PRESSURE TRANSDUCER AND HEAT FLUX GAGE CALIBRATION USING A SHOCK TUBE

Paulo Gilberto de Paula Toro (toro@ieav.cta.br)
Instituto de Aeronáutica e Espaço (IAE/CTA), Praça Marechal Eduardo Gomes, 50, São José dos Campos, CEP:12228-904

Valéria Serrano Faillace Oliveira Leite (valeria@ieav.cta.br)
Instituto de Estudos Avançados (IEAv/CTA), Rodovia dos Tamoios Km. 5,5, São José dos Campos, CEP: 12228-840

Abstract: *Simulations of high pressure and high temperature conditions at hypersonic speeds, close to those encountered during reentry of a space vehicle into the Earth's atmosphere, require the use of transient facilities, such as shock tubes and shock tunnels, to obtain experimental data. The short test times, which are on the order of a few milliseconds (at most), and the requirement for fast response measurement sensors, are two problems that arise with such facilities. In this paper the calibration procedures of pressure transducers and heat flux gages that will be used in experimental pressure and heat transfer investigations of hypersonic flow are presented. The pressure transducers and the heat gages are dynamically calibrated in a low-pressure shock tube. The pressure transducers are mounted radially and flush to the internal diameter of a test ring and the ring is attached to the test section of the shock tube. The heat flux gages are mounted perpendicularly and flush with the surface of a flat plate and the plate is installed into the test section of the shock tube at zero angle of attack. The traces of the voltage jump for both pressure transducers and heat flux gages are presented.*

Keywords: *Hypersonic Flow, Heat Transfer Measurement, Pressure Measurement, Shock Tube.*

1. INTRODUCTION

The feasibility of space flight is limited by phenomena such as aerodynamic drag and heating as well as related thermal management problems. Therefore, an efficient hypersonic space vehicle design has to combine a low drag coefficient (to maximize the net propulsive thrust) with low heat transfer rates (to minimize thermal protection system mass).

Shock tubes and shock tunnels have been widely used for high velocity and high temperature research since the early 1950s (Nagamatsu, 1958 and 1959). They are the most versatile experimental ground test facilities for very short test times, in which may be simulated the conditions encountered during the reentry of the space vehicle into the Earth's atmosphere. In these facilities very high temperature flows can be achieved briefly, where the duration of the uniform hot gas flow, varies from about 50 microseconds to about 10 milliseconds. Such facilities require specialized measurement techniques to obtain reliable pressure and temperature (heat flux) data. For short test times the pressure and the heat transfer measurement techniques have to be suited for transient conditions with a response time fast enough to trace variations caused by changing flow conditions. Since the fifties year, various research groups have been developed fast response heat gages to measure heat transfer rates (Nagamatsu, 1957; Olivier, 1992). These heat gages are basically resistance thermometers, where the temperature sensitive element is a thin metal film bonded to an insulating backing substrate.

The thin-film resistance thermometer (or thin-film heat gages) consists of a very thin metal film (gold, platinum, rhodium) applied to a backing substrate material of low thermal conductive (pyrex, alumina, macor). Film depositing techniques include platinum sputtering, evaporation under vacuum of gold, platinum and rhodium, or simply painting with appropriate paints, which are then baked at high temperatures. The film thickness has normally a few microns. Their response time is on the order of one microsecond, ideal for the short test times found in shock tubes and shock tunnels.

Both the pressure transducers and the thin-film heat gages may be dynamically calibrated in a low-pressure shock tube, under certain known pressure conditions. The calibration process, for the pressure transducer, is to find the relation between the mechanical pressure and its electrical output voltage and for the thin-film heat gages the calibration is to find the heat gage constant (β).

Four thin-film platinum heat gages were fabricated by bonding a platinum film strip on the flat surface of the macor cylindrical substrate, by painting process, creating the sensitive surface of the gage. The traces of the voltage jump for both pressure transducers and heat flux gages are presented.

2. THIN-FILM PLATINUM HEAT GAGE

The thin-film heat gage is essentially a surface thermometer with a negligible thermal inertia. In contrast, the thermal inertia of the backing or substrate material is very large. The response time, of the thin-film to temperature changes, is extremely fast. Since the transient time due to temperature changes inside the wall (substrate) is large, compared with the heat exchange time between the wall surface and the hot gas, the semi-infinite approximation to one-dimensional transient heat conduction may be used to compute the temperature field in the wall, and therefore, the heat flux at the wall.

2.1. Heat Gage Construction

The thin-film platinum heat gages were fabricated by bonding a platinum film strip on a flat surface of the macor cylindrical substrate (3.4 mm diameter and 25.0 mm long), by painting process, creating the sensitive surface of the gage (Toro, 1998; Leite, 2001). Silver contact tabs were placed, by sputtering, at the ends of the platinum band. Two 40.0 cm long electrical leads were soldered to the sides of the gages (on the silver contact tabs), using a 5% silver solder and low heat. The wires were soldered to the PCB connector adapters (model#70A09). Finally, a plastic shrink tubing was mounted to insulate the gage's cylindrical surface (solder and silver tabs) from the metal model material. The gage is shown in Figure 1, where the caliper is 10.0 mm open.

The thin-film heat gage is supplied with a constant current source, which provides 50 mA across the gage. The output signal of the gage was amplified with a gain of 450 times.

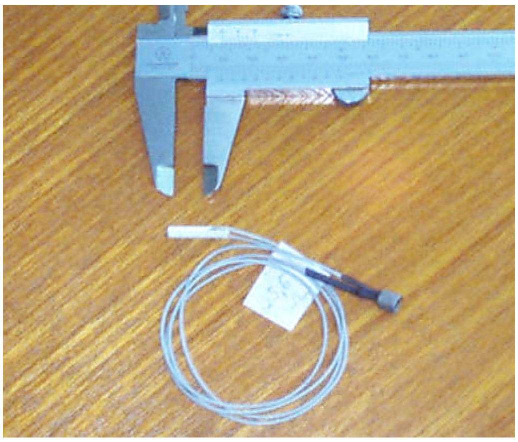


Figure 1: Thin-film platinum heat gage

2.2. Heat Gage Response

The thickness of the platinum film is very small (about one micron or less) compared to the length of the substrate or backing material (about 25.0 mm), as it can be seen in the schematic drawing of the gage (Figure 2).

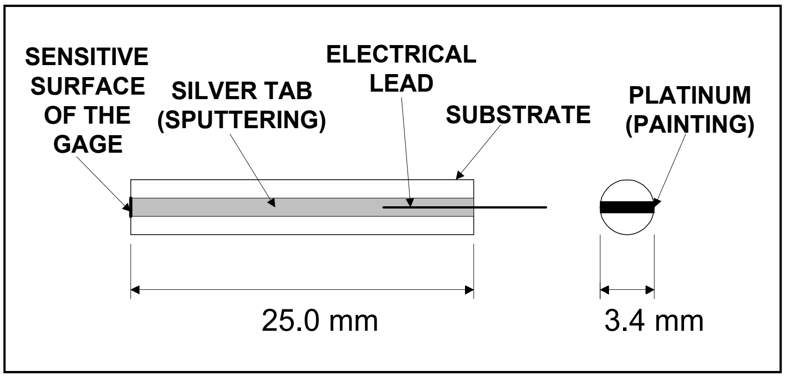


Figure 2: Schematic draw of the thin-film heat gage

Since the temperature gradient through the platinum film is very small, and may be neglected (Nagamatsu, 1957; Toro, 1998), and the time of exposure to the hot gas is very short (from 50 microseconds to 10 milliseconds), the classical one-dimensional heat conduction theory may be used, with the backing material acting like a semi-infinite one-dimensional solid heat sink.

For these conditions and making $\varepsilon \rightarrow 0$, where ε is the thin-film thickness, the one-dimensional transient heat conduction equations for the thin-film and the substrate material are given by

$$\frac{\partial^2 \theta}{\partial x^2} = \frac{1}{\alpha_s} \frac{\partial \theta}{\partial t} \tag{1}$$

with the initial and the boundary conditions given by

$$\theta(x,0) = 0 \tag{2}$$

$$-k_s \frac{\partial \theta(0,t)}{\partial x} = \dot{q}_w \tag{3}$$

$$\theta(\infty,t) = 0 \tag{4}$$

where: θ is the temperature change relative to the ambient temperature, $\theta(x,t) = T(x,t) - T_\infty$. Note that α_s is the thermal diffusivity of the material. The subscript s refers to substrate material and the subscript w refers to the wall or sensitive surface of the gage.

Using the Laplace transform to solve the governing equations, the surface temperature and the surface heat flux are given, respectively, by

$$\theta_w(t) = \frac{1}{\sqrt{\pi}} \frac{1}{\sqrt{(\rho c_p k)_s}} \int_0^t \frac{\dot{q}_w(\tau)}{(t-\tau)} d\tau \tag{5}$$

$$\dot{q}_w = \frac{\sqrt{(\rho c_p k)_s}}{\sqrt{\pi}} \left[\frac{\theta(t)}{\sqrt{t}} + \frac{1}{2} \int_0^t \frac{\theta(t) - \theta(\tau)}{(t-\tau)^{3/2}} d\tau \right] \tag{6}$$

where: ρ (density), c_p (specific heat at constant pressure) and k (thermal conductivity) are the thermal proprieties of the backing material or substrate.

The resistance of the platinum thin-film changes with the temperature variation as

$$R_f = R_0 \left[1 + \alpha_R \left(T_f - T_0\right)\right] \tag{7}$$

consequently, the driving voltage is changed, since $\Delta E = I \, \Delta R$. To measure the surface temperature, a constant current should be supplied to the heat gage. The voltage variation is the measured quantity in the shock tube or in the shock tunnel. For small temperature changes a linear relation between the temperature variation and the voltage variation may be assumed, and it is given by

$$E(t) - E_0 = \Delta E = I_0 (R_f - R_0) = I_0 \left[R_0 \alpha_R (T_f - T_0)\right] = I_0 R_0 \alpha_R \theta(t) \tag{8}$$

where: α_R is the thermal resistivity of the platinum thin-film and $E_0 = E(t=0) = 0$. Therefore, the relations between the temperature variation and the voltage variation may be given by

$$\theta(t) = \frac{E(t)}{I_0 R_0 \alpha_R} \tag{9}$$

Replacing the value of the temperature variation (Eq. 9) into the heat flux (Eq. 6), it becomes

$$\dot{q}_w = \frac{\beta}{\sqrt{\pi}} \frac{1}{I_0 R_0} \left[\frac{E(t)}{\sqrt{t}} + \frac{1}{2} \int_0^t \frac{E(t) - E(\tau)}{(t - \tau)^{3/2}} d\tau \right] \tag{10}$$

where β is the heat gage constant and given by

$$\beta = \frac{\sqrt{(\rho c_p k)_s}}{\alpha_R} \tag{11}$$

For the case where the integral in Eq. 10 is zero (i.e., the surface temperature $\theta(t)$ jumps to a constant value correspondent to E_c), the surface heat flux is given by

$$\dot{q}_w = \frac{\beta}{\sqrt{\pi}} \frac{1}{I_0 R_0} \frac{E_c}{\sqrt{t}} \tag{12}$$

3. SHOCK TUBE AND CALIBRATION APPARATUS

The shock tube may be used to dynamically calibrate the pressure transducers and the thin-film heat gages. The shock tube consists basically of a region of high-pressure gas, called driver section and a region of low-pressure gas, called driven section, separated by a single thin diaphragm. The diaphragm allows one to maintain different pressure in each part of the tube (Figure 3).

When the diaphragm is suddenly ruptured, compression waves are generated which coalesce into a normal shock wave. This shock then propagates into the low-pressure driven section while an expansion or rarefaction wave propagates into the high-pressure driver section. The shock wave arrives at the end wall of the driven tube and it is totally reflected. Different gases at different temperatures may be used in the driver and driven tubes. The quasi-steady motion may be studied around the model housed in the test section, which is placed at the end or close to the end of the driven section (Figure 3).

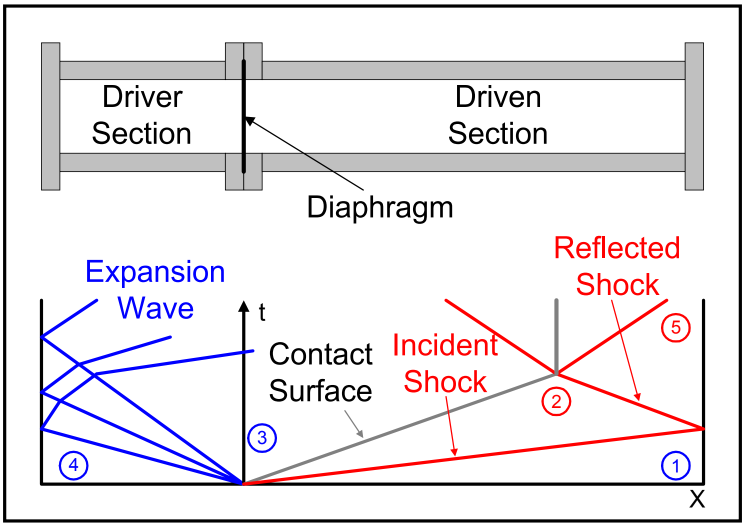


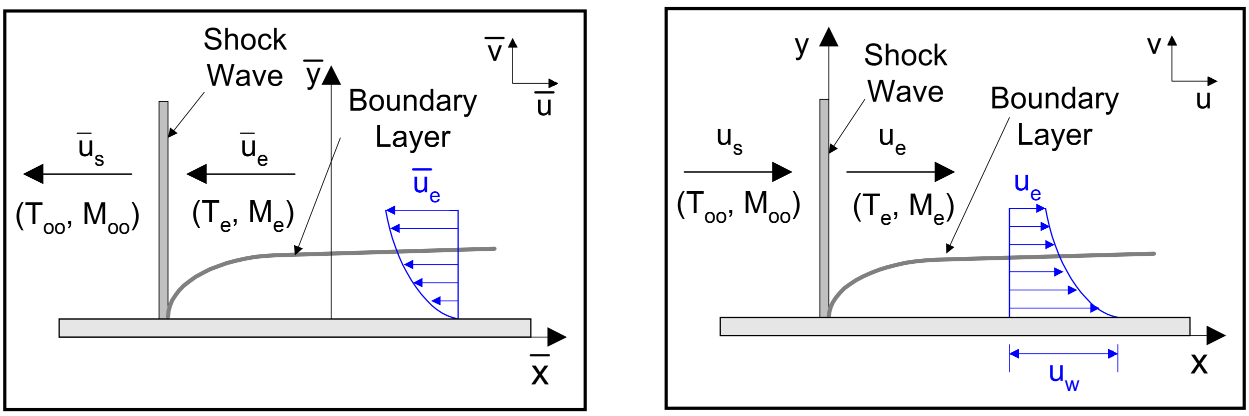
Figure 3: Shock tube operation

The one-dimensional incident shock wave moving into a stationary gas may be used to calculate the flow condition in the shock tube and the flow over the model. The unsteady laminar boundary layer behind the advancing shock wave (Mirels, 1955) and the one-dimensional heat conduction flux (Eqs. 1 - 4) into the thin-film may be used to calibrate the heat transfer gages.

3.1. Laminar Boundary Layer Behind Shock Advancing into Stationary Fluid

When a shock wave advances into a stationary fluid bounded by a wall, a boundary-layer flow is established along the wall behind the shock. This boundary layer is often important in studies of phenomena involving nonstationary shock waves (Mirels, 1955). In shock tube, for example, this boundary layer acts to attenuate the strength of the shock, which propagates through the low-pressure side of the tube.

A shock wave of constant strength is considered to move, parallel to a wall, into a stationary fluid. Considering a coordinate system fixed with respect to the wall or flat plate (Figure 4a), where $\bar{u}$ and $\bar{v}$ are the velocities parallel to the $\bar{x}$ and $\bar{y}$ coordinates, respectively, and where the flow is unsteady in this coordinate system. Considering now that (x, y) represent another coordinate system moving with the shock wave (Figure 4b), where u and v represent the velocities parallel to the x and y coordinates, respectively. In this last coordinate system the flow is steady.



(a) System fixed to the flat plate (b) System fixed to the shock wave

Figure 4: Coordinate systems

At time t = 0, the two coordinate systems coincide. Therefore $\bar{x}$ and x are related by $x = \bar{x} - \bar{u}_s t$, where $\bar{u}_s$ is the velocity of the shock wave relative to the coordinate system fixed in respect to the wall (the unsteady coordinate system). In the same way the velocities are related by $u = \bar{u} - \bar{u}_s$. Note that the wall moves at velocity u_w, in the coordinate system fixed with respect to the shock wave (the steady coordinate system), and $u_w = -\bar{u}_s$.

Assuming the flow over the flat plate, $dp/dx = 0$ and that the *Laminar Boundary-Layer* equations may be applied, the governing equations are:

continuity equation: $$\frac{\partial(\rho u)}{\partial x} + \frac{\partial(\rho v)}{\partial y} = 0 \tag{13}$$

x-momentum equation: $$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \frac{1}{\rho}\frac{\partial}{\partial y}\left(\mu\frac{\partial u}{\partial y}\right) \tag{14}$$

y-momentum equation: $$\frac{\partial p}{\partial y} = 0 \tag{15}$$

energy equation: $$\rho c_p\left(u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y}\right) = \frac{\partial}{\partial y}\left(k\frac{\partial T}{\partial y}\right) + \mu\left(\frac{\partial u}{\partial y}\right)^2 \tag{16}$$

and the equation of state for perfect gas: $$p = \rho RT \tag{17}$$

The boundary conditions need to solve the problem are

$$\begin{aligned} &u(x,0) = -\bar{u}_s && u(x,\delta) = u_e \\ &\quad v(x,0) = 0 && \\ &T(x,0) = T_w && T(x,\delta) = T_e \end{aligned} \tag{18}$$

where δ is the thickness of the boundary layer.

In order to satisfy the condition of zero slip at the wall, it is considered that the fluid at the wall moves with velocity $u(x,0) \equiv u_w = -\bar{u}_s$, as shown in the boundary conditions above. It is assumed that the wall temperature T_w is constant and the magnitudes of u_e, T_e and $\bar{u}_s$ (u_w) may be found from the normal shock relations

$$\frac{u_w}{u_e} = \frac{(\gamma+1)M_\infty^2}{(\gamma-1)M_\infty^2 + 2} \tag{19}$$

$$M_e^2 = \frac{2}{(\gamma+1)\frac{u_w}{u_e} - (\gamma-1)} \tag{20}$$

$$\frac{T_e}{T_\infty} = \frac{(\gamma+1)\frac{u_w}{u_e} - (\gamma-1)}{\frac{u_w}{u_e}\left[(\gamma+1) - (\gamma-1)\frac{u_w}{u_e}\right]} \tag{21}$$

Solving the Eqs. 13 to 17 with the boundary conditions defined in Eq. 18, one may find the heat flux by convection from the boundary layer over the flat plate that is given by

$$\dot{q}_{conv} = -k_w \left[\frac{\pi}{2} \frac{\bar{u}_s - \bar{u}_e}{\bar{u}_s} \frac{\rho_w}{\mu_w} \frac{1}{t} \right]^{1/2} (T_w - T_{w,i}) s'(0) \tag{22}$$

where: k_w, ρ_w, μ_w e T_w are the thermal conductivity, density, viscosity and temperature of the wall respectively and $T_{w,i}$ is the adiabatic wall temperature.

The factor $\frac{\bar{u}_s - \bar{u}_e}{\bar{u}_s} = \frac{u_e}{u_w}$ is given by the inverse of the normal shock relation (Eq. 19).

The adiabatic wall temperature is given by

$$\frac{T_{w,i}}{T_e} = 1 + \frac{\gamma - 1}{2} \left[\left(\frac{u_w}{u_e} - 1 \right) M_e \right]^2 r(0) \tag{23}$$

The subscript *e* refers to the flow behind the shock wave and external to the boundary layer.

The factors *r(0)* and *s'(0)* are given by the following equations (Hinckel, 1984)

$$r(0) = 0.8861 \left(\frac{\bar{u}_s}{\bar{u}_s - \bar{u}_e} \right)^{0.0217} \tag{24}$$

$$s'(0) = -0.6576 \left(\frac{\bar{u}_s}{\bar{u}_s - \bar{u}_e} \right)^{0.3764} \tag{25}$$

4. THIN-FILM HEAT GAGES CALIBRATION

The heat gage constant β must be evaluated for the thin-film platinum heat gage. There are two methods that may be used to characterize the thin-film platinum heat gage constant. One of the methods involves the material proprieties of the gage. By knowing the thermal proprieties and the amount of the material in the gage, the constant β may be found. The problem with this method is that it cannot account for impurities in the gage material and it can be extremely inaccurate. Since the material proprieties depend on the substance involved and the fabrication process used, the tabulation of these values is not always available. The other method, used to find the value of the gage constant β, consists in calibrating the heat gage dynamically. This dynamic calibration (Nagamatsu, 1959) is more conveniently done by considering the coupled effect of the substrate and platinum thermal proprieties. In this way, the diffusion of platinum into the substrate that takes place during the curing process is also accounted for. The calibration of the gages is done by imposing a known thermal condition to the heat gage and measuring the response.

The heat flux into the wall of the gage (i.e., the gage response), given by Eq. 12 and the heat flux from the compressible laminar boundary layer by convection (i.e., the known thermal condition imposed to the gage), given by Eq. 22, must be the same. This relation between the heat flux, by conduction, into the wall and the heat flux from the boundary layer, by convection, is given by the following equation (Nagamatsu, 1957; Toro, 1998)

$$\beta = \frac{\sqrt{(\rho c_p k)_s}}{\alpha_R} = -\frac{I_0 R_0}{\Delta E(t)} k_w \left[\frac{\pi}{2} \frac{\bar{u}_s - \bar{u}_e}{\bar{u}_s} \frac{\rho_w}{\mu_w} \right]^{1/2} (T_w - T_{w,i}) s'(0) \tag{26}$$

To calculate β, with the Eq. 26, the initial constant current given to the gage (I_0), the initial resistance of the gage (R_0), k_w, ρ_w, μ_w and T_w are known. The adiabatic temperature wall ($T_{w,i}$), the factor $(\bar{u}_s - \bar{u}_e / \bar{u}_s)$ and the coefficient *s'(0),* are functions of the Mach number of the incident shock wave (M_∞). So, the voltage output of the thin-film heat gage $\Delta E(t)$ and the time between the voltage jumps of the pressure transducer response and the thin-film heat gage response must be measured. The incident shock wave velocity may be calculated by the time measured and the known distance between the pressure transducer and the heat gage (Figure 5). In the calibration process one may obtain the two values, the time between the two signals and the output voltage of the gage (Figure 6).

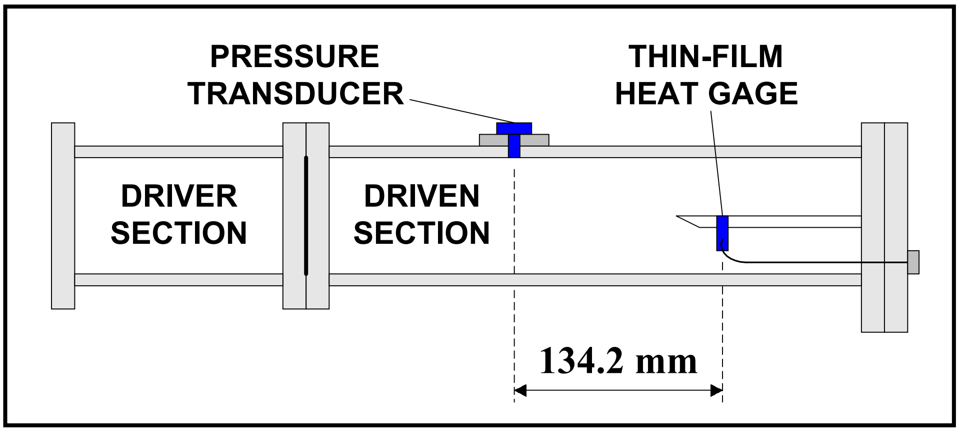


Figure 5: The thin-film heat gage calibration position inside the shock tube

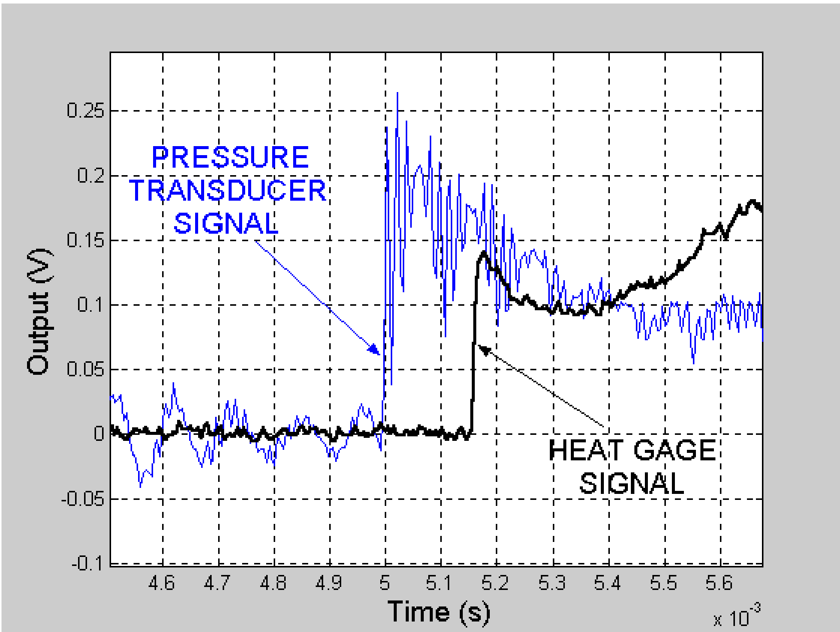


Figure 6: Output signal of the thin-film heat gage calibration

The Figure 7 shows that the heat gage constant β is a linear function of the pressure ratio p_2/p_1.

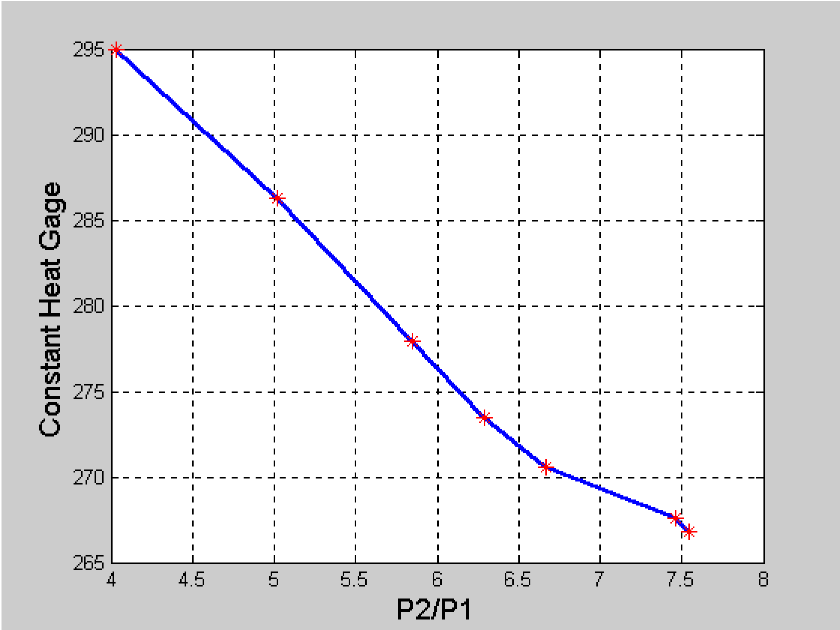


Figure 7: Calibration curve (values of the heat gage constant β)

5. PRESSURE TRANSDUCER CALIBRATION

Pressure transducers should also be dynamically calibrated in a low-pressure shock tube before being used in experimental investigations in the shock tunnel. This calibration is need to find the relation between the mechanical (pressure) and its electrical output (voltage) for the same pressure ratio that they will work in the shock tunnel. For this purpose the pressure transducers are installed radially and flush with internal diameter of the test ring (Figure 8) and the test ring is attached to the test section of the shock tube. Another section, 160.0 mm long, is clamped together with the ring to the driven tube end flange. This last section has its far end closed, and causes the incident shock wave to reflect off from it. As a result, each single run may generate two calibration points: one after the incident shock wave *(p_2)* and a second one after the reflected shock wave *(p_5)*.

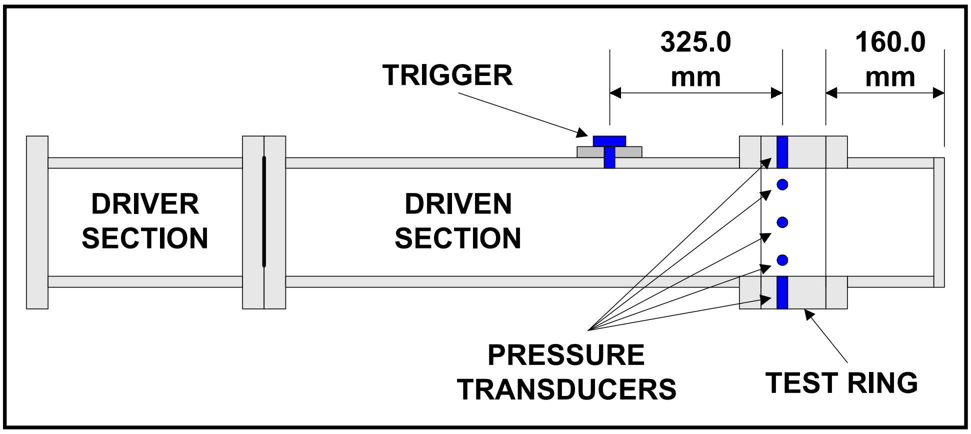


Figure 8: The pressure transducers calibration position inside the shock tube

For the calorically perfect gas the relation between the proprieties downstream and upstream of the shock wave is a function only of the shock wave Mach number. The ratio between the initial pressure on the driver section *(p_4)* and the initial pressure on the driven section *(p_1)*, called diaphragm pressure ratio (shock tube relation), is given by

$$\frac{p_4}{p_1} = \frac{p_2}{p_1}\left\{1 - \left[(\gamma_4 - 1)\frac{a_1(\gamma_4 - 1)}{a_4(\gamma_1 + 1)}\right]\left(\frac{M_\infty - 1}{M_\infty}\right)\right\}^{-\frac{2\gamma_4}{(\gamma_4 - 1)}} \tag{27}$$

where a is the sound velocity, $\gamma = c_p / c_v$ and

$$\frac{p_2}{p_1} = \frac{2\gamma_1 M_\infty^2 - (\gamma_1 - 1)}{\gamma_1 + 1} \tag{28}$$

where M_∞ is the Mach number of the incident shock wave given by

$$M_\infty = \frac{u_s}{a_1} \tag{29}$$

The subscript *1* refers to the conditions of the driven section, *4* to the conditions of the driver section, *2* to the conditions behind the shock wave and *s* and ∞ to the conditions ahead of the shock wave.

The value of p_1 (driven pressure) is known, therefore the pressure of the incident shock wave (p_2) can be calculated from Eq. 28.

Figure 9 shows the signals of the pressure transducer, located in the test ring, and the trigger.

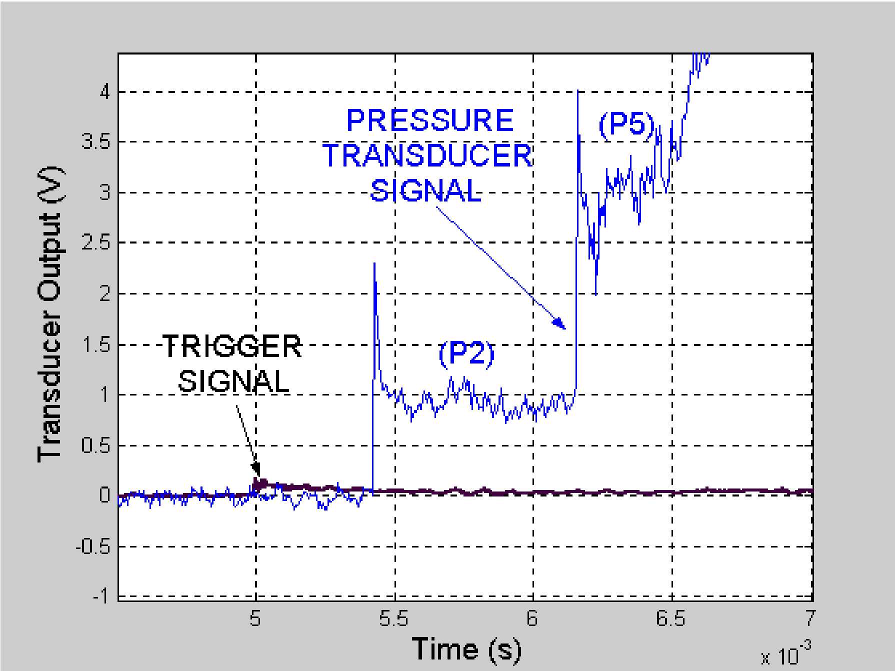


Figure 9: Output signal of the pressure transducer calibration

6. CONCLUSIONS

Shock tubes and shock tunnels are the most versatile experimental ground test facilities for very short test times, in which may be simulated the conditions encountered during the reentry of the space vehicle into the Earth's atmosphere. The calibrations of the thin-film platinum heat gage and the pressure transducer using a low-pressure shock tube are presented. Both the pressure transducers and the thin-film heat gages were dynamically calibrated in a low-pressure shock tube, under certain known pressure conditions that are the same conditions that will be worked with in the shock tunnel.

The calibration process for the thin-film heat gages is to find the heat gage constant (β). The calibration of the heat gages is done by imposing a known thermal condition to the gage and measuring its response. The voltage output $\Delta E(t)$ of the thin-film heat gage, mounted flush with the surface of a flat plate, and its calibration curve are presented.

The calibration process for the pressure transducer is to find the relation between the mechanical (pressure) and its electrical signal (voltage output). The voltage output $\Delta E(t)$ of the pressure transducer, corresponding to the pressure after the incident shock wave (p_2), is presented.

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