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BUBBLE DYNAMICS TAKING INTO ACCOUNT IRREVERSIBLE PHASE CHANGE TRANSFORMATIONS

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Abstract. *This paper presents a continuum thermo-mechanical coherent model to describe isothermal flows of incompressible liquid with cavities filled with vapor and a non-condensable gas. The motions of the constituents inside the cavities are described with the aid of the continuum mixture theory, while the liquid motion description is accomplished by the classical continuum theory. The concepts of the singular surface theory are used to provide the additional interfacial balance equations. Irreversible phase change transformations are accounted for in such a way that the interfacial entropy inequality is always satisfied, regardless the singular surface shape and the initial and boundary conditions. The model applicability is tested by solving numerically the single bubble dynamics problem. In particular, it is investigated the influence of the phase change process on the bubble collapse.*

Key-words: *bubble dynamics, phase change, irreversible process, mixture theory, singular surface.*

1. INTRODUCTION

The theoretical research interest in single bubble dynamics and cavitation goes back to the beginning of the twenty century, when Besant (1913) first investigated the pressure developed by a collapsing cavity. Almost at the same time, experimental interest arose from the finding that cavitation was associated with erosion, which was first reported on ship propellers (Silberrad, 1912). Soon after, Rayleigh (1917) has made the first consistent and relevant analysis in cavitation and bubble dynamics by solving the problem of the collapse of an empty cavity in an unbounded, incompressible, non-viscous liquid medium. The growth and collapse of the bubbles - also referred

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to in the literature as bubble dynamics- in contact with solid surfaces might have damaging effects on the solid and so has been of great interest in mechanics.

Sixty years later, Plesset and Prosperetti (1977) have included in the analysis liquid viscosity and surface-tension effects by assuming, at first, that the medium filling the cavity was essentially a permanent non-condensable gas. Viscous, thermal and acoustic components of the growth and collapse of a bubble have been analyzed, when it was subjected to small-amplitudes and non-linear oscillations. Later on that and in another work (Plesset and Prosperetti, 1976), other effects such as the liquid compressibility, mass-diffusion processes across the bubble liquid interface, energy flow into and out of the bubble as well as the stability of the interface have been considered.

Although significant progress has been achieved along the years, a growing interest in such a problem has motivated further development in this subject due to its applications not only on mechanics but also in the biomedical area. From the 80's to the present time, a large amount of theoretical and experimental works have been carried out, in which emphasis has been placed on phase change process (evaporation-condensation) and shock formation effects (Vuong et al and references therein., 1999).

In the great majority of these problems the bubble dynamics is analytically treated by the traditional thermostatic theory, which assumes there is no dissipation associated to the phase change process. However, it is well accepted that this approach may lead to unsatisfactory results (Freitas Rachid, 2002), what motivates the search for more physically realistic and mathematically coherent models.

The theoretical analysis of the phase change process in the context of irreversible thermodynamics has been originally carried out by Bornhorst and Hatsopoulos (1967), who have derived general rate equations across the non-equilibrium region existing at a phase interface. However, their study is limited to phase change problems involving only linear phenomenological equations without taking a third constituent (a non-condensable gas) into account.

This paper presents a logically consistent thermo-mechanical model to describe the isothermal bubble dynamics problem. The continuum mixture theory is used to describe the motions of the vapor and of a non-condensable gas inside the cavity, while the traditional continuum theory is used to model the Newtonian behavior of the liquid outside the cavity. Interfacial balance equations of mass, linear momentum as well as an entropy inequality are considered with the aid of the concepts of singular surface theory. The macroscopic dissipative effects of the liquid-vapor transformation are accounted for in such a way the interfacial entropy inequality is always satisfied. The potentiality of the model as well as its basic features are illustrated through a numerical example in which it is investigated the influence of the phase change process in the cavity collapse.

2. THEORETICAL MODEL

Consider a flow of a liquid in which there are cavities filled with its vapor and a non-condensable gas (air, for example). The liquid and the air-vapor filled cavities occupy open regions of the three-dimensional space denoted by Ω_l and Ω_g, respectively, so that the liquid-cavity interfaces are characterized by the surfaces described by $\overline{\Omega}_l \cap \overline{\Omega}_g$. It is implicitly assumed hereinafter that these domains have the necessary regularity in order to assure that the forthcoming analysis makes sense.

The flow is supposed to be isothermal, so that the three constituents (liquid, vapor and air) have all a same temperature at any spatial position $\boldsymbol{x} \in \Re^3$ and at any time instant t. The motion of an isothermal mixture of vapor and air, which is confined to the domain Ω_g, is described here with the aid of the Continuum Theory of Mixtures. In this theory, the constituents have independent superimposed kinematics. It means that every spatial position $\boldsymbol{x}$ in the mixture is occupied simultaneously by both air and vapor, for each time instant *t.* To each constituent it is associated a local density $\rho_\alpha, \alpha \in \Xi = \{g, v\}$, which is defined as being the ratio between the mass of the

constituent α and the total volume of the mixture, and a spatial velocity field $\boldsymbol{v}_\alpha$, with $\alpha \in \Xi$. For $\alpha = g$, the properties of the constituent are associated with the air and for $\alpha = v$ the properties are referred to the vapor. In the absence of a gravitational field and also by considering that the air and the vapor do not react with each other, the balance equations of mass and linear momentum for both constituents can be expressed as (Rajagopal and Tao, 1995):

$$\begin{aligned} &\frac{\partial}{\partial t}\rho_\alpha + \nabla\cdot\left(\rho_\alpha \boldsymbol{v}_\alpha\right) = 0 \ \text{ in } \ \Omega_{\mathrm{g}} \ \text{ for } \ \alpha \in \Xi \\ &\frac{\partial}{\partial t}\left(\rho_\alpha \boldsymbol{v}_\alpha\right) + \nabla\cdot\left(\rho_\alpha \boldsymbol{v}_\alpha \otimes \boldsymbol{v}_\alpha\right) = \nabla\cdot\boldsymbol{T}_\alpha + \boldsymbol{f}_\alpha \ \text{ in } \ \Omega_{\mathrm{g}} \ \text{ for } \ \alpha \in \Xi \end{aligned} \tag{1}$$

In the above equations, $\boldsymbol{T}_\alpha$ stands for the partial stress tensor, $\boldsymbol{f}_\alpha$ the internal force of interaction between the two constituents and the symbol $\otimes$ denotes the usual tensorial product.

To ensure that the balance of linear and angular momentum be satisfied for the mixture as a whole, the internal force of interaction and the partial stress tensor are to subjected to the following restriction:

$$\begin{aligned} &\sum_{\substack{\alpha \\ \alpha\in\Xi}} \boldsymbol{f}_\alpha = 0 \\ &\boldsymbol{T}_\alpha = \boldsymbol{T}_\alpha^T \quad \text{for} \quad \alpha \in \Xi \end{aligned} \tag{2}$$

It should be noticed that the last equation is far restrictive than that required by the classical Theory of Continuum Mixtures (Rajagopal and Tao, 1995). It is further assumed that both the air and the vapor behave as a compressible Newtonian fluid whose constitutive behavior is given by:

$$\boldsymbol{T}_\alpha = -p_\alpha \boldsymbol{I} + \boldsymbol{S}_\alpha \tag{3}$$

in which p_α is the thermodynamic pressure ($p_\alpha > 0$) and $\boldsymbol{S}_\alpha = 2\mu_\alpha \boldsymbol{D}_\alpha$ is the extra stress tensor due to motion. The dynamic viscosity of the air and the vapor is represented by μ_α and $\boldsymbol{D}_\alpha = 1/2\left(\nabla \boldsymbol{v}_\alpha + \left(\nabla \boldsymbol{v}_\alpha\right)^T\right)$ is the rate of strain of these constituents. As usual in Continuum Theory Mixture, we adopt the following form for the interaction force between the constituents:

$$\boldsymbol{f}_v = -\boldsymbol{f}_g = k\left(\boldsymbol{v}_v - \boldsymbol{v}_g\right) \tag{4}$$

in which k is a constitutive constant.

The liquid occupies the domain Ω_l and is supposed to be incompressible and Newtonian so that the balance equations of mass and linear momentum which governs its motion are described by (Germain and Muller, 1995):

$$\begin{aligned} &\nabla\cdot\boldsymbol{v}_l = 0 \quad \text{in} \quad \Omega_l \\ &\rho_l\left(\frac{\partial \boldsymbol{v}_l}{\partial t} + \nabla\cdot\left(\boldsymbol{v}_l \otimes \boldsymbol{v}_l\right)\right) = \nabla\cdot\boldsymbol{T}_l \quad \text{in} \quad \Omega_l \end{aligned} \tag{5}$$

in which $\boldsymbol{T}_l = -p_l + \boldsymbol{S}_l$ is the Cauchy stress tensor, with p_l being the non-thermodynamic pressure and $\boldsymbol{S}_l = 2\mu_l \boldsymbol{D}_l$ the extra stress tensor due to motion. The dynamic liquid viscosity is represented

by μ_l and $\boldsymbol{D}_l = 1/2\left(\nabla \boldsymbol{v}_l + (\nabla \boldsymbol{v}_l)^T\right)$ is the rate of strain of the liquid. The liquid density, which is a constant, is denoted by ρ_l.

The balance equations presented above can be applied to the domains Ω_g and Ω_l up to an interface but not across it. Particular forms of the balance equations should be used at an interface in order to take into account the sharp changes (discontinuities) in various variables. By considering that the interface is a singular surface whose motion is described by $\xi(\boldsymbol{x},t) = 0$, for all time instant and spatial position $\boldsymbol{x}$, then the normal component of the surface velocity $\boldsymbol{v}_i$ is given:

$$\boldsymbol{v}_i \cdot \boldsymbol{n} = -\frac{\partial \xi / \partial t}{\|\nabla \xi\|} \tag{6}$$

in which $\boldsymbol{n}$ represents the outward unit vector to the interfacial surface. Since the three constituents are present at the interface, it becomes convenient to introduce the set $\Gamma = \{g, v, l\}$.

To complete the model description, local interfacial balance equations are derived next following the ideas presented by Ishii (1975). By neglecting the mass density of the interface, kinetic energy and momentum fluxes across it, molecular diffusion fluxes along it as well as surface sources such as radiation effects, it can be shown that the interfacial balance equations of mass, momentum and the second law of thermodynamics can be written for an isothermal field ($\dot{\theta} = 0$ and $\nabla\theta = 0$), θ being the absolute temperature, as (Ishii, 1975 and Drew and Passman, 1998):

$$\begin{aligned}
&\sum_{\substack{\alpha \\ \alpha \in \Gamma}} \dot{m}_\alpha = 0 \qquad \text{across} \quad \overline{\Omega}_l \cap \overline{\Omega}_g \\
&\sum_{\substack{\alpha \\ \alpha \in \Gamma}} \left[\dot{m}_\alpha \hat{\boldsymbol{v}}_\alpha - \hat{\boldsymbol{T}}_\alpha \boldsymbol{n}_\alpha \right] + H\sigma \boldsymbol{n} = \boldsymbol{0} \qquad \text{across} \quad \overline{\Omega}_l \cap \overline{\Omega}_g \\
&\sum_{\substack{\alpha \\ \alpha \in \Gamma}} \dot{m}_\alpha \hat{g}_\alpha - \sum_{\substack{\alpha \\ \alpha \in \Gamma}} \left(\left(\boldsymbol{I} - \boldsymbol{n} \otimes \boldsymbol{n} \right) \hat{\boldsymbol{S}}_\alpha \boldsymbol{n} \right) \cdot \left(\left(\boldsymbol{I} - \boldsymbol{n} \otimes \boldsymbol{n} \right) \left(\hat{\boldsymbol{v}}_\alpha - \boldsymbol{v}_i \right) \right) \geq 0 \qquad \text{across} \quad \overline{\Omega}_l \cap \overline{\Omega}_g
\end{aligned} \tag{7}$$

in which $\dot{m}_\alpha = \hat{\rho}_\alpha \boldsymbol{n}_\alpha \cdot (\hat{\boldsymbol{v}}_\alpha - \boldsymbol{v}_i)$ stands for the interfacial mass efflux of the α constituent, g_α represents the Gibbs free energy per unit mass, σ is the surface tension (a constant) and H is the mean curvature of the singular surface. In the past equations, the superimposed hat means that the variable is evaluated at the interface. Since air and vapor are assumed to coexist at one side of the interface while liquid flows alone at the other side, it comes out that:

$$\boldsymbol{n}_g = \boldsymbol{n}_v = -\boldsymbol{n}_l \tag{8}$$

Equation (7a) establishes that the total flux of mass through the interface must be equal to zero. Since we consider by hypothesis that the air can not be dissolved into nor comes out from the liquid, then it results that:

$$\dot{m}_g = 0 \tag{9}$$

The interfacial balance of momentum (7b) states that the flux of momentum through the interface is compensated by the surface traction. Finally, the second law of thermodynamics establishes a distinction between possible and impossible processes. Possible processes are those for which the inequality is satisfied while impossible ones are characterized by the violation of this

inequality. The possible processes are classed as reversible and irreversible. In the reversible processes the equal sign prevails and in the irreversible ones the unequal sign holds strictly.

To ensure that the second law of thermodynamics is always verified, we assume as constitutive assumptions that the no-slip condition holds at the interface and also that the rate of liquid-vapor phase change is proportional to the difference of the Gibbs free energy of the vapor and of the liquid:

$$\begin{aligned} &\left(\boldsymbol{I}-\boldsymbol{n}\otimes\boldsymbol{n}\right)\hat{\boldsymbol{v}}_g=\left(\boldsymbol{I}-\boldsymbol{n}\otimes\boldsymbol{n}\right)\hat{\boldsymbol{v}}_v=\left(\boldsymbol{I}-\boldsymbol{n}\otimes\boldsymbol{n}\right)\hat{\boldsymbol{v}}_l=\left(\boldsymbol{I}-\boldsymbol{n}\otimes\boldsymbol{n}\right)\boldsymbol{v}_i \\ &\dot{m}=\dot{m}_v=-\dot{m}_l=\beta\left(\hat{g}_v-\hat{g}_l\right) \end{aligned} \tag{10}$$

Equation (10a) states that the tangential velocities of all constituents (gas, vapor and liquid) at the interface are equal to the tangential velocity of the interface. By assuming that β is a non-negative constant ($\beta\geq 0$), it is easy to show that with the above constitutive assumptions the second law of the thermodynamics (7c) is always satisfied regardless the initial and boundary conditions as well as the geometric nature of the interface.

To finally complete the modeling, state equations are required for the air and vapor constituents. For the sake of simplicity, we consider in this paper that both air and vapor behave as an ideal gas. For such kind of behavior, the equations of state are

$$p_\alpha=\rho_\alpha R_\alpha\theta \quad \text{for} \quad \alpha\in\Xi, \tag{11}$$

and the Gibbs free energy of the liquid and the vapor are given by:

$$\begin{aligned} &g_l=-c_l\theta\,\boldsymbol{Log}(\theta) \\ &g_v=-c_v\theta\,\boldsymbol{Log}(\theta)+R_v\theta\,\boldsymbol{Log}(\rho_v)+R_v\theta+L(\theta) \end{aligned} \tag{12}$$

in which the R_α, c_α and $L(\theta)$ represent the gas constant, the specific heat at constant volume and the latent heat associated with the liquid-vapor phase change, respectively. The latent heat is an affine function of the temperature.

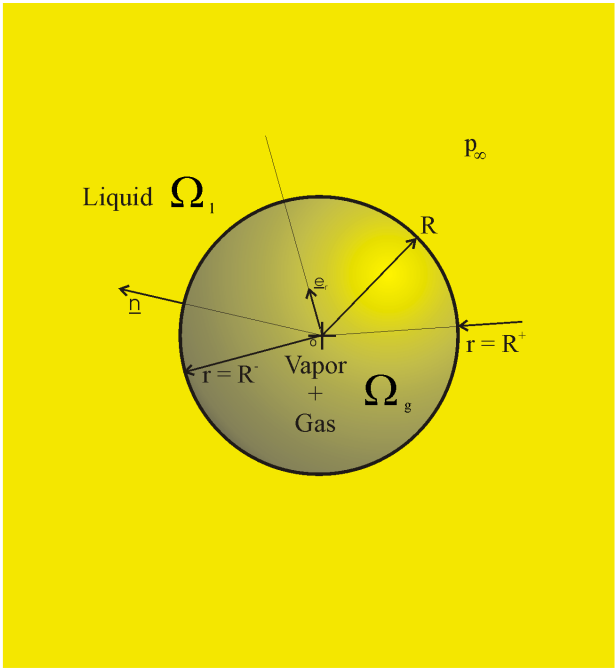

Figure 1: Geometry of the single bubble dynamics problem.

Equations (1) along with (3-12), completed by appropriated boundary and initial conditions, form a complete set of equations to describe the motion of air-vapor cavities in flows of incompressible liquids taking into account irreversible mechanisms associated with the isothermal phase change transformation.

Whatever the form of the singular surface, it can be seen that at the mechanical equilibrium (i.e. $\dot{m} = 0$ and $\nabla \boldsymbol{v}_\alpha = \boldsymbol{0}$, for $\alpha \in \Gamma$) one must have from (10b) that $\hat{g}_v = \hat{g}_l$. By using (11) and (12), this relationship shows that there exists, for each fixed temperature, an equilibrium vapor pressure p_{sv} - called the saturated vapor pressure - for which $p_v = p_{sv}(\theta)$.

3. SINGLE BUBBLE DYNAMICS PROBLEM

Aiming to evaluate the capability of the model described in the past section to properly describe liquid flows with air-vapor cavities taking into account irreversible phase change transformations, we consider, as a first attempt, the classical single bubble dynamics problem. Due its simplicity, such a problem has been studied by many investigators using several different approaches in the past fifty years. Special mention is made here to the consecrated works of Plesset and Prosperetti (1977) and Knapp and Hollander (1948), in which emphasis has been placed on the cavity collapse phenomenon.

Consider a single bubble filled with air and water vapor immersed in an unbounded incompressible liquid medium. The liquid far away from the bubble is at rest and its pressure, denoted by p_∞, can be varied in time according to a known function $p_\infty(t)$. No matter this input function looks like, the shape of the cavity is assumed to remain spherical with radius $R(t)$ centered at a fixed point, chosen as being the origin of our spherical frame of reference (see Fig. 1).

Under these circumstances, the interfacial singular surface motion described by the function $\xi(\boldsymbol{x},t) = 0$ takes the form

$$\xi(r,t) = r - R(t) \tag{13}$$

and, consequently, the surface velocity is expressed as:

$$\boldsymbol{v}_i = \dot{R}\,\boldsymbol{e}_r \tag{14}$$

in which r is the radial coordinate and $\boldsymbol{e}_r$ is the unit vector in the radial direction. Denoting by $v_l = v_l(r,t)$ the magnitude of radial component of the liquid velocity field, the integration of Eq. (5a) in the radial coordinate between the limits $r = R^+$ and $r \to \infty$ yields:

$$v_l = \frac{R^2}{r^2}\hat{v}_l \tag{15}$$

in which $\hat{v}_l = \hat{v}_l(t)$ stands for the liquid velocity at the interface (i.e., $r = R^+$). The application of the same procedure to Eq. (5b), taking into account (15), allows one to rewrite the balance of linear momentum for the liquid as:

$$\frac{d\hat{v}_l}{dt}R + \frac{3}{2}\hat{v}_l^{\,2} = \frac{1}{\rho_l}\left[\frac{4\mu_l}{3R}\hat{v}_l + \hat{p}_l - p_\infty\right] \tag{16}$$

By neglecting the inertia of the air and the vapor, the viscous effects and also the interaction force between these constituents and carrying out the integration of Eq. (1a), for $\alpha \in \Xi$, in the radial coordinate between $r = 0$ and $r = R^-$ we obtain:

$$\hat{v}_v = -\frac{R}{3\hat{\rho}_v}\frac{d\hat{\rho}_v}{dt}$$
$$\hat{p}_g = \hat{p}_{g_0}\frac{R_0^3}{R^3} \tag{17}$$

in which $\hat{p}_{g_0}$ and R_0 represents the air pressure and the radius of the cavity at the initial time instant $t = 0$. By employing the assumptions made so far, the interfacial balance of mass and linear momentum (Eqs. (7a) and (7b)) can be written as:

$$\dot{m} = \hat{\rho}_v\left(\hat{v}_v - \dot{R}\right) = \hat{\rho}_l\left(\hat{v}_l - \dot{R}\right)$$
$$\hat{p}_l - \hat{p}_g - \hat{p}_v = -\frac{2\sigma}{R} - \dot{m}^2\left(\frac{1}{\hat{\rho}_l} - \frac{1}{\hat{\rho}_v}\right) - 4\mu_l\hat{v}_l\frac{1}{R} \tag{18}$$

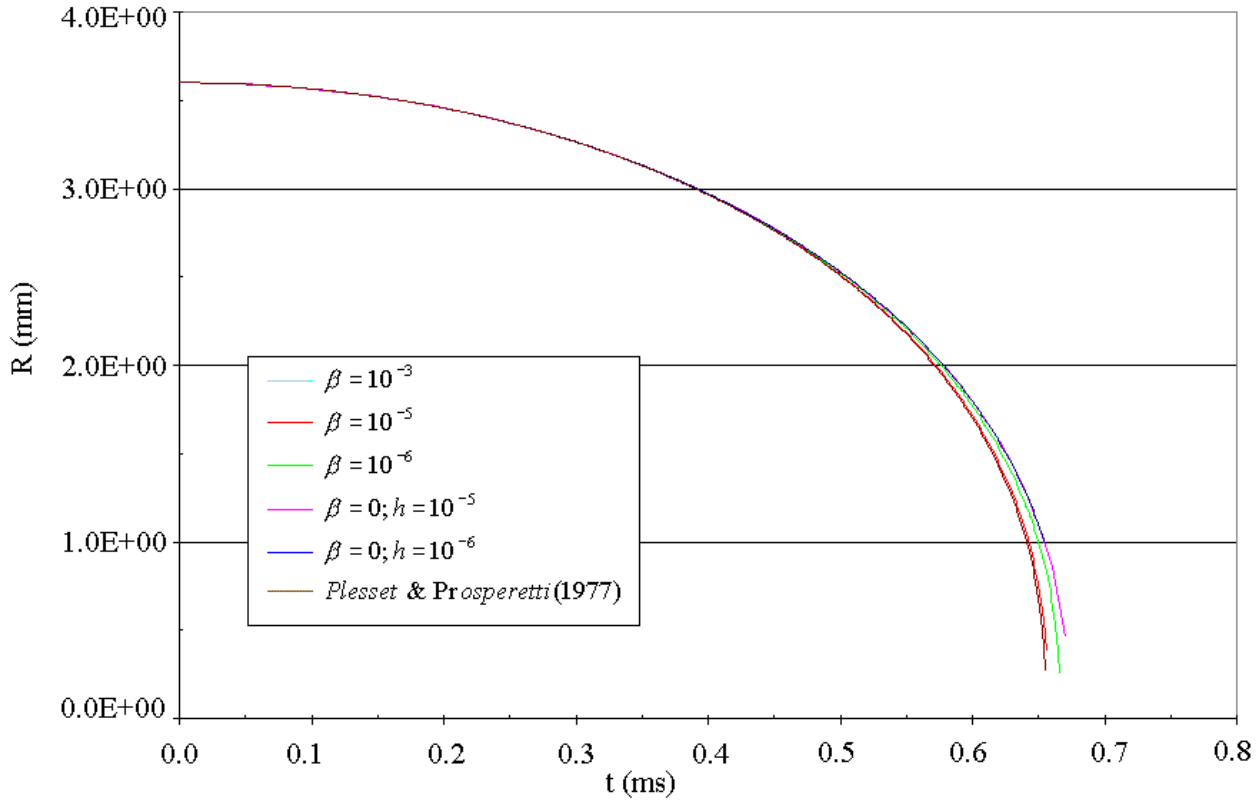

Figure 2: Cavity radius prediction as a function of time for different simulations.

When the constitutive equations (11) and (12) are evaluated at the interface they can be written as

$$\dot{m} = \beta R_v \theta \, \mathbf{Log}\left(\frac{\hat{p}_v}{\hat{p}_{vs}}\right)$$
$$\hat{p}_v = \hat{\rho}_v R_v \theta \qquad (19)$$
$$\hat{p}_g = \hat{\rho}_g R_g \theta$$

Equations (16) to (19) form a complete set of nine equations for the nine unknowns $R, \hat{v}_l, \hat{v}_v, \hat{p}_l, \hat{p}_v, \hat{p}_g, \hat{\rho}_v, \hat{\rho}_g, \dot{m}$. It is worth mention that if we disregard the phase change process, what can be artificially done by setting $\dot{m} = 0$, the classical equation of the single bubble dynamics of Plesset and Prosperetti (1977) is recovered. In such a case, the vapor inside the cavity remains saturated at the saturated vapor pressure ($\hat{p}_v = \hat{p}_{sv} = const$) since there is no mass transfer through the bubble interface. As a result, the liquid and vapor velocities at the interface are always equal to the interfacial velocity; $\hat{v}_v = \hat{v}_l = \dot{R}$.

4. NUMERICAL RESULTS

In order to assess the influence of the phase change process on the collapse of a spherical cavity, we have employed the model described in the past section and compared its predictions with the ones of the Plesset and Prosperetti's model (1977). To do so, we have considered a bubble of water vapor and air having an initial radius $R = 0.0036m$. The vapor is at the beginning of the simulation at a saturated vapor pressure of $\hat{p}_v = \hat{p}_{sv} = 2340Pa$, which corresponds to a temperature $\theta = 25^{\circ}\mathrm{C}$, while the partial pressure of the air is $\hat{p}_g = 80\mathrm{Pa}$. The liquid pressure far away from the bubble, which is at static equilibrium at $t = 0$, is suddenly raised to $p_\infty = 27.6\mathrm{kPa}$ and kept constant. These particular parameters correspond to the classical experimental work of Knapp and Hollander (1948), which has been considered for years as a standard test.

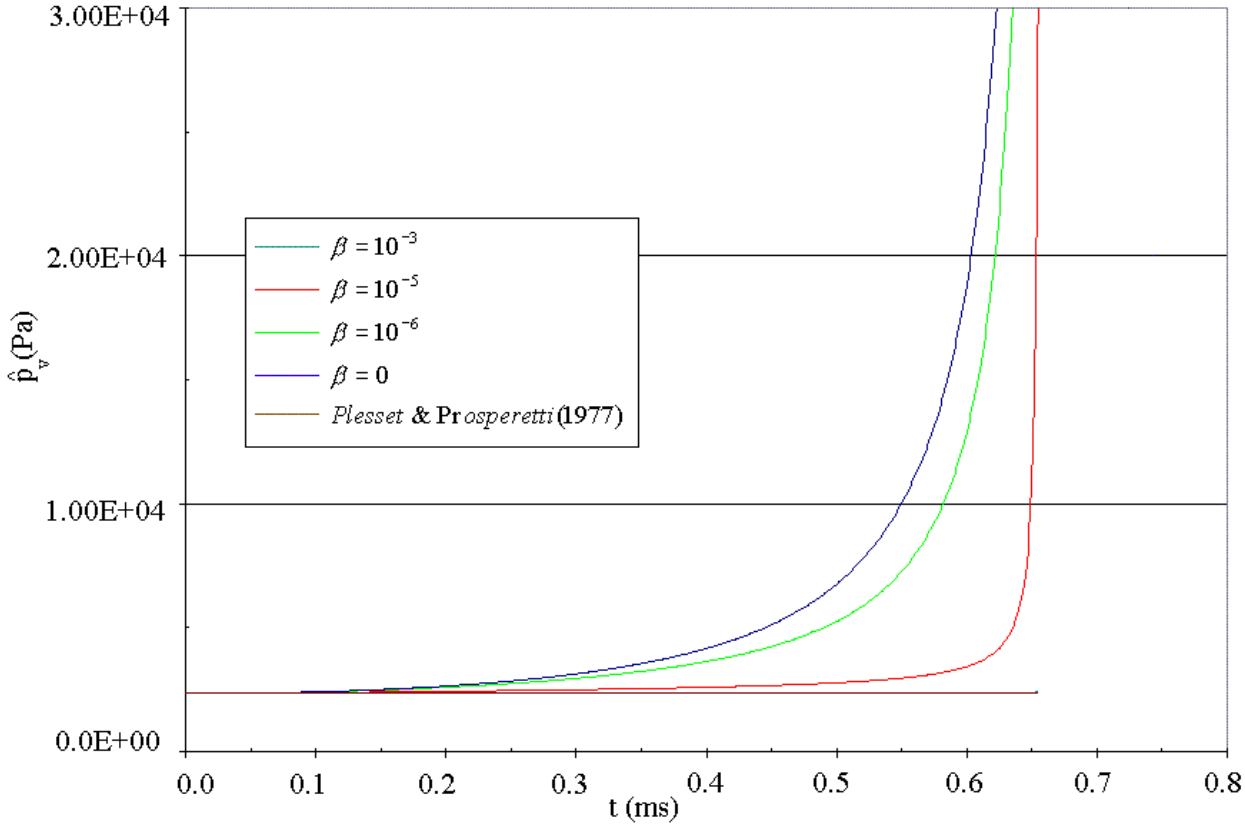

Figure 3: Vapor pressure inside the cavity as a function of time for different simulations.

The surface tension, liquid viscosity and density at this temperature are $\sigma = 0.073\text{N/m}$, $\rho_l = 998\text{kg/m}^3$ and $\mu_l = 1.01\times10^{-3}\text{Pa.s}$, respectively. The product $R_v\theta$ is equal to 135kJ/kg. The influence of phase change transformation on the bubble collapse is then investigated by attributing to the material constant β several different values: $\beta = 0$, $\beta = 10^{-3}$, $\beta = 10^{-5}$ and $\beta = 10^{-6}$ $\text{kg}^2.\text{m}^{-2}.\text{kW}^{-1}$. When $\beta = 0$, the vapor inside the bubble is allowed to contract by raising its pressure, without transforming into liquid. On the other extreme, when a large value is assigned to it ($\beta = 10^{-3}$) the vapor remains almost all time at the saturated vapor pressure during the cavity collapse. This mechanical behavior is the same one predicted by the Plesset and Prosperetti's (1977) model, although in such a model the phase change process is not taken into account. Any value different from zero attributed to the material constant (such as, $\beta = 10^{-3}$, $\beta = 10^{-5}$ and $\beta = 10^{-6}$) gives the model a more realistic physical behavior since one should expect that the vapor be transformed into liquid during the cavity collapse by dissipating some amount of energy. On physical grounds, this energy dissipation is associated with the irreversibility of the phase change process.

Figure 2 displays the cavity radius against time until the cavity collapse. To make sure the time step h used to integrate the equations is adequate, we have carried out simulations using $h = 10^{-5}\text{s}$ and $h = 10^{-6}\text{s}$ for $\beta = 0$. As can be seen in Fig 2, there is almost no distinction between these two curves which are the right most curves. Based on this result, we have carried out all the other numerical simulations with a time step of $h = 10^{-6}\text{s}$.

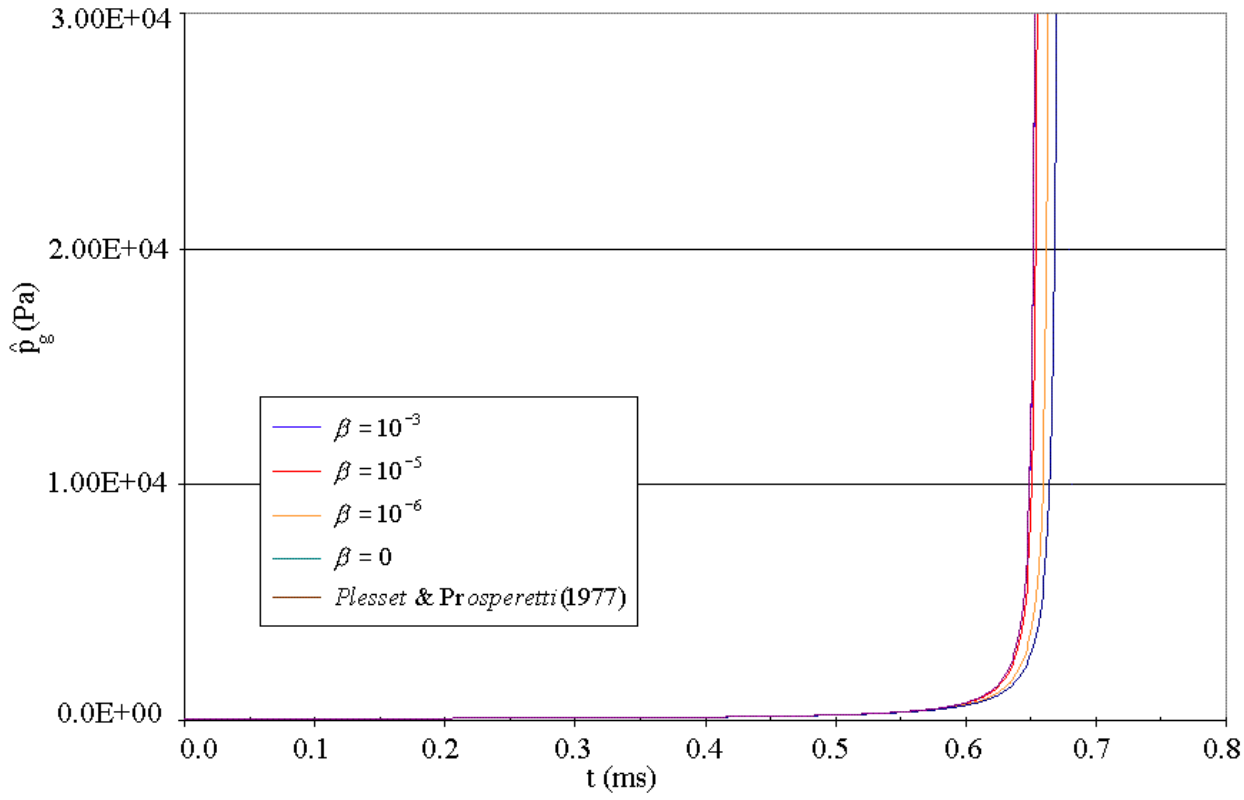

Figure 4: Air pressure inside the cavity as a function of time for different simulations.

The left most curves in Fig 2 are those related to simulations carried out by using the model presented in this paper with $\beta = 10^{-3}$ and by employing the Plesset and Prosperetti's model. As it has been anticipated, the Plesset and Prosperetti model is recovered as a particular case of the model

proposed herein when large values are assigned to β, such as $\beta = 10^{-3}$. Note in Fig 2 that there is virtually no distinction between these two curves. The curves associated with $\beta = 10^{-5}$ and

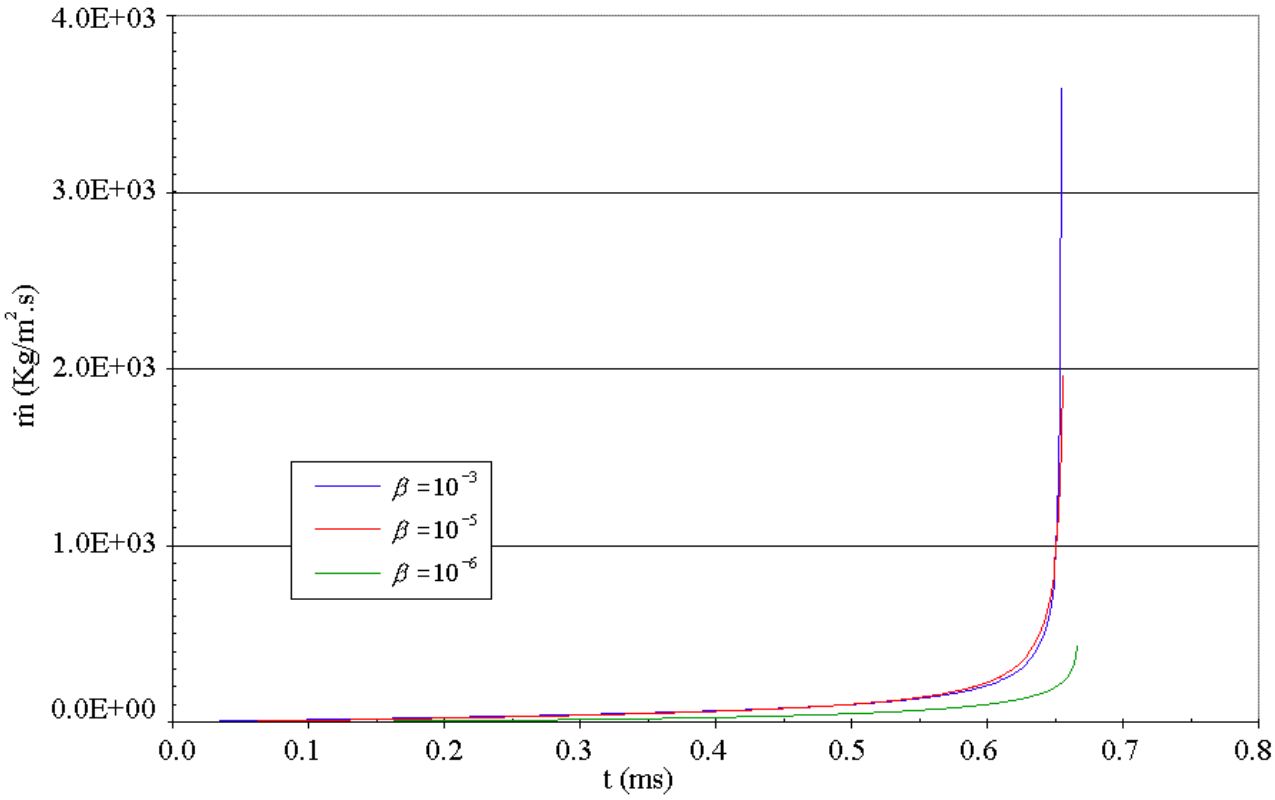

Figure 5: Generation rate of vapor mass as a function of time for different simulations.

$\beta = 10^{-6}$ represent intermediate results between the limiting cases obtained with the Plesset and Prosperetti model (or, equivalently with the present model taking $\beta = 10^{-3}$) and with the model presented here taking $\beta = 0$. Although the difference among these responses is not so significant, one can see in Fig 2 that as β is decreased, the cavity collapse takes place at later time instants.
The differences in the cavity collapse behavior predicted by these simulations become more apparent when the vapor pressure is plotted against the time, as illustrated in Fig 3. For $\beta = 10^{-3}$, the vapor pressure remains constant and equal to the saturated vapor pressure until the cavity collapse. This same behavior is achieved when the Plesset and Prosperetti's model is employed. By assigning decreasing values to β, the vapor pressure inside the cavity begins to vary with the time less and less abruptly. Since there is no energy dissipation associated with the liquid-vapor phase change in the simulation for $\beta = 0$, the vapor pressure increase is more stringent than in the other cases in order to absorb elastically the energy from the liquid medium.

The air pressure as a function of the time is depicted in Fig 3 for the different simulations carried out. Although the difference among the cases is not so significant, one can note that the relative behavior of the curves is in reverse order when compared to those curves plotted in Fig 3. That is, as the value of β is reduced the air pressure raises at later time instants.

Finally, it is shown in Fig 5 the mass rate of phase change versus the time for the three different values of $\beta = 10^{-3}, \beta = 10^{-5}$ and $\beta = 10^{-6}$. The case for which $\beta = 0$ implies $\dot{m} = 0$, and so has not been plotted. Whatever value different from zero β assumes, one can see that the phase change process is mainly restricted to the final stages of the bubble collapse. By looking at both Fig 3 and

Fig 5, we can see that although there is not any difference in $\dot{m} = 0$ for $\beta = 10^{-3}$ and $\beta = 10^{-5}$, the largest value of β imposes a vapor pressure that is almost constant during the simulation and jumps abruptly at the imminence of the collapse. If we admit that such a discontinuity in vapor pressure should not be expected, then the phase change process acts by reducing the bubble time collapse. Of course, experimental evidence is required prior to any conclusion be definitively made.

5. FINAL REMARKS

A continuum thermo-mechanical model has been proposed in this paper to describe the flow of a liquid having cavities filled with vapor and a non-condensable gas. Irreversible phase change transformations are taken into account in such a way that the interfacial entropy inequality is always satisfied, regardless the surface geometry. The theory has then been applied to investigate the influence of the phase change process on collapse of a single bubble immersed in a liquid medium. Numerical simulations carried out have shown that the phase change transformation takes place at the final stages of the bubble collapse and may reduce the bubble collapse time.

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