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FULLY DEVELOPED LAMINAR FLOW IN CONCENTRIC CURVE CIRCULAR DUCTS WITH FOUR RECTANGULAR FINS

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Abstract. *This work presents a numerical investigation about the secondary flow effect on the heat transfer and friction factor for fully developed laminar flow through curved concentric annular duct with four rectangular fins disposed orthogonally each other. The outer tube surface is thermally insulated and the inner tube and fins has a specified peripherally constant temperature as thermal condition. The mass conservation (Poisson equation), momentum and energy partial differential equations are numerically solved by the Galerkin finite element method. Numerical simulations are carried for the water flow (Prandtl number equal to 2.5) with a constant annular duct curvature ratio. The Nusselt number and friction factor are obtained as a function of the Dean number varying the fins angle position. In addition, the velocity and temperature fields on the tube cross-section are also determined.*

Keywords: *concentric curved tube, heat transfer, friction factor, fins*

1. INTRODUCTION

Technology has led to a demand for high-performance, light-weight, and compact heat transfer components. To accommodate this demand, finned walls configurations are commonly used to increase the heat transfer rate between an internal duct and the surrounding fluid in heat exchangers and refrigeration equipments. The finned surface efficiency effect is determined by the analysis of the heat transfer friction factor and finned region results.

The first analytical investigation on flow in a coil tube was performed by Dean (1927). These results showed that the centrifugal forces induce a secondary circulation, represented by two vortices perpendicular to the main axial flow.

According to Shah and Joshi (1987), the curved ducts have a higher heat transfer rates than equivalent straight ducts. It occurs due to secondary flows that increase the momentum and energy exchanges. A large number of works into curved ducts have been developed using the toroidal coordinate system.

Kalb and Seader (1972) studied the viscous flow in curved circular tubes considering fully developed velocity and temperature fields under the thermal boundary conditions of axially uniform wall heat flux with peripherally uniform wall temperature. These results are showed to Dean-number range from 1 to 1200.

Sillekens et al. (1998) studied the development of mixed convection in a coiled heat exchanger using a finite difference method. Their results showed that heat transfer process is highly influenced by secondary flow induced by curvature effect and buoyancy force.

Several authors have focused heat transfer problem in straight annular ducts. Nieckele and Patankar (1985) present a numerical study of the fully developed laminar flow and heat transfer in a concentric annulus, in which the heating from the inner cylinder leads to significant buoyancy-induced secondary flow.

Xin et al. (1997) present an experimental investigation in annular helicoidal pipes. The coil geometry and flow rates effects on single-phase and two-phase flow pressure drop were experimentally studied. After, they establish correlations and compared with the Lockhart-Martinelli parameter for the two-phase flow case.

Silva et al. (1997) reported the flow rate influence on the flow and heat transfer in fully developed region in a steady laminar annular flow. Authors considered two immiscible liquids inside horizontal and slightly curved tube with constant circular cross-section and the solution is obtained by the finite volume method.

An analytical study of heat transfer in fully developed laminar annular ducts with peripherally varying heat flux has been carried out by Buyruk et al. (1999). The axsymmetric temperature distribution is imposed and the perturbation temperature solution is numerically obtained using a point-matching method.

An theoretical-numerical analysis of longitudinal and annular fins and spines was presented by Laor and Kalman (1996). These authors present a study using rectangular, triangular and parabolic profiles for the fins and cylindrical, conical and parabolic for the spines.

Braga and Saboya (1999) determined experimentally average heat transfer coefficient and frictions factor results for turbulent flow through annular ducts with continuous longitudinal rectangular fins. In addition, the fin efficiency was also determined by means of a numerical two-dimensional heat transfer analysis. These data are presented in a dimensionless form, and the of the average Nusselt number, friction factor and fin efficiency are plotted as functions of the Reynolds number.

Pantaleão et al (2001) studied the secondary flow influence on the heat transfer and pressure drop results for the fully developed laminar flow in curved concentric annular ducts. It was observed that the secondary flow induced by the annular duct curvature increases the heat transfer rate and the pressure drop. This effect is more accentuated for small radii ratios due to a stronger secondary flow in the cross-section free-flow area.

The purpose of the present work is to analyze the heat transfer rate and friction factor for the fully developed laminar flow in concentric annulus. Rectangular fins are disposed orthogonally each other at the inner duct and the centrifugal force influence will be also considered.

Numerical solution is obtained employing the Galerkin finite element technique with an unstructured and adaptative mesh. Nusselt number and friction factor results are presented as a function of the Dean-number parameter, varying the fins angle position for different angle (0, 30°, 45° and 60°). Four fins are attached to the internal tube wall and their thicknesses are maintained constant.

2 MAHEMATICAL FORMULATION

Steady-state laminar incompressible water-flow in a curved annular duct with fins is analyzed using a toroidal coordinate system showed in Fig. (1). The flow is both hydrodynamically and thermally fully developed, with negligible viscous dissipation and axial conduction. All fluid properties are considered constant.

The fully developed flow and the constant axial temperature gradient assumptions result in the following conditions for velocity and temperature profiles:

$$\frac{\partial u}{\partial z} = \frac{\partial v}{\partial z} = \frac{\partial w}{\partial z} = 0 \;\; ; \;\; \frac{\partial T}{\partial z} = \frac{dT_w}{dz} = \frac{dT_b}{dz} = \text{constant} \tag{1}$$

where z is the axial coordinate (main flow), w is the velocity component in the z direction, u and v are the velocity components in the transversal section (secondary flow), T_w is the wall temperature and T_b is the fluid bulk mean temperature.

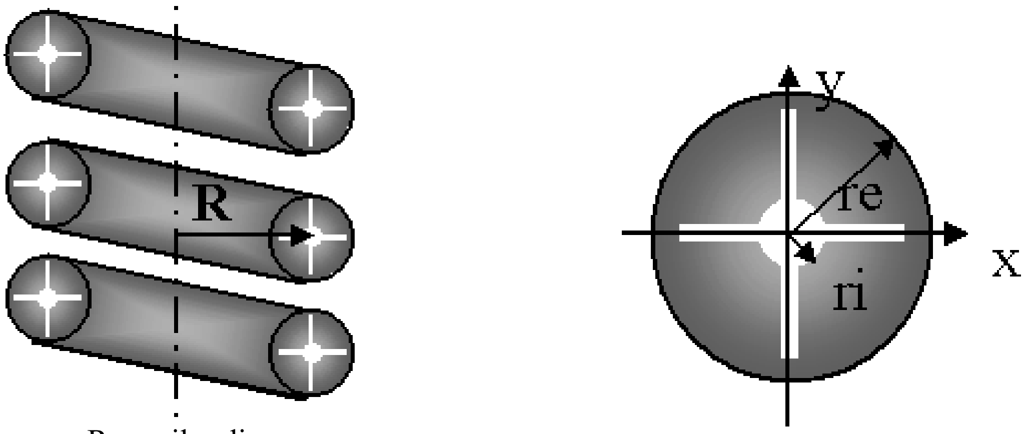


Figure 1. Schematic representation of the curved annular duct with four rectangular fins.

The total pressure field P'(x, y, z) is decoupled in an axial contribution and in a part corresponding to the transversal one as:

$$P'(x,y,z) = \overline{P}(z) + P(x,y) \tag{2}$$

The governing equations (continuity, energy, x, y and z-momentum) are represented by:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{u}{R}\left[\frac{1}{1+(x/R)}\right] = 0 \tag{3}$$

$$\begin{aligned}\rho\left(u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} - \frac{w^2}{R(1+x/R)}\right) = & -\frac{\partial P}{\partial x} + \frac{1}{(x+R)}\frac{\partial}{\partial x}\left[(x+R)\left(2\mu\frac{\partial u}{\partial x}\right)\right] \\ & -\frac{(2\mu u)}{(x+R)^2} + \frac{\partial}{\partial y}\left[\mu\left(\frac{\partial v}{\partial x} + \frac{\partial u}{\partial y}\right)\right]\end{aligned} \tag{4}$$

$$\rho\left(u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y}\right) = -\frac{\partial P}{\partial y} + \frac{1}{(x+R)}\frac{\partial}{\partial x}\left[\mu(x+R)\left(\frac{\partial v}{\partial x} + \frac{\partial u}{\partial y}\right)\right] + \frac{\partial}{\partial y}\left(2\mu\frac{\partial v}{\partial y}\right) \tag{5}$$

$$\rho\left[u\frac{\partial w}{\partial x}+v\frac{\partial w}{\partial y}+\frac{uw}{(x+R)}\right]=-\frac{1}{(x+R)}\frac{\partial\overline{P}}{\partial z}+\frac{1}{(x+R)^2}\frac{\partial}{\partial x}\left\{(x+R)^3\left[\mu\frac{\partial}{\partial x}\left(\frac{w}{x+R}\right)\right]\right\}+\frac{\partial}{\partial y}\left(\mu\frac{\partial w}{\partial y}\right) \tag{6}$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}+\frac{w}{(1+x/R)}\frac{dT_b}{dz}=\frac{k}{\rho\, C_p}\left[\nabla^2 T+\frac{1}{R(1+x/R)}\frac{\partial T}{\partial x}\right] \tag{7}$$

where: R is the duct curvature radius, ρ is the fluid density, μ is the fluid viscosity, C_p is the fluid constant pressure specific heat and k is the fluid thermal conductivity. From the fluid properties the Prandtl number (Pr) can be defined as:

$$Pr=\frac{\mu\, Cp}{k} \tag{8}$$

The boundary conditions for the problem are:

$$u = v = w = 0 \text{ and } T=T_w \text{ at } r = r_i = r_e\, r^* \tag{9}$$

$$u = v = w = 0 \text{ and } \frac{dT}{dn}=0 \text{ at } r=r_e \text{ in the fins external surface.} \tag{10}$$

where n is the normal surface outward unit vector; r_i and r_e are the radii of the inner and outer tubes, respectively.

After numerically determining the axial velocity (w) and the temperature field (T), the average velocity (w_m) and the Reynolds number (Re) are calculated as:

$$w_m = \frac{1}{A}\int w\, dA \quad \text{and} \quad Re=\left(\frac{\rho\, w_m\, D_h}{\mu}\right) \tag{11}$$

where A is the net free-flow area and D_h is the hydraulic diameter given by:

$$D_h = 2\ (r_e - r_i) \tag{12}$$

The Dean number (De) and the duct curvature ratio (RC) are calculated as follows:

$$De = Re\sqrt{\frac{D_h}{R}} \quad \text{and} \quad RC=\frac{R}{D_h} \tag{13}$$

The Nu (Nusselt number) is obtained as:

$$Nu = \frac{h\, D_h}{k} \tag{14}$$

where the convection coefficient h is defined as:

$$h = \frac{q_{wm}}{(T_w - T_b)}, \quad q_{wm} = \frac{(dT_b/dz)\,\rho\, C_p w_m}{(1 + x/R)} \left[\frac{(r_e^{\,2} - r_i^{\,2})}{2\, r_i} \right] \tag{15}$$

The *fRe* parameter (the friction coefficient and Reynolds number product) is expressed by:

$$f\,Re = \left(\frac{f_i\, r_i + f_e\, r_e}{r_i + r_e} \right) Re \tag{16}$$

with

$$f_i\,Re = \left(-\frac{1}{\mu} \right) \left(\frac{d\,p}{d\,z} \right) \left(\frac{Dh}{w_m} \right) \left(\frac{r_m^{\,2} - r_i^{\,2}}{r_i} \right) \tag{17}$$

and

$$f_e\,Re = \left(-\frac{1}{\mu} \right) \left(\frac{d\,p}{d\,z} \right) \left(\frac{Dh}{w_m} \right) \left(\frac{r_e^{\,2} - r_m^{\,2}}{r_i} \right) \tag{18}$$

where r_m is calculated by:

$$r_m = \sqrt{\frac{(1 - r^{*2})}{(2\,Ln\left(\frac{1}{r^*} \right)}} \tag{19}$$

2 SOLUTION METHODOLOGY

The numerical solution is obtained employing a program based upon the Galerkin finite element method with an unstructured mesh as presented in Fig. (2). It has an adaptive mesh refinement that can be controlled by default or a user refinement control parameter (tolerance). This parameter represents an error limit obtained by the following manner: as each iteration is completed the program calculates the error limit each patch and subdivides only those patches where the error exceeds the tolerance value.

The mass conservation restriction for the annular tube cross-section secondary flow is imposed by the pressure Poisson equation that is derived by combining Eq. (4) and Eq.(5).

The problem was solved using a quadratic interpolation polynomial to convert the continuous partial differential equations, represented by the equations (4) to (7) and the Poisson equation, into discrete nodal equations. The algebraic equations system has been solved through the iterative conjugate-gradient method. The mesh refinement is automatically processed and presents more intense refinement in regions which have large curvature or that are subjected to high temperature gradients. Fig. (2) presents the curved concentric annulus cross-section and an intermediary mesh in the solution process. This program allows the velocity field visualization, identifying the regions where the secondary flow is more intense (Fig. (3)).

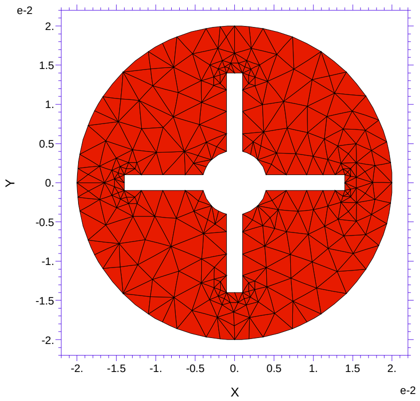


Figure 2. Computational domain and an intermediary mesh in the solution process.

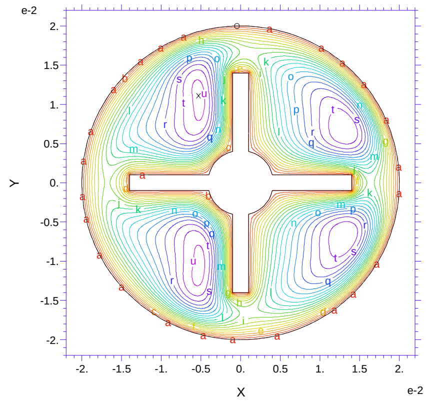


Figure 3. The average velocity (w/w_m) for Dean number = 127 and fin angle = 0.

4 RESULTS AND CONCLUSION

To validate the numerical code a comparison with the literature data (Shah and Joshi, 1987) for straight tube concentric annular duct was presented in Pantaleão et al. (2001) and showed a good agreement. At the present work the curved concentric annulus case is analyzed with four rectangular attached to the internal tube wall. It was verified that the solution convergence is reached for a tolerance parameter smaller than 1.0 10^{-6}.

The numerical simulations were carried out for water flow with $Pr = 2.5$ (Eq. (8)) and a constant duct curvature ratio $R/D_h = 10$ indicated in Eq. (13). The internal to external radii ratio $r^* = r_i/r_e$ was considered equal the 0.2.

Fig. (4) shows the secondary flow patterns for four different fin angles at the curved duct cross-section and Dean number =70.

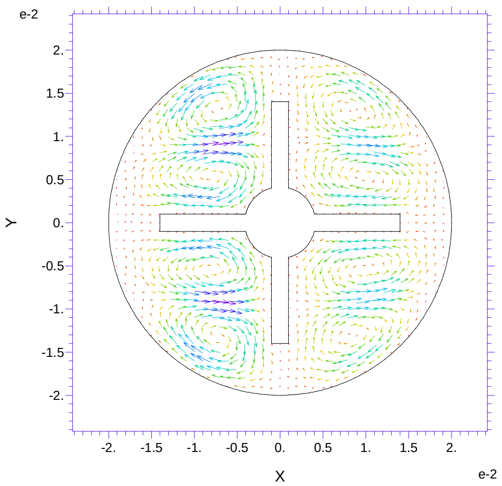


Figure 4a. Secondary flow in the curved annular duct cross- section at Dean number = 70 and angle = 0°

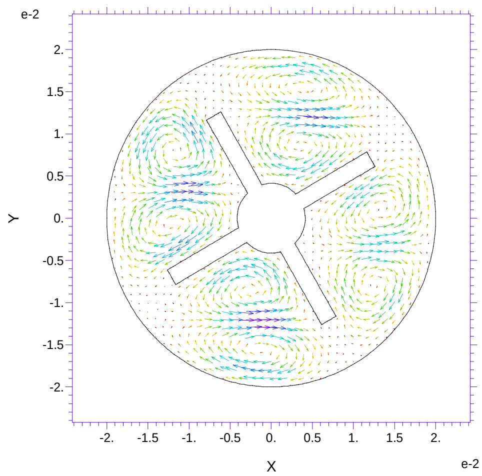


Figure 4b. Secondary flow in the curved annular duct cross- section at Dean number = 70 and angle = 30°

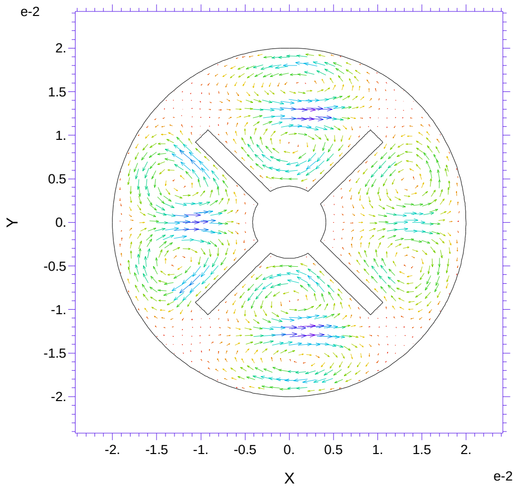


Figure 4c. Secondary flow in the curved annular duct cross- section at Dean number = 70 and angle = 45°

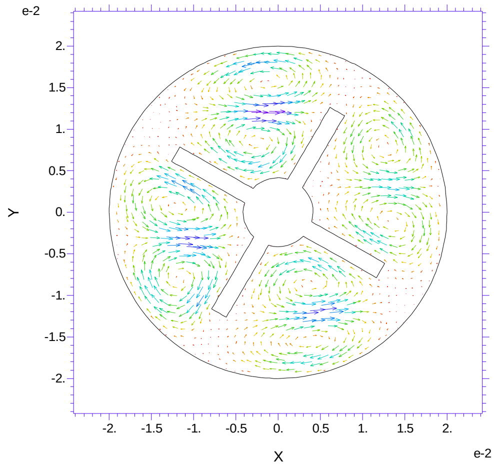


Figure 4d. Secondary flow in the curved annular duct cross- section at Dean number = 70 and angle = 60°

The results presented in Fig (4) show the secondary flow induced by the centrifugal force. In Fig. (4) is observed the formation of two counter-rotating cells in the secondary flow between two adjacent fins. Such cells are induced by the duct curvature and enhance the momentum transfer at the duct cross-section. The fin angle variation only displaces the secondary flow position but doesn't change significantly its intensity. The flow is maintained weaker close to the fin tip.

Fig. **(**5) presents the temperature field for four different fin angles and Dean number =0.9. The temperature gradients are concentrated near to the fins surface, where the high temperature value is prescribed. The low temperature region is located between two adjacent fins close to the external tube wall. It is observed that these temperature fields are similar for all inclination angles considered.

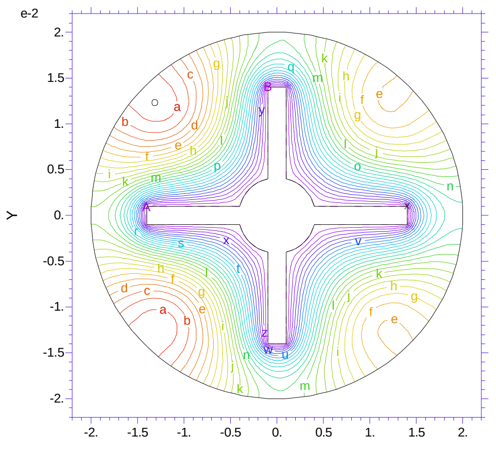


Figure 5a. Temperature field at Dean number = 0.9 and angle = 0

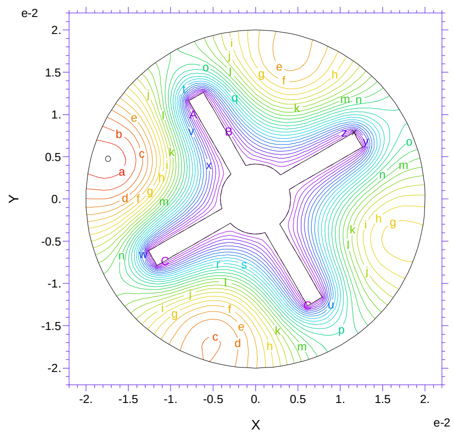
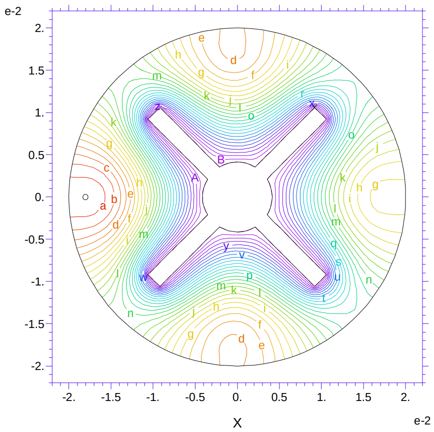


Figure 5b. Temperature field at Dean number = 0.9 and angle = 30°



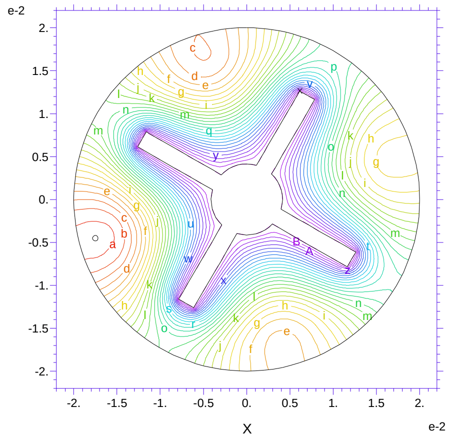
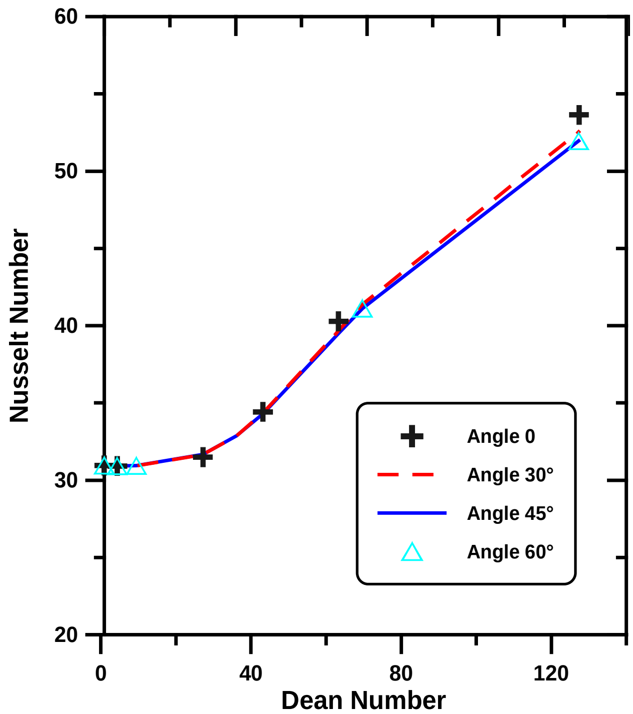


Figure 5c. Temperature field at Dean number = 0.9 and angle = 45°

Figure5d. Temperature field at Dean number = 0.9 and angle = 60

The Nusselt number and friction factor results are plotted respectively in Fig. (6) and Fig. (7) as a function of the Dean number. For *De* < 30 the *Nu* and *fRe* values are practically constant. As the *De* number increases these results also elevate for the four configuration angle simulated. When *De* = 120, the heat transfer rate is almost six times more intense than the straight concentric annulus without fins, where the Nusselt number is equal to 8.5 (Shah and Joshi, 1987; Pantaleão et al, 2001). However, these heat transfer enhancement is punished by a pressure drop elevation.



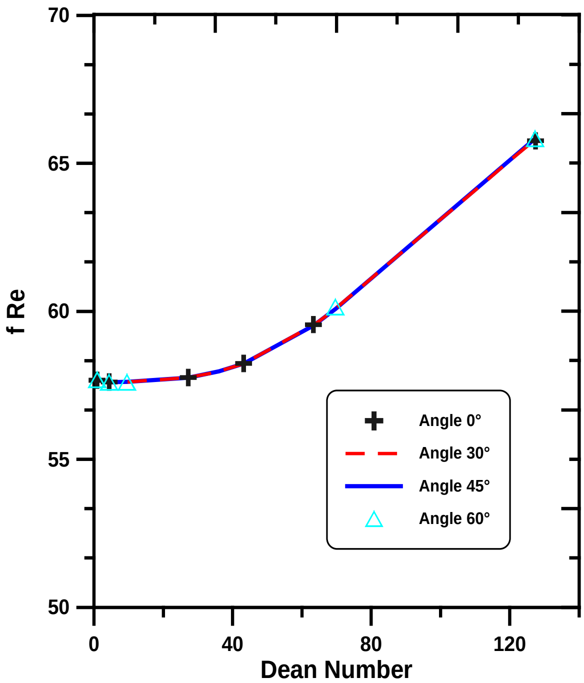


Figure 6 Nusselt number results for concentric curve circular ducts with four rectangular fins.

Figure 7. Friction factor results for the concentric curve circular ducts with four rectangular fins.

The results showed that both Nusselt number and friction factor don't differ significantly for the fins angles here considered (angle equal the 0, 30°, 45° and 60°) in the Dean number range analyzed. Such results allow conclude that heat transfer rate and the pressure drop aren't sensible to the fins rotation. In spite of the secondary flow intensity reduction (in comparison with the unfinned internal tube), the fin intensifies the heat transfer rate mainly due to the increase in the heat transfer area.

5 ACKNOWLEDGES

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