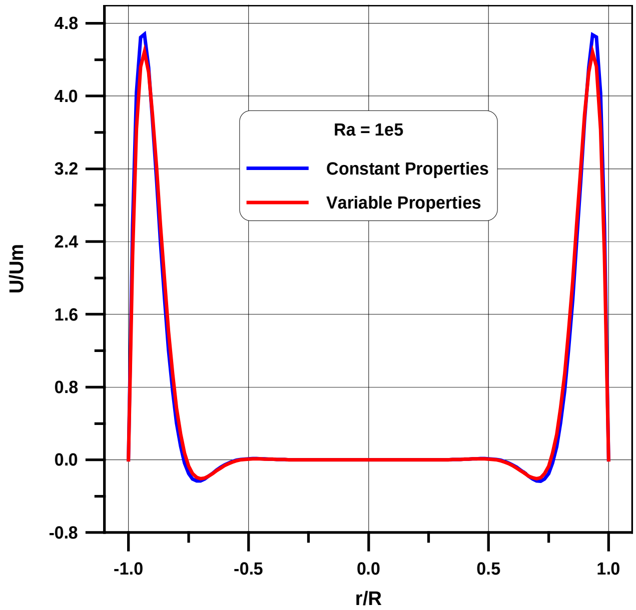


Figure 8. Dimensionless velocity profile at Ra = 10^5 and θ =0.02 along the tube transversal section

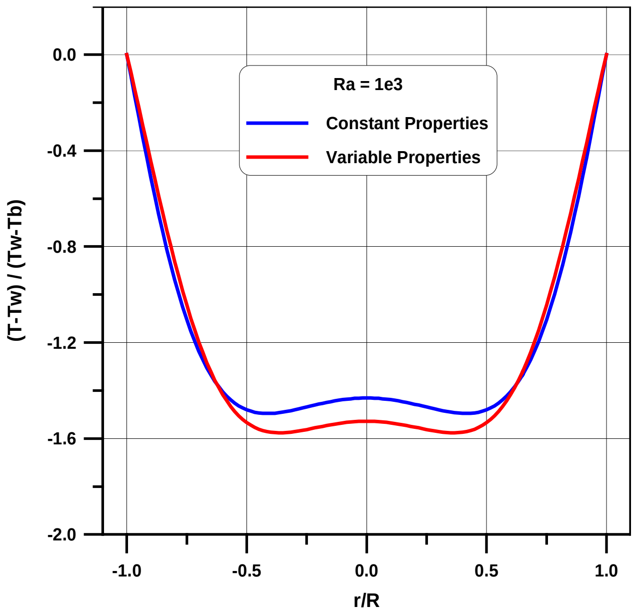


Figure 9. Dimensionless velocity profiles at Ra = 10^3 and θ =0.02 along the tube transversal section

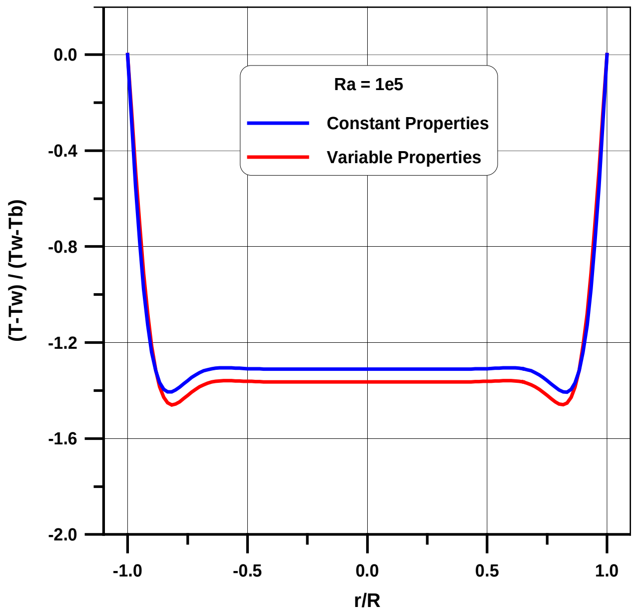


Figure 10. Dimensionless temperature profiles at Ra = 10^5 and θ =0.02 along the tube transversal section

A comparison between the dimensionless temperature profiles for constant and variable properties simulations is shown in Figs. (9) and (10). It is noted that when all thermophysical properties varies as a function of the temperature the main differences between the two formulations are located at the tube central region. The effect of the thermal properties variation is to elevate the bulk mean temperature (T_b), increasing the heat transfer rate and consequently the Nusselt number as presented in Tab. (2).

Table 2: Comparison between constant and variable properties

	Ra = 10^3		Ra = 10^6	
θ =0.04	T_b	Nu	T_b	Nu
Constant properties	373 K	2.17	375 K	13.19
Variable properties	376 K	2.42	380 K	14.72

The difference between constant and variable properties Nusselt values exhibited in Tab. (2) is almost 12 %. As the Rayleigh number increases this difference remains constant since the temperature ratio (θ) indicated by Eq. (12) doesn't change.

Fig. (11) presents Nusselt number results for constant and variable properties with θ =0.02. Both cases the heat transfer rate elevates as the Ra values increases but when the viscosity, specific heat and thermal conductivity depend on the temperature this process is more accentuated reaching, when Ra = 10^6, more than 14 times the Nusselt number corresponding to the forced convection (Nu = 4.364).

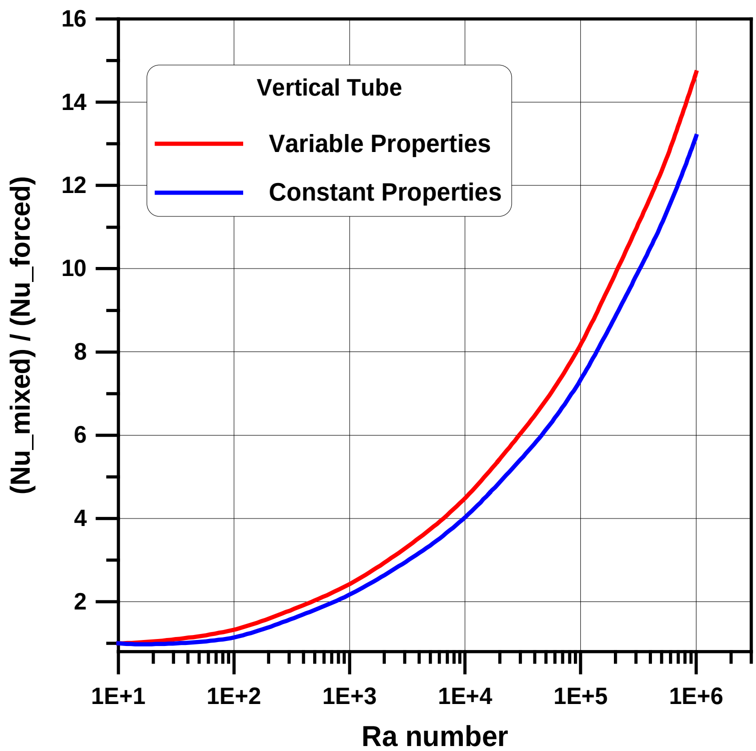


Figure 11. Nu number for the mixed convection in vertical tube as a function of Ra for θ =0.02

As the Ra number increases the natural convection phenomenon is intensified modifying the pure forced convection velocity and temperature profiles. (see the near-parabolic temperature distribution shown in Fig. (12) for Ra = 10^2). For higher Ra values the maximum velocity values (Fig. (12)) are displaced towards the tube wall. Also occurs a negative values region (descending flow) when Ra = 10^5.

Temperature distributions presented in Fig. (13) shows that the temperature gradients are concentrated close to the tube wall. As the Rayleigh number elevates the central region, where the temperature values constant, tend to increase along the tube transversal section.

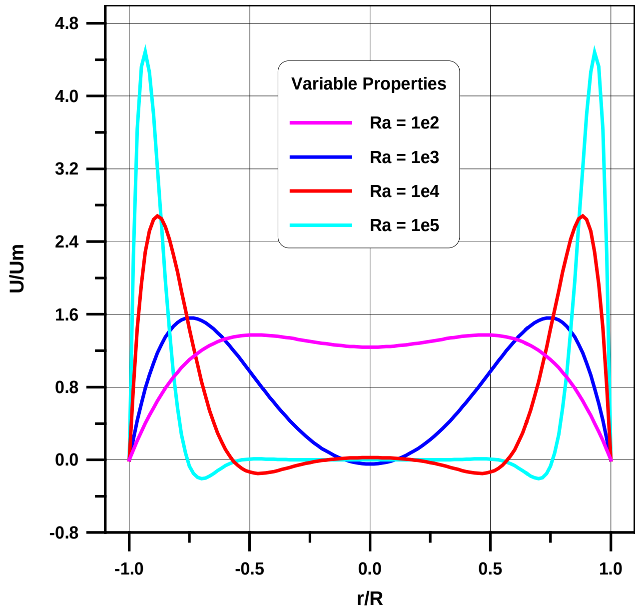


Figure 12. Dimensionless velocity profile with θ =0.02 along the tube transversal section

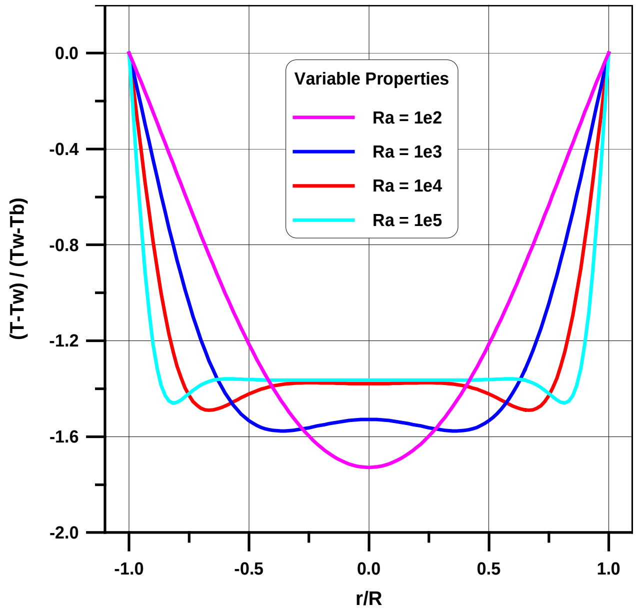


Figure 13. Dimensionless temperature profile with θ =0.02 along the tube transversal section

When the viscosity, specific heat and thermal conductivity varies as a function of the temperature, the θ parameter (Eq. (12)) also affects the heat transfer enhancement process. This influence is presented in Tab. (3) and shows that as the temperature ratio elevates the Nusselt number values decreases. This occurs because for higher temperature ratio values the temperature profile is modified reducing the temperature gradients neat the tube wall as can be observed in Fig. (14) for Ra = 10^3 and two different θ values.

Table 3: Variable properties: influence of the temperature ratio on the heat transfer rate

Ra = 10^3	θ = 0.01	θ = 0.02	θ = 0.04	θ = 0.08	θ = 0.1
Nusselt	2.45	2.44	2.42	2.2	1.9

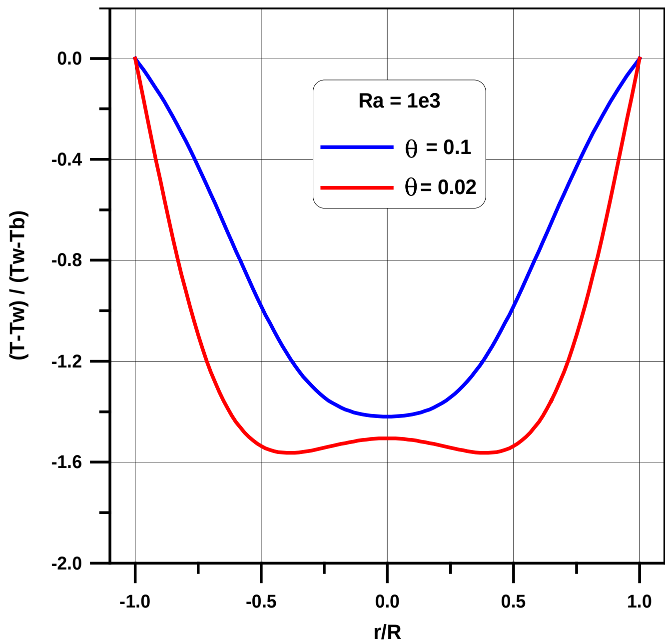


Figure 14. The influence of the temperature ratio (θ) on the temperature profile along the tube transversal section.

The elevation of the temperature ratio has an effect on the heat transfer rate similar of that occurred when the Rayleigh number simulated is lower (see Fig. (13)), with an only maximum value located at the tube central region. For $\theta = 0.02$ the temperature profile show two maximum values that migrate towards the tube wall. This behavior was also presented when the Ra values increase (Fig. (14)).

5. CONCLUSIONS

At this work the mixed convection in a vertical tube was numerically studied. Nusselt number results for buoyancy-assisted flow were obtained using two formulations: a constant properties case (where only the density varies linearly with the temperature under the Boussinesq approach) and variable properties one. At this last approach the temperature-varying of all thermophysical properties were considered.

It was shown that for variable properties case (for Pr = 0.7) the heat transfer rate associated with the mixed convection problem depends on both the Rayleigh number and a dimensionless parameter involving the tube wall and the fluid bulk mean temperatures (temperature ratio).

Numerical results for Nusselt number with constant properties presented a good agreement with literature data. Besides, the comparison with the variable case showed that when the temperature dependence is taking account the Nusselt number values increase (for the same Ra and temperature ratio values) and the velocity and temperature profiles along the tube transversal section are also affected by the thermal properties variation.

6. REFERENCES

Aung, W. and Worku, G., 1987, "Mixed Convection in Ducts with Asymmetric Wall Heat Fluxes", *Journal of Heat Transfer*, 109, 947-951.

Cheng, C.H. and Yu, J.H., 2000, "Buoyancy-Assisted Flow Reversal in Vertical Circular Channels within an Asymmetrically-Heated Slab", *Int. Comm. Heat Mass Transfer*, vol. 27, pp. 645-654.

Churchill, S. W. , 1992, "*Handbook of Heat Exchanger Design*", Single-Phase Convective Heat Transfer: Combined Free and Forced Convection in Channels, 2.5.10.1-2.5.10.12.

Hallman, Theodore M., 1961, "Experimental Study of Combined Forced and Free Laminar Convection in aVertical Tube", *NASA*, TN D-1104.

Joye, D. D., 1996, "Comparison of Aiding and Opposing Mixed Convection Heat Transfer in a Vertical Tube with Grashof Number Variation", *Int. J. Heat and Fluid Flow*, 17**,** 96-101.

Incropera, F.P. and DeWitt, D.P., 1981, *Fundamentals of Heat Transfer*", John Wiley & Sons, Inc. New York.

Laplante G. and Bernier, M. A., 1997, "Convection Mixte Défavorable et Conjugueé dans un Tube Vertical", *Int. J. Heat Mass Transfer*, 15**,** 3527-3536.

Leontiev, A. , 1985, "*Théorie des Échanges de Chaleur et de Masse*", Mir.

Petukhov B. S. and Poliakov A. F.,1988, "*Heat Transfer in Turbulent Mixed Convection*", Hemisphere Publishing.

Zamora, B and Hernández, J., 1997, "Influence of Variable Property Effects on Natural Convection Flows in Asymmetric-Heated Vertical Channels", *Int. Comm. Heat Mass Transfer*, vol. 24, pp. 1153-1162.