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# DETERMINATION OF THE THERMAL CONDUCTIVITY TENSORS DUE TO DISPERSION AND TORTUOSITY FOR LAMINAR AND TURBULENT FLOWS IN POROUS MEDIA

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***Abstract.*** *In this work some correlations for the Dispersion and Tortuosity thermal conductivity tensors are obtained for laminar and turbulent flows in porous media. These correlations are obtained simulating the flow in the porous medium through an infinite array of unit cells with periodic boundary conditions for the flow and an imposed temperature gradient through the unit cell. The solutions obtained for the hydrodynamic and thermal fields inside the unit cell for a broad range of Reynolds number based on the cell dimensions are then utilized to obtain the above mentioned correlations. Two types of boundary conditions for the thermal field are investigated and compared to the results of Nakayama and Kuwahara (1999). It is found that only one type of boundary conditions is suitable due to the repeatability of the temperature pattern in successive cells.*

***Keywords:*** *Porous Medium, Turbulence, Heat Transfer, Numerical Methods.*

## 1. INTRODUCTION

Due to the growing interest on the applications of porous media in several areas of engineering and science, a better understanding of the phenomena occurring in laminar and turbulent flows as well as heat transport in porous media is desirable. As examples of applications, one can mention filtration, catalytic reactors, combustion in porous matrices, electronic device cooling, oil engineering, etc.

The aim of this work is to explore a methodology to obtain the thermal conductivity tensors due to dispersion and tortuosity. These tensors arise in the heat transport equation for porous media and are due to the presence of the porous matrix. These conductivity tensors are present in both laminar and turbulent flows.

Here, the porous medium is represented by an infinite array of unit cells and the calculations are performed for one unit cell with periodic boundary conditions for the flow and an imposed temperature gradient. Two types of boundary conditions for the temperature are considered, namely *a*) prescribed temperatures at the cell boundaries; and *b*) prescribed temperature difference between the cell boundaries. The results are then compared with the ones obtained by Nakayama and Kuwahara (1999) and some conclusions are drawn.

## 2. MATHMATICAL MODEL

To arrive at the macroscopic one-equation model for the heat transport in a porous medium, one starts with the microscopic energy equations for the fluid and the porous matrix (solid) and then one applies the time average procedure followed by the volume average operator, or vice-versa, making use of the local thermal equilibrium hypothesis.

### 2.1. Microscopic Energy Equations

For an incompressible flow in a rigid, homogeneous and saturated porous medium, without internal sources, one can write:

$$\left(\rho c_p\right)_f \left\{\frac{\partial T_f}{\partial t} + \nabla \cdot \left(\mathbf{u} T_f\right)\right\} = \nabla \cdot \left(k_f \nabla T_f\right) \tag{1}$$

$$\left(\rho c_p\right)_s \frac{\partial T_s}{\partial t} = \nabla \cdot \left(k_s \nabla T_s\right) \tag{2}$$

where Eqs.(1) and (2) refer to the fluid and solid, respectively. Here $\rho$ is the density, $c_p$ is the specific heat, $k$ is the thermal conductivity, $T$ is the temperature, $\mathbf{u}$ is the fluid velocity and the subscripts $(\,)_f$ and $(\,)_s$ refer to the fluid and solid matrix, respectively.

### 2.2. Macroscopic Energy Equation

Applying the time average followed by the volume average, or vice-versa, to the microscopic energy equations for the fluid and the solid, using the *double decomposition* concept introduced by Pedras e de Lemos (1999a) and taking into account the local thermal equilibrium hypothesis, one gets:

$$\begin{aligned}
\left\{\left(\rho c_p\right)_f \phi + \left(\rho c_p\right)_s (1-\phi)\right\} \frac{\partial \langle \overline{T} \rangle^i}{\partial t} + \left(\rho c_p\right)_f \nabla \cdot \left(\overline{\mathbf{u}}_D \langle \overline{T} \rangle^i\right) = \\
\nabla \cdot \left\{\left[k_f \phi + k_s (1-\phi)\right] \nabla \langle \overline{T} \rangle^i\right\} + \\
\nabla \cdot \underbrace{\left[\frac{1}{\Delta V} \int_{A_i} \mathbf{n}\left(k_f \overline{T_f} - k_s \overline{T_s}\right) dS\right]}_{I} - \\
\left(\rho c_p\right)_f \nabla \cdot \left[\phi \left(\underbrace{\overline{\langle \mathbf{u}' \rangle^i \langle T_f' \rangle^i}}_{II} + \underbrace{\langle {}^i\overline{\mathbf{u}}\, {}^i\overline{T_f} \rangle^i}_{III} + \underbrace{\langle \overline{{}^i\mathbf{u}'\, {}^iT_f'} \rangle^i}_{IV}\right)\right]
\end{aligned} \tag{3}$$

where use has been made of the fact that $\langle \overline{T}_f \rangle^i = \langle \overline{T}_s \rangle^i = \langle \overline{T} \rangle^i$ and $\overline{\mathbf{u}}_D = \langle \overline{\mathbf{u}} \rangle^v = \phi \langle \overline{\mathbf{u}} \rangle^i$.

Equation (3) is the macroscopic one-equation energy model for the heat transport in a porous medium. In order to use this equation to calculate the temperature field in a porous medium, the underscored terms on the right hand side need to be modeled in terms of the macroscopic velocity and temperature, $\overline{\mathbf{u}}_D$ and $\langle \overline{T} \rangle^i$. This is accomplished using a gradient type diffusion model for each term as follow:

I) Tortuosity-

$$\left[\frac{1}{\Delta V}\int_{A_i}\mathbf{n}\left(k_f\overline{T_f}-k_s\overline{T_s}\right)dS\right]=\underline{\underline{K}}_{tor}\cdot\nabla\langle\overline{T}\rangle^i \tag{4}$$

II) Turbulent heat flux-

$$-\left(\rho c_p\right)_f\left(\phi\ \overline{\langle\mathbf{u}'\rangle^i\langle T_f'\rangle^i}\right)=\underline{\underline{K}}_t\cdot\nabla\langle\overline{T}\rangle^i \tag{5}$$

III) Thermal dispersion-

$$-\left(\rho c_p\right)_f\left(\phi\ \langle{}^i\overline{\mathbf{u}}\,{}^i\overline{T_f}\rangle^i\right)=\underline{\underline{K}}_{disp}\cdot\nabla\langle\overline{T}\rangle^i \tag{6}$$

IV) Turbulent thermal dispersion-

$$-\left(\rho c_p\right)_f\left(\phi\ \langle\overline{{}^i\mathbf{u}'\,{}^iT_f'}\rangle^i\right)=\underline{\underline{K}}_{disp,t}\cdot\nabla\langle\overline{T}\rangle^i \tag{7}$$

Thus, using the diffusivity models given above, the macroscopic one-equation model for the heat transport in a porous medium can be written as:

$$\left\{\left(\rho c_p\right)_f\phi+\left(\rho c_p\right)_s(1-\phi)\right\}\frac{\partial\langle\overline{T}\rangle^i}{\partial t}+\left(\rho c_p\right)_f\nabla\cdot\left(\mathbf{u}_D\langle\overline{T}\rangle^i\right)=\nabla\cdot\left\{\underline{\underline{K}}_{eff}\cdot\nabla\langle\overline{T}\rangle^i\right\} \tag{8}$$

where the effective thermal conductivity tensor, $\underline{\underline{K}}_{eff}$, is given by:

$$\underline{\underline{K}}_{eff}=\left[\phi k_f+(1-\phi)k_s\right]\underline{\underline{I}}+\underline{\underline{K}}_{tor}+\underline{\underline{K}}_t+\underline{\underline{K}}_{disp}+\underline{\underline{K}}_{disp,t} \tag{9}$$

## 2.3. Obtension of the Thermal Conductivity Tensors

The thermal conductivity tensors due to the turbulent heat flux and turbulent thermal dispersion, $\underline{\underline{K}}_t$ and $\underline{\underline{K}}_{disp,t}$, given in Eqs. (5) and (7), respectivelly, are modeled through the macroscopic eddy diffusivity concept, as presented in Rocamora and de Lemos (2001), as:

$$\underline{\underline{K}}_t+\underline{\underline{K}}_{disp,t}=\phi(\rho c_p)_f\frac{\nu_{t_\phi}}{\sigma_T}\underline{\underline{I}} \tag{10}$$

where $\nu_{t_\phi}$ express the macroscopic version of the eddy viscosity, $\mu_{t_\phi}=\rho_f\,\nu_{t_\phi}$ ,given by:

$$\mu_{t_\phi}=\rho_f\,C_\mu\frac{\langle k\rangle^{i\,2}}{\langle\varepsilon\rangle^i} \tag{11}$$

To obtain the dispersion and tortuosity conductivity tensors, $\underline{\underline{K}}_{disp}$ and $\underline{\underline{K}}_{tor}$, which are present in both laminar and turbulent flows, a unit cell like the one shown in Fig. 1 is used.

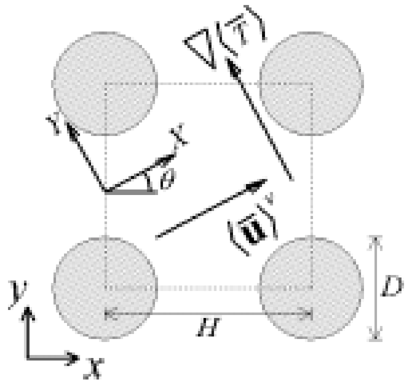



Figure 1 - Unit cell for microscopic calculation.

For the microscopic calculations of the velocity and temperature fields in the unit cell, periodic boundary conditions for the flow and an imposed macroscopic temperature gradient are used. These boundary conditions are defined in the equations bellow:

$$\begin{aligned} \overline{\mathbf{u}}\big|_{x=0} &= \overline{\mathbf{u}}\big|_{x=H} \\ \overline{\mathbf{u}}\big|_{y=0} &= \overline{\mathbf{u}}\big|_{y=H} \end{aligned} \tag{12}$$

$$\begin{aligned} \left.\int_0^H \overline{u}\, dy\right|_{x=0} &= \left.\int_0^H \overline{u}\, dy\right|_{x=H} = H\left|\langle\overline{\mathbf{u}}\rangle^v\right|\cos\theta \\ \left.\int_0^H \overline{v}\, dx\right|_{y=0} &= \left.\int_0^H \overline{v}\, dx\right|_{y=H} = H\left|\langle\overline{\mathbf{u}}\rangle^v\right| sin\theta \end{aligned} \tag{13}$$

and

$$\begin{aligned} T\big|_{x=0} &= T\big|_{x=H} + \Delta T\, sin\theta \\ T\big|_{y=0} &= T\big|_{y=H} - \Delta T\cos\theta \end{aligned} \quad \text{: transversal components,} \tag{14}$$

$$\begin{aligned} T\big|_{x=0} &= T\big|_{x=H} - \Delta T\cos\theta \\ T\big|_{y=0} &= T\big|_{y=H} - \Delta T\, sin\theta \end{aligned} \quad \text{: longitudinal components.} \tag{15}$$

where

$$\langle\overline{\mathbf{u}}\rangle^v = \left|\langle\overline{\mathbf{u}}\rangle^v\right|\left(\cos\theta\,\vec{i} + sin\theta\,\vec{j}\right) \tag{16}$$

$$\nabla\langle\overline{T}\rangle = \frac{\Delta T}{H}(-sin\theta\,\vec{i} + \cos\theta\,\vec{j}) \quad \text{: transversal components,} \tag{17}$$

$$\nabla\langle\overline{T}\rangle = \frac{\Delta T}{H}(\cos\theta\,\vec{i} + sin\theta\,\vec{j}) \quad \text{: longitudinal components.} \tag{18}$$

From the microscopic unit cell results the dispersion and tortuosity thermal conductivity tensors' components are then calculated through the expressions obtained from Eqs. (4) and (6) [Nakayama and Kuwahara (1999)], which read:

$$\left(\underline{\underline{K}}_{tor}\right)_{XX}=\frac{-\left(\frac{k_s-k_f}{V}\int_{A_i}\overline{T}\,d\vec{A}\right)\cdot\left(cos\theta\;\vec{i}+\sin\theta\;\vec{j}\right)}{\left(\Delta T/H\right)} \tag{19}$$

$$\left(\underline{\underline{K}}_{disp}\right)_{XX}=\frac{-\frac{(\rho c_p)_f}{H^2}}{\left(\Delta T/H\right)}\int_0^H\int_0^H\left(\overline{T}-\langle\overline{T}\rangle\right)\left(\overline{\mathbf{u}}-\langle\overline{\mathbf{u}}\rangle^i\right)dxdy\cdot\left(\cos\theta\;\vec{i}+\sin\theta\;\vec{j}\right) \tag{20}$$

for the longitudinal components, and

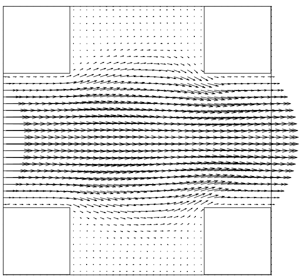
$$\left(\underline{\underline{K}}_{tor}\right)_{YY}=\frac{-\left(\frac{k_s-k_f}{V}\int_{A_i}\overline{T}\,d\vec{A}\right)\cdot\left(-sin\theta\;\vec{i}+cos\theta\;\vec{j}\right)}{\left(\Delta T/H\right)} \tag{21}$$

$$\left(\underline{\underline{K}}_{disp}\right)_{YY}=\frac{-\frac{(\rho c_p)_f}{H^2}}{\left(\Delta T/H\right)}\int_0^H\int_0^H\left(\overline{T}-\langle\overline{T}\rangle\right)\left(\overline{\mathbf{u}}-\langle\overline{\mathbf{u}}\rangle^i\right)dxdy\cdot\left(-sin\theta\;\vec{i}+\cos\theta\;\vec{j}\right) \tag{22}$$

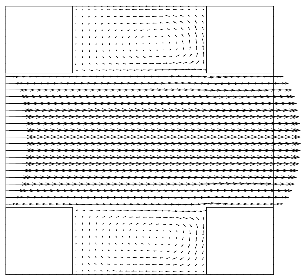
for the transversal components.

## 3. RESULTS

The unit cell used to obtain the correlations for the thermal conductivity tensors' components is composed of square rods in order to compare with the results of Kuwahara and Nakayama (1998). Here, two types of boundary conditions for the energy equation are used. The first type is prescribed temperature at the cell boundaries and the second is prescribed temperature difference between the cell boundaries. Figure 2 shows the velocity fields obtained for Reynolds numbers, $Re_H$, of 10 and 1000 for $\theta$=0$^o$ and $\phi$=0.75.



*a)* $Re_H$ =10.

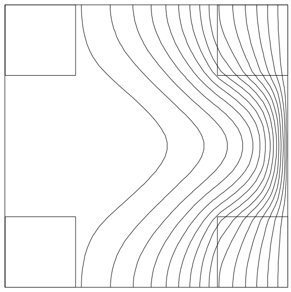


*b)* $Re_H$ =1000.

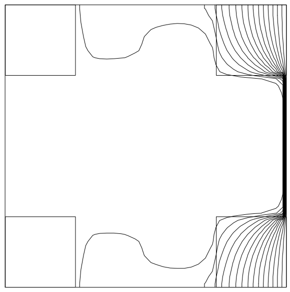
Figure 2 - Velocity Fields for the unit cell ($\phi$=0,75; $\theta$=0°). *a)* $Re_H$ =10 and *b)* $Re_H$ =1000.

For the unit cell it was considered *H*=0.01 m, $k_s/k_f=2.0$ and $\nabla T=50^oC$.

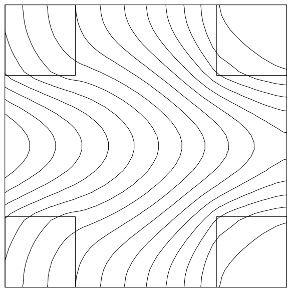
The corresponding temperature fields using both types of boundary conditions described above are shown in Fig. 3 bellow.



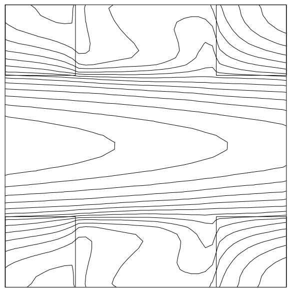
*a*) $Re_H$ =10.



*b*) $Re_H$ =1000.



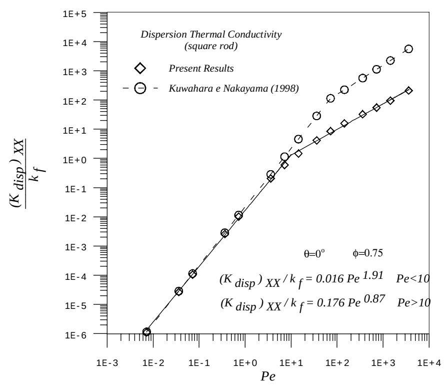
*c*) $Re_H$ =10.



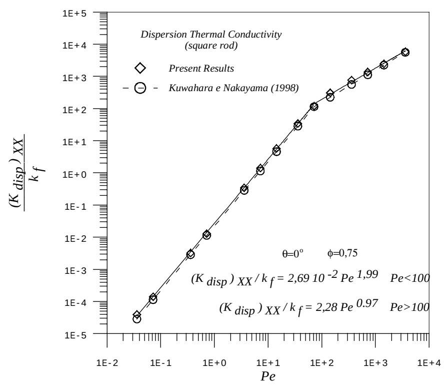
*d*) $Re_H$ =1000.

Figure 3 - Isotherms for the unit cell ($\theta$ =0°, $\phi$=0.75) for longitudinal temperature gradient (*X* direction): *a*) and *b*)- Specified temperature; *c*) and *d*)- Specified temperature difference.

As can be observed from Fig. 3, the temperature fields for the two types of boundary conditions considered differ substantially. The correlations obtained for the longitudinal component of the dispersion thermal conductivity tensor, $\left(\underline{\underline{K}}_{disp}\right)_{XX}$ , are shown in Fig. 4 bellow. Also shown are the results of Kuwahara and Nakayama (1998).



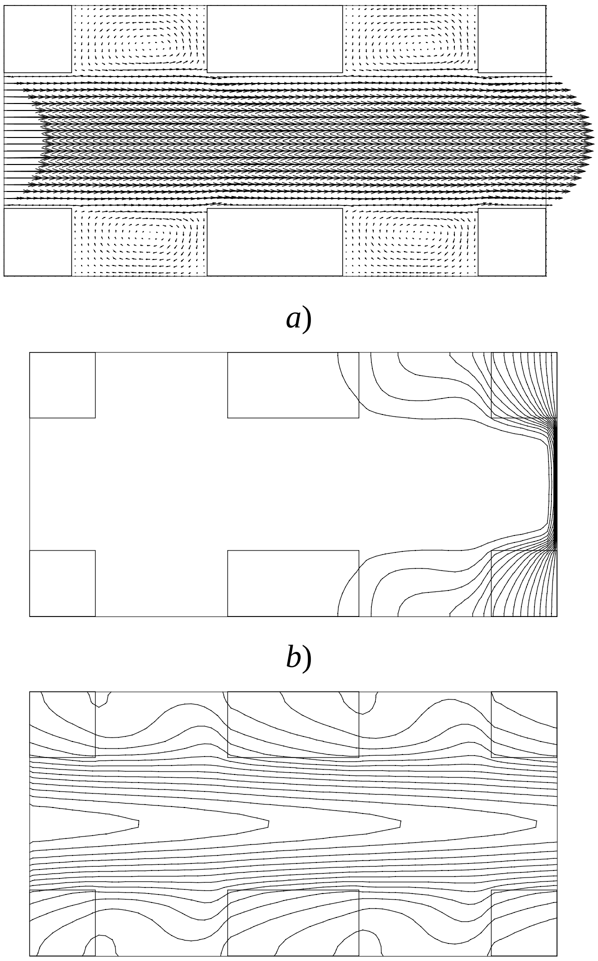

*a*)




*b*)

Figure 4 - Dispersion thermal conductivity tensor component in the *X* direction ($\phi$=0.75; $\theta$ =0°): *a*) Specified temperature; *b*) Specified temperature difference.

It is observed that the results obtained using the first type of boundary conditions, *i.e.*, prescribed temperature, show a difference of about two orders of magnitude in the high *Pe* range compared to Kuwahara and Nakayama (1998) results. On the other hand, for the second type of boundary conditions, *i.e.*, prescribed temperature difference, the results seam to match very closely Kuwahara and Nakayama (1998) results. In order to investigate the reason for this behavior, a run was made for two cells in a sequence for both types of boundary conditions. The results obtained are shown in Fig. 5 .



*a*)

*b*)

*c*)

Figure 5 - Velocity and temperature Fields for two cells in a row ( $Re_H$ =180.). Cases *a*) Velocity Field, *b*) Specified Temperature and *c*) Specified Temperature Difference.

It is apparent from Fig. 5(*b*) that the temperature field obtained for the first type of boundary condition results in a very different pattern for the two consecutive cells. This would cause the conductivity tensor obtained for one cell to be different from the one obtained for the other cell. On the other hand, Fig. 5(*c*) shows that for the second type of boundary conditions the temperature pattern is just about the same for both cells. This would result in the same values for the conductivity tensors for both cells. As the idea is to simulate the porous medium as an infinite array of unit cells, this leads to the conclusion that the first type of boundary conditions for the energy equation is not suitable for calculating the thermal conductivity tensors' components. Nevertheless, it should be pointed out that for both types of boundary conditions, a macroscopic temperature gradient is being imposed to a microscopic problem (unit cell).

## 4. CONCLUSIONS

In this work two procedures for obtaining the dispersion and tortuosity thermal conductivity tensors' components using a unit cell to represent the porous medium were analyzed. It was shown that the imposition of a macroscopic temperature gradient through the specification of the temperatures at the cell boundaries is not suitable to calculate the temperature field in the cell. This was accomplished using two unit cells in a sequence and analyzing the temperature fields obtained by both procedures. It was also shown that for the second type of boundary conditions the results obtained for the thermal conductivity tensors' components compared very well with the ones obtained by Kuwahara and Nakayama (1998).

## 5. ACKNOWLEDGEMENTS

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