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# ON THE CONTINUUM MECHANICS MODELING AND SUPG APPROXIMATION OF CREAM INJECTION INTO SKIMMILK

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***Abstract.*** *Standardization of milk involves the controlled injection of cream with a high percentage of fat in the skimmilk, in order to obtain a final product with standard fat content. In modern milk processing plants, direct in-line standardization is commonly used. In order to calculate some Engineering variables, as pressure drop, local velocities and species mass fractions, a mathematical formulation of the problem is needed. The main goal of this paper is to give a simple and consistent formulation of the problem of cream into skimmilk through a continuum mechanics view. Some assumptions have to be made, and the relevant equations have to be developed in a way such to model the problem and make it solvable with a numerical tool, such as the finite element method. Also, a finite element Streamline Upwind/Petrov-Galerkin (SUPG) approximation is presented. Some numerical qualitative results for the velocity and mass fraction fields are shown to illustrate the application of this problem's modeling.*

***Keywords:*** *Continuum Mechanics, SUPG, Food Processing.*

## 1. PRELIMINARIES

A preliminary step in solving an Engineering problem is to formulate its mechanical model, identifying the physical laws and parameters that rule the studied phenomena.

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Before performing a numerical simulation or even an experimental study, one must analyze the relevant equations and variables involved, in order plan the experiment taking into account its most important features. Problems in Food Engineering have been solved much using experimental techniques, due to the food industry has still a strong basis on empiricism and a lack of Engineering research. Even though, Computational Fluid Dynamics (CFD) also finds a great application in this area. It makes possible to predict the behavior of foods during their process without a great expense of money or raw materials.

In this paper a mechanical modeling of the injection of cream in skimmilk is proposed. In dairy industries, this process occurs right after milk centrifugation, where the high percentage fat part, the milk cream, is separated from the low fat part, the skimmilk. The proportions to get a final fat content are calculated by the control system before the cream is re-injected into the skimmilk. Changes in flow rates through the centrifugal separator and variations of the incoming whole milk fat content are compensated by an accurate density control combined with constant pressure control at the skimmilk outlet on the separator, ensuring the necessary conditions for the re-mixing (Tetra Pak, 1995).

Modeling the cream injection and convection along the skimmilk line is the object of study of this paper. A solution of the proposed model, using the Streamline Upwind/Petrov-Galerkin (SUPG) approximation of the Finite Element Method (Brooks and Hughes, 1982), is presented with some preliminary results. One is the velocity field along the flow, indicating velocity profiles that are responsible for the convection of one component into another. Other is the pressure field, giving the pressure drop, an important Engineering parameter. The mass fraction field calculated along the flow gives a good idea of how the cream is convected into skimmilk in this process.

The problem will be modeled using the Continuum Mechanics Theory. According to that, matter can be considered as a continuum media with average macroscopic properties. Besides it is less realistic than a microscopic approach, it makes the model much simpler. Also, in problems involving scales of much higher order than the mean free path of the molecules, as here, there is no lost of accuracy considering the Continuum theory (Slattery, 1999). The notation of mathematical theory of continuum will be used to make this text compatible with others involving this approach.

## 1.1 MASS BALANCE IN THE CONTROL SYSTEM

In modern milk processing plants, direct in-line standardization is usually combined with separation. The whole process consists of separating skimmilk from cream and then re-mixing them in a controlled rate. Control valves, flow and density meters and a computerized control loop are used to adjust the fat content of milk and cream to desired values. This system controls the amount of cream injected in the skimmilk, which depends on their fat levels after separation. This amount is calculated as follows.

Let $X_m$ be the desired fat percentage in the standardized milk. Let $X_c$ and $X_s$ be the fat percentage of cream and skimmilk after separation, and $F_c$, $F_s$ and $F_m$ the momentary flows of each component cream, skimmilk and standardized milk. A mass balance over the problem's domain represented in Figure (1) gives:

$$\begin{aligned} F_c X_c + F_s X_s &= F_m X_m \\ F_c + F_s &= F_m \end{aligned} \tag{1}$$

As the quantities $X_c$ and $X_s$ are measured by an in-line density meter, Eqs. (1) give enough information to the controller to set the amount of cream to be injected.

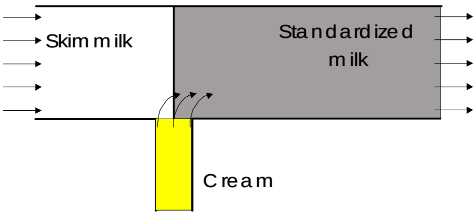


Figure (1): Two-dimensional representation of the problem

The Reynolds number is determined based on the mean velocity of the whole mixture, considered the bulk fluid:

$$\mathrm{Re} = \frac{D_h \bar{U} \rho}{\mu} \tag{2}$$

in which $A$ is the pipe's cross section and $\rho$ and $\mu$ are the density and the viscosity of the mixture.

The bulk fluid physical properties should be evaluated as a pondered average of the fluids flowing in the considered region. As the local mass fractions are not known before solving the problem, these properties are taken as the main fluid (skimmilk) properties. This is possible because the two fluids have very similar properties. It is important to have a good estimate of the Reynolds number before performing any numerical or practical experiment, because if the flow is not laminar (for Re>2000), some different aspects regarding turbulence should be taken into account.

## 2. A REVIEW IN THERMOMECHANICS MODELING

### 2.1. A Continuum Formulation

In order to allow a mechanical formulation of the problem through the continuum mechanical theory, it is necessary to the fluid to be thought as a continuous media, with average physical properties. As the mass fractions of each cream and skimmilk vary inside the domain, the average properties could be taken as locally averaged with the local mass fractions, such as

$$\begin{aligned} \rho &= \rho_{(c)}\omega_{(c)} + \rho_{(s)}\omega_{(s)}; \\ \mu &= \mu_{(c)}\omega_{(c)} + \mu_{(s)}\omega_{(s)} \end{aligned} \tag{3}$$

where $\omega_{(i)}$ is the local mass fraction of the $i$th species, $\rho_{(i)}$ and $\mu_{(i)}$ are the species' density and viscosity.

Instead of that, the concept of bulk fluid is going to be used, and the average properties are considered constant all over the problem's domain.

### 2.2. Principle of Mass Conservation

According to the mass conservation postulate, *the mass of a continuous body* $M$*, which can be any solid, liquid or gas, is time independent, or does not vary with time* (Slattery, 1999). This law is true even after a series of translations, rotations and deformations that the body may come to suffer. The postulate of mass conservation is mathematically written as Eq. (4), where mass is integrated on the material volume $V_m$ occupied by the body in its deformed configuration.

$$\overset{\circ}{M} = \frac{d}{dt}\int_{V_m} \rho dV = 0 \tag{4}$$

Let $V$ be a continuous portion of a mechanical body. Applying the Reynolds transport theorem (Truesdell and Toupin, 1960) for the density ρ:

$$\int_V \left( \frac{D\rho}{Dt} + \rho \operatorname{div} \mathbf{u} \right) = 0 \tag{5}$$

where **u** is the velocity vector field and the operator $D(\cdot)/Dt$ denotes the material derivative.

As this equation is true for any body of any portion of a body, it may be extracted the local forms of the continuity equation:

$$\begin{aligned} &\frac{D\rho}{Dt} + \rho \operatorname{div} \mathbf{u} = 0 \qquad \text{or} \\ &\frac{\partial \rho}{\partial t} + \operatorname{div}(\rho \mathbf{u}) = 0 \end{aligned} \tag{6}$$

For an incompressible material, which density does not vary with time, the motion is called isochoric, and div **u** = 0. This is true for most liquids considered in practice, and Eq. (6) is one of a set of equations that should be used to solve the velocity field of the problem of cream injection.

### 2.3. Momentum Balance

The momentum balance for a body is given by Euler's First Law, which says that *a body's time rate of change of the momentum relative to an inertial frame of reference is equal to the sum of forces acting on the body* (Truesdell and Toupin, 1960). It can then be expressed mathematically as:

$$\frac{d}{dt}\int_{Vm} \rho \mathbf{u} dV = \int_{Sm} \mathbf{t} dS + \int_{Vm} \rho \mathbf{f} dV \tag{7}$$

where **f** stands for the field of external and mutual forces per unit mass (Slattery, 1999) and **t** is the stress vector per unit area, representing the contact forces. Here, mutual forces can be neglected and the only external force field that may be considered is the gravity field.

The stress vector **t** comes from the definition of contact forces. They are not function of position, but represent the force exerted by one portion of the material upon another, beyond any mutual forces that could exist (Slattery, 1999). This force per unit area is exerted by every complementary portion of the body on its own portion, and **t** can be considered as **t** = **t**(**x**, *P*), with **x** standing for the position vector.

The nature of the contact load is given by the stress principle, which says that *there is a vector-valued function* **t** = **t**(**x**, *P*) *defined for all points* ***x*** *in a body B and for all unit vectors* **n** *directed outwardly the closed bounding of P such that* **t** = **t**(**x**, *P*) = **t**(**x**, **n**) (Truesdell and Noll, 1965). If the momentum balance is applied for two neighboring portions of a continuous body, considering each portion and their common surface, Cauchy's lemma can be deduced, giving **t** = **t**(**x**, **n**) = -**t**(**x**, -**n**), which means that *the stress vectors acting upon opposite sides of the same surface at a given point are equal in magnitude and opposite in direction* (Truesdell and Noll, 1965).

In order to improve the momentum balance formulation, a stress tensor **T** can be defined as $\mathbf{T}=T_{ij}\mathbf{e}_i\otimes\mathbf{e}_j$, in the tensorial basis $\mathbf{e}_i\otimes\mathbf{e}_j$, where each component $T_{ij}$ is the ith-component of the stress vector acting upon the positive side of the plane $x_j$ = cste (Slattery, 1999). In this way, Cauchy's theorem postulates that

$$\mathbf{t}=\mathbf{T}\cdot\mathbf{n}, \quad \text{with} \quad \mathbf{T}=\mathbf{T}^T \tag{8}$$

Substituting Eq. (8) in Eq. (7), considering an incompressible fluid, rearranging and applying Green's transformation, and taking advantage of the fact that a portion of a body is also a body, the remaining integrand of this equation equals to zero, resulting in the differential momentum balance, or Cauchy's First Law (Slattery, 1999):

$$\rho\frac{D\mathbf{u}}{Dt}=\operatorname{div}\mathbf{T}+\rho\mathbf{f} \tag{9}$$

### 3.4. The Material Behavior

It is obvious that Eq. (9) can describe any system where the material can be considered as a continuous media. Although it is an intuitive experience that all materials do not behave in the same way in response to the same conditions. The difference lies in the fact of how the stresses are distributed along a body, or how the contact forces in a body depend upon the motion and deformation of the body. This description is given by the variation of the stress tensor **T** with motion and deformation.

The motions that cause stresses are the components of the symmetric part of the velocity gradient tensor $\nabla\mathbf{u}$, called rate of deformation tensor (**D**). The way that **D** affects **T** is the so called constitutive equation of the material. For a newtonian incompressible fluid (Truesdell and Noll, 1965), the constitutive equation is given by

$$\mathbf{T}=-p\mathbf{I}+2\mu\mathbf{D} \tag{10}$$

For a generalized newtonian fluid (Slattery, 1999), **T** is given by

$$\mathbf{T} = -p\mathbf{I} + 2\eta(\gamma)\mathbf{D}, \text{ with } \gamma \equiv \sqrt{2\overline{II}_D} = \sqrt{2\,\mathrm{tr}(\mathbf{D}\cdot\mathbf{D})} \tag{11}$$

where $\eta(\gamma)$ is a function that depends on the model that is being used, and $\overline{II}_D$ stating the second principal invariant of **D** (Slattery, 1999). For further details on fluid models, see Slattery (1999) and Astarita and Marrucci (1974).

Defining a viscous portion of the stress tensor (Slattery, 1999), or the deviatoric stresses tensor (Panton, 1996) as **S** = **T** + $p$**I** it is possible to write the differential momentum balance, or Cauchy's First Law (Eq. (9)) as follows:

$$\frac{\partial \mathbf{u}}{\partial t} + (\nabla \mathbf{u})\mathbf{u} = -\frac{\nabla p}{\rho} + \frac{1}{\rho}\mathrm{div}\,\mathbf{S} + \mathbf{f} \tag{12}$$

### 3.5. Species Mass Balance

The density and the mass-averaged velocity of a multicomponent system, and the mass fraction of a species in this system are given by:

$$\rho = \sum_{i=1}^{n} \rho_{(i)}; \quad \mathbf{u} = \frac{1}{\rho}\sum_{i=1}^{n} \rho_{(i)}\mathbf{u}_{(i)}; \quad \omega_{(i)} = \frac{\rho_{(i)}}{\rho} \tag{13}$$

For each species of a multicomponent mixture, it is possible to formulate a mass balance involving the many forms of how a particular species may have variations in its quantity. In a simple way, the time rate of change of the mass of a body of species $i$ is equal to the rate at which the mass of $i$ is produced by chemical reactions, that may be in a volumetric rate $r_{(i)}$ or a rate per unit interface area $r_{(i)}^{\sigma}$ (Slattery, 1999):

$$\frac{d}{dt}\int_{V_m} \rho_{(i)} dV = \int_{V_m} r_{(i)} dV + \int_{S_m} r_{(i)}^{\sigma} dS \tag{14}$$

Applying the transport theorem (Truesdell and Toupin, 1960) on the left-hand side and considering a mass flux outwardly the interface with the species velocity $\mathbf{u}_{(i)}$, and the bulk velocity field **u**:

$$\int_{V_m}\left(\frac{D\rho_{(i)}}{Dt} + \rho_{(i)}\,\mathrm{div}\,\mathbf{u}_{(i)} - r_{(i)}\right)dV = \int_{S_m}\left\{\left[\rho_{(i)}\left(\mathbf{u}_{(i)} - \mathbf{u}\right)\cdot\mathbf{n}\right] - r_{(i)}^{\sigma}\right\}dS \tag{15}$$

Employing a localization argumentation, analogous to the one applied to Eq. (5), we obtain the local form of motion equation form each species:

$$\frac{\partial \rho_{(i)}}{\partial t} + \mathrm{div}\left(\rho_{(i)}\mathbf{u}_{(i)}\right) - r_{(i)} = 0 \tag{16}$$

Defining the mass flux of species $i$ with respect to the bulk velocity as $\mathbf{j}_i = \rho_{(i)} (\mathbf{u}_{(i)} - \mathbf{u})$, and employing the definitions for $\mathbf{j}_i$ and $\omega_{(i)}$ in Eq. (16), it comes that

$$\rho \frac{D\omega_{(i)}}{Dt} + \operatorname{div} \mathbf{j}_{(i)} - r_{(i)} = 0 \tag{17}$$

The equation for the mass transfer rate for a binary mixture, or the Fick's Law, given by Eq. (18), makes it possible to formulate a simpler way of Eq. (17), Eq. (19), if it is considered that the density of the mixture is constant:

$$\mathbf{j}_{(i)} = \rho D_{ij} \nabla \omega_{(i)} \tag{18}$$

$$\frac{D\rho\omega_{(i)}}{Dt} + \operatorname{div}\left(\nabla \rho D_{ij} \omega_{(i)}\right) - r_{(i)} = 0 \tag{19}$$

where $D_{ij}$ is the coefficient of binary diffusion of species $i$ and $j$, which may vary with temperature and mass fractions, while $\rho$ is always locally defined because of the variation of species mass fractions along the domain.

## 4. THE MECHANICAL MODEL OF THE CREAM INJECTION PROBLEM

The balances of the previous section, performed over the problem's domain $\Omega \in \mathbb{R}^2$, give the necessary equations to solve the velocity and pressure fields, so as the mass fraction field, if initial and boundary conditions (on the usual frontiers $\Gamma_h$ and $\Gamma_g$), are also given. Although, it may become a too complex system.

To make the problem simpler, some hypothesis can be considered. The first one is that the cream and skimmilk fluxes are constant (steady state). The second is that $D_{ij}$ is constant, which does not fall far from reality. The third hypothesis is to consider a bulk fluid with properties $\rho$ and $\mu$, in order to solve the momentum equations, and also for the species balance, where $\rho$ would no more be a function of local mass fractions. This hypothesis is a good one for two similar fluids, such as cream and skimmilk. Using those suppositions, a set of equations are proposed to model the cream injection:

$$\begin{aligned}
&(\nabla \mathbf{u})\mathbf{u} + \frac{\nabla p}{\rho} - \frac{1}{\rho}\operatorname{div}\mathbf{S} - \mathbf{f} = 0 && \text{on } \Omega \\
&\mathbf{S} = 2\eta(\gamma)\mathbf{D} && \text{on } \Omega \\
&\operatorname{div}\mathbf{u} = 0 && \text{on } \Omega \\
&\mathbf{u}\cdot\nabla\omega_{(i)} - D_{ij}\operatorname{div}\left(\nabla\omega_{(i)}\right) - R_{(i)} = 0 && \text{on } \Omega
\end{aligned} \tag{20}$$

where $R_{(i)}$ is the ith component mass production rate per unit bulk mass of bulk fluid.

The boundary conditions are: no-slippery condition at the walls; prescribed velocity $\mathbf{u}_g$ and prescribed mass fraction field $\omega_{(i)g}$ at the beginning of each pipeline; free-traction $\mathbf{S}_h$ and free mass-flux $j_{(i)h}$ at the outlet:

$$\begin{aligned} &\mathbf{u} = \mathbf{u}_g && \text{on } \Gamma_g \\ &\omega_{(i)} = \omega_{(i)g} && \text{on } \Gamma_g \\ &\left[-p\mathbf{I} + 2\eta(\gamma)\mathbf{D}\right]\mathbf{n} = \mathbf{S}_h && \text{on } \Gamma_h \\ &\left[\rho D_{ij}\nabla\omega_{(i)}\right]\cdot\mathbf{n} = j_{(i)h} && \text{on } \Gamma_h \end{aligned} \tag{21}$$

## 5. FINITE ELEMENT MODEL

In order to perform some numerical experiment using the above modeling, it was chosen to use the finite element method with SUPG formulation, applying the common finite subspaces $\mathbf{V}_h$, $P_h$ (Brooks and Hughes, 1982), and $W_h$ (Franca et al., 1992):

Find the triple $(\mathbf{u}_h, p_h, \omega_{(i)h}) \in \mathbf{V}_h \times P_h \times W_h$ such that

$$B(\mathbf{u}_h, p_h, \omega_{(i)h}; \mathbf{v}, q, w) = F(\mathbf{v}, q, w), \quad (\mathbf{v}, q, w) \in \mathbf{V}_h \times P_h \times W_h \tag{22}$$

where

$$\begin{aligned} B(\mathbf{u}_h, p_h, \omega_{(i)h}; \mathbf{v}, q, w) &= \left([\nabla\mathbf{u}]\mathbf{u}, \mathbf{v}\right) + (2\nu\mathbf{D}(\mathbf{u}), \mathbf{D}(\mathbf{v})) - (\nabla\cdot\mathbf{v}, p) - (\nabla\cdot\mathbf{u}, q) \\ &+ \left(\mathbf{u}\cdot\nabla\omega_{(i)}, w\right) + \left(D_{ij}\nabla\omega_{(i)}, \nabla w\right) \\ &+ \sum_{K\in C_h} \left([\nabla\mathbf{u}]\mathbf{u} + \nabla p - 2\nu\nabla\cdot\mathbf{D}(\mathbf{u}), \tau(\mathrm{Re}_K)([\nabla\mathbf{v}]\mathbf{v} - \nabla q)\right)_K \\ &+ \sum_{K\in C_h} \left(\mathbf{u}\cdot\nabla\omega_{(i)} - D_{ij}\Delta\omega_{(i)}, \alpha(\mathrm{Pe}_K^m)\mathbf{u}\cdot\nabla w\right)_K \end{aligned} \tag{23}$$

and

$$\begin{aligned} F(\mathbf{v}, q, w) &= (\mathbf{f}, \mathbf{v}) + (\mathbf{S}_h, \mathbf{v})_{\Gamma_h} + \left(R_{(i)}, w\right) + (j_{(i)h}, w)_{\Gamma_h} \\ &+ \sum_{K\in C_h} (\mathbf{f}, \tau(\mathrm{Re}_K)([\nabla\mathbf{v}]\mathbf{v} - \nabla q))_K + \sum_{K\in C_h} \left(R_i, \alpha(\mathrm{Pe}_K^m)\mathbf{u}\cdot\nabla w\right)_K \end{aligned} \tag{24}$$

with the stability parameter $\tau(\mathrm{Re}_K)$, dependent on the element Reynolds number $\mathrm{Re}_K$ is defined as in Franca and Frey (1992), and the stability parameter $\alpha(\mathrm{Pe}^m{}_K)$, dependent on the grid *Peclet for mass transfer* (Slattery, 1999) number $\alpha(\mathrm{Pe}^m{}_K)$ is defined as in Franca et al. (1992).

## 6. NUMERICAL APPLICATION

A two-dimensional model of a geometry, using similar to industrial pipes measures, was used to implement the formulation described previously.

The numerical simulations for obtaining the velocity field problem were performed at the National Center of Supercomputation (CESUP), using “Silicon Graphics ORIGIN 200 Workgroup Server” with two processors and 256 Mb RAM and “Silicon Graphics Octane

Workstation” with two processors and 128 Mb RAM. The software used was Flotran (ANSYS 5.7, Ansys Inc.).

The simulations of the advection-diffusion problem were performed at the Laboratory of Applied and Computational Fluid Mechanics (LAMAC), using a Intel Pentium III with 1.1 GHz processor and 1 Gb RAM, in a programmed software. The visual post processing was made with use of the software Ensight 7.

Some qualitative results are given for the velocity field and mass fraction field obtained for the simulation of a ½ inch pipe injecting cream into a 1 inch skimmilk tube. A 12466 finite elements mesh was used, employing quadrilateral 4 node elements (Q1). Figure (2) shows the velocity field and the pressure profile obtained. Figure (3) shows the cream mass fraction field, distributed along the tubes. Figure (4) gives the isovalue lines for the cream mass fraction, in a detailed region. Still some light numerical instabilities appear in the SUPG solution, which is to be corrected along the future works in this research.

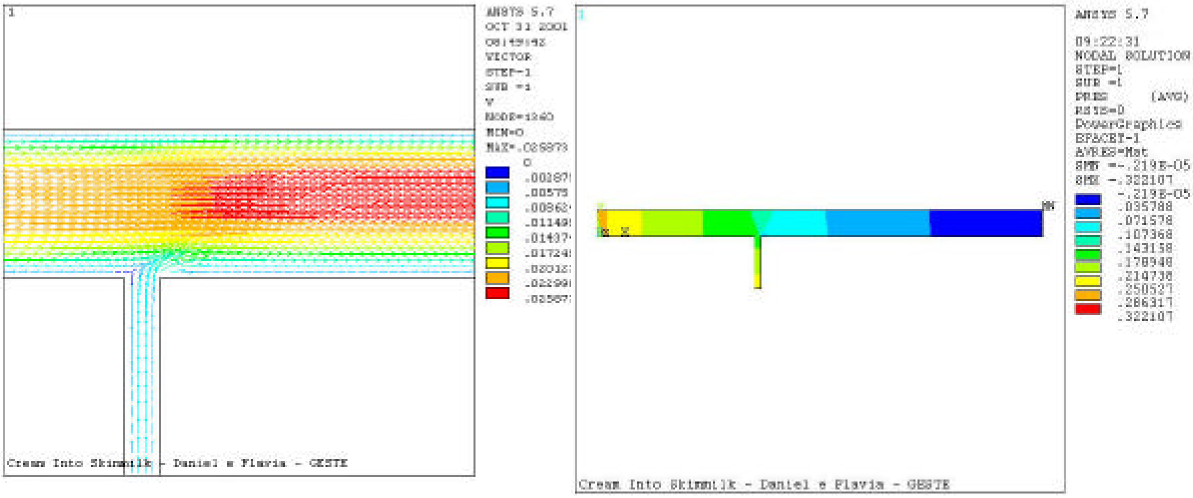


Figure (2): Velocity field and pressure profile

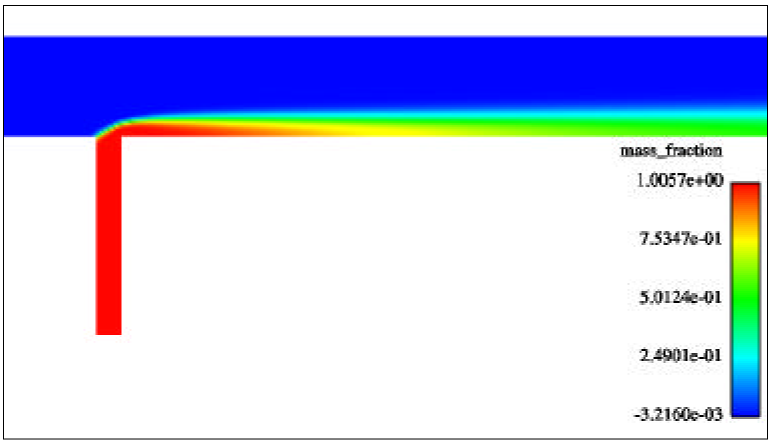


Figure (3): Cream mass fraction field

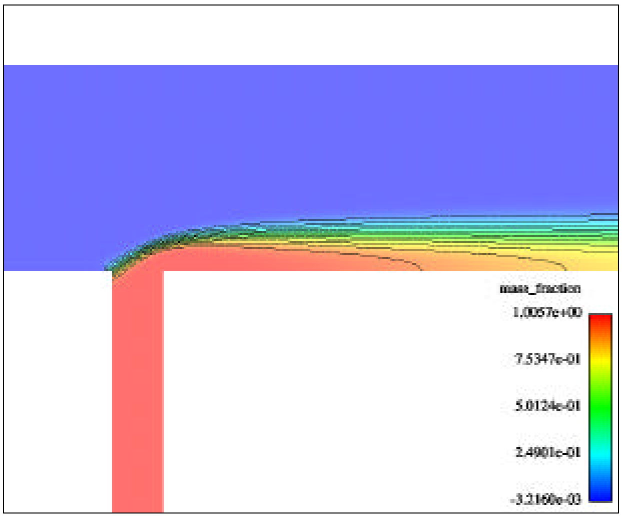


Figure (4): Isovalues for cream mass fractions

## 7. ACKNOWLEDGEMENTS

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