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FREE CONVECTION INSIDE HORIZONTAL ANNULAR CONCENTRIC CHANNELS BY USING INTEGRAL TRANSFORM TECHNIQUE

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Abstract. *This work deals with free convection heat transfer inside horizontal annular concentric channels using the Generalized Integral Transform Technique – GITT, a hybrid numerical-analytical method that has been successfully applied to different classes of convection-diffusion problems. This method allows for error controlled solutions without requirements on grid generation strategies. The two-dimensional energy and Navier-Stokes equations in cylindrical coordinates are solved making use of the streamfunction-only formulation. Several sets of results for different values of aspect ratio and Rayleigh number are calculated and then critically compared with previously reported experimental data.*

Key-words: *Free Convection, Annular Ducts, Hybrid Methods, Integral Transforms.*

1. INTRODUCTION

Free convection inside cavities has received an increasing attention of the thermal science researchers because of its wide applicability in industrial processes. The precise knowledge of the heat transfer between the cavity walls and the fluid is extremely important in the choice of adequate materials and in the optimum design of thermal equipment. In particular, the flow in the annular region comprehended by circular concentric ducts is of special interest in thermal engineering applications. This flow model occurs, for instance, in double pipe heat exchangers, in nuclear reactors cooling, thermal storage tanks, cylindrical thermal insulation, and various other applications.

The present research intends to add some reference information to the literature by providing results for steady laminar buoyancy induced flow within horizontal annular concentric cavities, making use of the Generalized Integral Transform Technique (GITT) (Cotta, 1993; Cotta and Mikhailov, 1997 and Cotta, 1998). The aim of this paper is to demonstrate the suitability of this technique as a tool in obtaining engineering or benchmark results in problems of natural convection inside annular concentric horizontal ducts and cavities.

The problem of natural convection in horizontal concentric annular channels was studied by Kuehn and Goldstein (1976), who performed an experimental and theoretical study of the same problem here proposed for analysis. They presented results for the temperature distribution and for the local heat transfer coefficients. Tsui and Trambley (1984) considered both steady and transient regimen using the ADI scheme. Rao et al. (1985) investigated transient two-dimensional and steady three-dimensional situations. Mahony et al. (1986), using finite differences, solved a variable properties model.

The present contribution, through integral transformation and its inherent automatic error control capability, provides sets of reference results for validation purposes, here employed in critical comparisons against some of the above cited previous works. The present analysis is a natural extension in the development of this hybrid numerical-analytical approach for heat and fluid flow problems, and some of the most representative previous contributions, related to the present work, may now be cited: Pérez Guerrero and Cotta (1992, 1996), Pereira et al. (1998, 1999), Leal et al. (1999), Pérez Guerrero et al. (2000) and Pereira (2000).

2. ANALYSIS

2.1 Problem Formulation

The physical problem under consideration is related to an annular horizontal channel fulfilled with a Newtonian fluid, according to Fig. (1). The annular space is formed by two infinitely long concentric cylinders with radii R_1 and R_2, for the internal and external cylinders, respectively. The cylinders walls are maintained at constant and uniform temperatures, with $T_1 > T_2$. The fluid flow is assumed laminar and occurs only by density differences (buoyancy effects) caused by the different sidewalls temperatures. Fluid flow variations along the axial direction are neglected allowing for the assumption of a two-dimensional flow situation. The symmetry in the flow related to the vertical plane is taken into consideration; besides, the Boussinesq hypothesis is adopted. The mathematical representation for this problem is given by the conservation of mass, momentum and energy, which in steady state and dimensionless form, are written as:

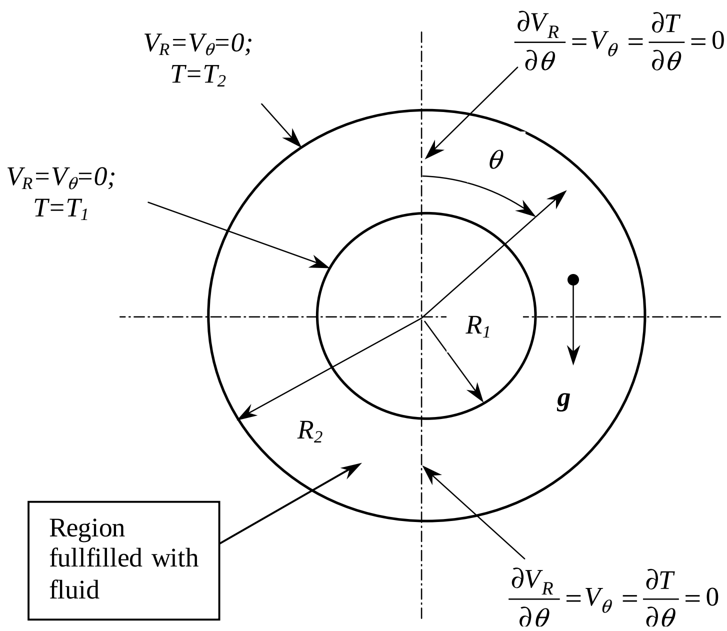


Figure 1. Geometry and coordinate system.

$$\frac{\partial v_r}{\partial r}+\frac{v_r}{r}+\frac{1}{r}\frac{\partial v_\theta}{\partial \theta}=0 \tag{1}$$

$$v_r\frac{\partial v_r}{\partial r}+\frac{v_\theta}{r}\frac{\partial v_r}{\partial \theta}-\frac{v_\theta^2}{r}=-\frac{\partial p}{\partial r}-Ra_{\mathrm{L}}\,\mathrm{Pr}\,Cos\,\theta\;\Theta+\mathrm{Pr}\left(\nabla^2 v_r-\frac{v_r}{r^2}-\frac{2}{r^2}\frac{\partial v_\theta}{\partial \theta}\right) \tag{2}$$

$$v_r\frac{\partial v_\theta}{\partial r}+\frac{v_\theta}{r}\frac{\partial v_\theta}{\partial \theta}+\frac{v_r v_\theta}{r}=-\frac{1}{r}\frac{\partial p}{\partial \theta}+\mathrm{Ra}_{\mathrm{L}}\,\mathrm{Pr}\,Sin\,\theta\;\Theta+\mathrm{Pr}\left(\nabla^2 v_\theta-\frac{v_\theta}{r^2}+\frac{2}{r^2}\frac{\partial v_R}{\partial \theta}\right) \tag{3}$$

$$v_r\frac{\partial \Theta}{\partial r}+\frac{v_\theta}{r}\frac{\partial \Theta}{\partial \theta}=\nabla^2\Theta \tag{4}$$

and the following dimensionless boundary conditions:

$$\left.\begin{matrix} v_r=0 \\ v_\theta=0 \\ \Theta=1 \end{matrix}\right\}\ \text{for } r=r_1 \text{ and } 0<\theta<\pi\ , \qquad \left.\begin{matrix} v_r=0 \\ v_\theta=0 \\ \Theta=0 \end{matrix}\right\}\ \text{for } r=r_2 \text{ and } 0<\theta<\pi \tag{5.a-f}$$

$$\left.\begin{matrix} \dfrac{\partial v_r}{\partial \theta}=0 \\ v_\theta=0 \\ \dfrac{\partial \Theta}{\partial \theta}=0 \end{matrix}\right\}\ \text{for } \theta=0 \text{ and } r_1<r<r_2\,, \qquad \left.\begin{matrix} \dfrac{\partial v_r}{\partial \theta}=0 \\ v_\theta=0 \\ \dfrac{\partial \Theta}{\partial \theta}=0 \end{matrix}\right\}\ \text{for } \theta=\pi \text{ and } r_1<r<r_2 \tag{5.g-l}$$

The dimensionless groups used to write Eqs. (1-4) and the boundary conditions (Eqs. 5) are defined as:

$$v_r=\frac{V_R L}{\alpha};\qquad v_\theta=\frac{V_\theta L}{\alpha};\qquad r=\frac{R}{L};\qquad \Theta=\frac{T-T_2}{T_1-T_2};\qquad p=\frac{P}{\rho\alpha^2/L^2} \tag{6.a-e}$$

$$Ra_L=\frac{\rho g\beta(T_1-T_2)L^3}{\mu\alpha};\qquad Pr=\frac{\mu c_p}{k} \tag{6.f-g}$$

where V_R and V_θ are the dimensional radial and angular velocity components, respectively; R and θ are the dimensional radial and angular coordinates, P is the dimensional absolute pressure; T is the dimensional temperature; T_1 and T_2 are the dimensional temperatures of the internal and external cylinders walls; g is the gravity acceleration; α is the thermal diffusivity; ρ is the fluid specific mass; β is the thermal expansion coefficient; μ is the absolute viscosity; c_p is the specific heat at constant pressure; k is the fluid thermal conductivity; Ra_{L} is the Rayleigh number, based on the cavity width; and Pr is the Prandtl number. The following additional dimensionless parameters are then defined:

$$r_1=\frac{R_1}{L};\qquad r_2=\frac{R_2}{L};\qquad \varpi=\frac{r_2}{r_1};\quad \text{where, } L=R_2-R_1; \tag{6.h-k}$$

with r_1 and r_2 being the dimensionless positions of the internal and external cylindrical walls, respectively; ϖ is the radii ratio; L is the cavity width (distance between the internal and external cylinders walls).

The momentum equations can be represented in the streamfunction-only formulation to eliminate the pressure terms and automatically satisfy the continuity equation. Therefore, using the same procedure adopted by Pereira et al. (1998), i.e., making use of the definition of the velocity components in terms of the streamfunction, given by:

$$v_r = \frac{1}{r}\frac{\partial \psi}{\partial \theta}; \qquad v_\theta = -\frac{\partial \psi}{\partial r} \tag{7.a,b}$$

the following dimensionless coupled partial differential equations are generated:

$$\nabla^4 \psi = \frac{1}{Pr}\left[\frac{1}{r}\frac{\partial \psi}{\partial \theta}\frac{\partial(\nabla^2\psi)}{\partial r} - \frac{1}{r}\frac{\partial \psi}{\partial r}\frac{\partial(\nabla^2\psi)}{\partial \theta}\right] + Ra_L\left(Sin\theta\frac{\partial \Theta}{\partial r} + \frac{1}{r}Cos\theta\frac{\partial \Theta}{\partial \theta}\right) \tag{8}$$

$$\nabla^2 \Theta = \frac{1}{r}\frac{\partial \psi}{\partial \theta}\frac{\partial \Theta}{\partial r} - \frac{1}{r}\frac{\partial \psi}{\partial r}\frac{\partial \Theta}{\partial \theta} \tag{9}$$

with the following boundary conditions:

$$\left.\begin{array}{l} \psi = K_1 \\ \dfrac{\partial \psi}{\partial r} = 0 \\ \Theta = 1 \end{array}\right\}, \quad \text{for } r = r_1 \text{ and } 0 < \theta < \pi, \qquad \left.\begin{array}{l} \psi = K_2 \\ \dfrac{\partial \psi}{\partial r} = 0 \\ \Theta = 0 \end{array}\right\}, \quad \text{for } r = r_2 \text{ and } 0 < \theta < \pi \tag{10.a-f}$$

$$\left.\begin{array}{l} \psi = K_3 \\ \dfrac{\partial^2 \psi}{\partial \theta^2} = 0 \\ \dfrac{\partial \Theta}{\partial \theta} = 0 \end{array}\right\}, \quad \text{for } \theta = 0 \text{ and } r_1 < r < r_2, \qquad \left.\begin{array}{l} \psi = K_4 \\ \dfrac{\partial^2 \psi}{\partial \theta^2} = 0 \\ \dfrac{\partial \Theta}{\partial \theta} = 0 \end{array}\right\}, \quad \text{for } \theta = \pi \text{ and } r_1 < r < r_2 \tag{10.g-l}$$

where

$$\nabla^2 \equiv \frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r} + \frac{1}{r^2}\frac{\partial^2}{\partial \theta^2}; \qquad \nabla^4\psi = \nabla^2(\nabla^2\psi) \tag{11.a,b}$$

The fact that no flow occurs across the channel boundaries makes it possible to take $K_1 = K_2 = K_3 = K_4 = 0$ without loss of generality.

2.2 Solution Methodology

According to the integral transformation approach, the first step is to choose auxiliary eigenvalue problems for the momentum and energy equations. Similarly to the forced convection situation (Pereira et al., 1999), we have here used the same forth order eigenvalue problem introduced by Chandrasekhar and Reid (1957) as the auxiliary problem to solve the streamfunction equation, which is written as:

$$\left(\frac{d^2}{dr^2}+\frac{1}{r}\frac{d}{dr}\right)^2 \mathrm{X}_i(r)=\alpha_i^4\,\mathrm{X}_i(r) \tag{12.a}$$

$$\left.\begin{aligned}\mathrm{X}_i&=0\\ \frac{d\mathrm{X}_i}{dr}&=0\end{aligned}\right\},\ \text{at } r=r_1;\qquad \left.\begin{aligned}\mathrm{X}_i&=0\\ \frac{d\mathrm{X}_i}{dr}&=0\end{aligned}\right\},\ \text{at } r=r_2 \tag{12.b-e}$$

where $\mathrm{X}_i(r)$ and α_i are the eigenfunctions and eigenvalues, respectively.

The solution is shown in detail in Pereira (2000) and its general form is given by:

$$\mathrm{X}_i(r)=A_{1i}J_0(\alpha_i\, r)+A_{2i}\,Y_0(\alpha_i\, r)+A_{3i}\frac{I_0(\alpha_i\, r)}{I_0(\alpha_i r_2)}+A_{4i}\frac{K_0(\alpha_i\, r)}{K_0(\alpha_i r_1)} \tag{13}$$

It can also be noticed that one of the boundary conditions for the temperature in the 'r' direction (Eq. 10.c), is non-homogeneous, and generates an additional term in the final system. Therefore, it is convenient to make use of a filtering strategy with the goal of homogenizing the boundary conditions. Thus, the problem of heat conduction in a hollow cylinder was utilized to achieve this purpose. In this sense, the filtering function problem statement is:

$$\frac{1}{r}\frac{d}{dr}\left[r\frac{d\phi(r)}{dr}\right]=0;\quad \phi(r_1)=1;\quad \phi(r_2)=0 \tag{14.a-c}$$

The original temperature field is then filtered through the following solution:

$$\Theta(r,\theta)=\Phi(r,\theta)+\phi(r),\quad \text{where } \phi(r)=1-\frac{\ln(r/r_1)}{\ln(\varpi)} \tag{15.a,b}$$

Making use of this result in Eqs. (8-10), the new set of equations to be solved is obtained as:

$$\nabla^4\psi=\frac{1}{Pr}\left[\frac{1}{r}\frac{\partial\psi}{\partial\theta}\frac{\partial(\nabla^2\psi)}{\partial r}-\frac{1}{r}\frac{\partial\psi}{\partial r}\frac{\partial(\nabla^2\psi)}{\partial\theta}\right]+Ra_L\left[Sin\theta\left(\frac{\partial\Phi}{\partial r}-\frac{1}{r\ln(\varpi)}\right)+\frac{1}{r}Cos\theta\frac{\partial\Phi}{\partial\theta}\right] \tag{16}$$

$$\nabla^2\Phi=\frac{1}{r}\frac{\partial\psi}{\partial\theta}\frac{\partial\Phi}{\partial r}-\frac{1}{r}\frac{\partial\psi}{\partial r}\frac{\partial\Phi}{\partial\theta}-\frac{1}{\ln(\varpi)}\frac{1}{r^2}\frac{\partial\psi}{\partial\theta} \tag{17}$$

with the new filtered thermal boundary conditions:

$$\left.\begin{aligned}\Phi(r_1,\theta)&=0\\ \Phi(r_2,\theta)&=0\end{aligned}\right\},\ \text{for } 0<\theta<\pi\ ,\qquad \left.\begin{aligned}\Phi(r,0)&=0\\ \Phi(r,\pi)&=0\end{aligned}\right\},\ \text{for } r_1<r<r_2 \tag{18.a-d}$$

Thus, the energy equation is rewritten in terms of a new potential $\Phi(r,\theta)$ representing the filtered potential for the convection problem temperature profile, after extracting a heat conduction-type analytical solution as explicit filter (Pereira, 2000).

In the case of the energy equation, the auxiliary problem for the resulting filtered energy equation is of the Sturm-Liouville type, written as:

$$\frac{d^2\Gamma_m(r)}{dr^2}+\frac{1}{r}\frac{d\Gamma_m(r)}{dr}+\xi_m^2\Gamma_m(r)=0\,;\quad r_1<r<r_2\,,\quad m=1,2,3,\ldots,\infty \tag{19.a}$$

$$\Gamma_m(r_1)=0\,;\qquad \Gamma_m(r_2)=0 \tag{19.b,c}$$

where $\Gamma_m(r)$ and ξ_m are the eigenfunctions and the eigenvalues, respectively.

The solution of this auxiliary problem and the orthogonality property are readily available in Ozisik (1993) and given by

$$\Gamma_m(r)=J_0(\xi_m r)Y_0(\xi_m r_2)-J_0(\xi_m r_2)Y_0(\xi_m r) \tag{20}$$

The next step in the solution procedure is to determine the integral transform pairs. Making use of the orthogonality property of the eigenfunctions, the following integral transform pairs for the streamfunction and temperature equations are obtained, respectively:

$$\overline{\psi}_i(\theta)=\int_{r_1}^{r_2}\,\mathrm{r}\,\tilde{\mathrm{X}}_i(r)\,\psi(r,\theta)\,dr\,,\quad \text{(transform)} \tag{21.a}$$

$$\psi(r,\theta)=\sum_{i=1}^{\infty}\tilde{\mathrm{X}}_i(r)\,\overline{\psi}_i(\theta)\,,\qquad \text{(inverse)} \tag{21.b}$$

$$\overline{\Phi}_m(\theta)=\int_{r_1}^{r_2} r\,\tilde{\Gamma}_m(r)\,\Phi(r,\theta)\,dr\,,\quad \text{(transform)} \tag{22.a}$$

$$\Phi(r,\theta)=\sum_{m=1}^{\infty}\tilde{\Gamma}_m(r)\,\overline{\Phi}_m(\theta)\,,\qquad \text{(inverse)} \tag{22.b}$$

Using the transformation rules, given by Eqs. (21.a and 22.a), the coupled partial differential equations with their respective boundary conditions are transformed resulting in the following coupled ordinary differential system:

$$\sum_{j=1}^{\infty}A_{ij}^{*}\overline{\psi}_j^{\,iv}=-\sum_{j=1}^{\infty}\left\{\left[\delta_{ij}\alpha_j^4\overline{\psi}_j+2B_{ij}^{*}\overline{\psi}_j''\right]+\frac{1}{Pr}\left[\sum_{k=1}^{\infty}A_{ijk}^{*}\overline{\psi}_j'\overline{\psi}_k+B_{ijk}^{*}\overline{\psi}_j'\overline{\psi}_k''-C_{ijk}^{*}\overline{\psi}_j'''\overline{\psi}_k\right]-\right.$$
$$\left.Ra_L\left[A_i\,\frac{\mathrm{sen}\,\theta}{\ln(\varpi)}-\sum_{n=1}^{\infty}\left(Sin\theta\;A_{in}^{*}\overline{\Phi}_n+Cos\theta\;B_{in}^{*}\overline{\Phi}_n'\right)\right]\right\} \tag{23.a}$$

$$\sum_{n=1}^{\infty}A_{mn}^{*}\,\overline{\Phi}_n''=\sum_{n=1}^{\infty}\left\{\delta_{mn}\,\xi_n^2\,\overline{\Phi}_n+\sum_{j=1}^{\infty}\left(A_{mnj}^{*}\,\overline{\Phi}_n\,\overline{\psi}_j'-B_{mnj}^{*}\,\overline{\Phi}_n'\,\overline{\psi}_j\right)\right\}-\frac{1}{\ln(\varpi)}\sum_{j=1}^{\infty}C_{mj}^{*}\,\overline{\psi}_j' \tag{23.b}$$

with the transformed boundary conditions in the angular direction:

$$\left.\begin{array}{l}\overline{\psi}_i(\theta)=0\\ \overline{\psi}_i''(\theta)=0\\ \overline{\Phi}_m'(\theta)=0\end{array}\right\}\ \text{at}\ \theta=0\qquad\text{and}\qquad \left.\begin{array}{l}\overline{\psi}_i(\theta)=0\\ \overline{\psi}_i''(\theta)=0\\ \overline{\Phi}_m'(\theta)=0\end{array}\right\}\ \text{at}\ \theta=\pi \tag{24.a-f}$$

The integral coefficients appearing in Eqs.(23.a,b), which result from the integral transformation procedure, after applying the inversion operators (Eqs. 21.b and 22.b) in the non-transformable terms, are defined as (Pereira, 2000):

$$A^*_{ijk} = \int_{r_1}^{r_2} \tilde{X}_i \left(\tilde{X}_j \tilde{X}'''_k + \frac{1}{r} \tilde{X}_j \tilde{X}''_k - \frac{1}{r^2} \tilde{X}_j \tilde{X}'_k - \tilde{X}''_j \tilde{X}'_k - \frac{1}{r} \tilde{X}'_j \tilde{X}'_k \right) dr \,; \quad A^*_{ij} = \int_{r_1}^{r_2} \frac{1}{r^3} \tilde{X}_i \tilde{X}_j dr \qquad (25.a,b)$$

$$A^*_{mnj} = \int_{r_1}^{r_2} \tilde{\Gamma}_m \tilde{\Gamma}'_n \tilde{X}_j dr \,; \quad A^*_{mn} = \int_{r_1}^{r_2} \frac{1}{r} \tilde{\Gamma}_m \tilde{\Gamma}_n dr \,; \quad A^*_{in} = \int_{r_1}^{r_2} r \tilde{X}_i \tilde{\Gamma}'_n dr \,; \quad A^*_i = \int_{r_1}^{r_2} \tilde{X}_i dr \qquad (25.c\text{-}f)$$

$$B^*_{ijk} = \int_{r_1}^{r_2} \tilde{X}_i \left(\frac{1}{r^2} \tilde{X}_j \tilde{X}'_k - \frac{2}{r^3} \tilde{X}_j \tilde{X}_k \right) dr \,; \quad B^*_{mnj} = \int_{r_1}^{r_2} \tilde{\Gamma}_m \tilde{\Gamma}_n \tilde{X}'_j dr \,; \quad B^*_{in} = \int_{r_1}^{r_2} \tilde{X}_i \tilde{\Gamma}_n dr \qquad (25.g\text{-}i)$$

$$B^*_{ij} = \int_{r_1}^{r_2} \tilde{X}_i \left(\frac{2}{r^3} \tilde{X}_j - \frac{1}{r^2} \tilde{X}'_j + \frac{1}{r} \tilde{X}''_j \right) dr \,; \; C^*_{ijk} = \int_{r_1}^{r_2} \frac{1}{r^2} \tilde{X}_i \tilde{X}_j \tilde{X}'_k dr \,; \; C^*_{mj} = \int_{r_1}^{r_2} \frac{1}{r} \tilde{\Gamma}_m \tilde{X}_j dr \qquad (25.j\text{-}l)$$

3. RESULTS AND DISCUSSION

The truncated form of the above ordinary differential system, for the transformed potentials, is solved by using readily available subroutines with automatic error control, such as the subroutine BVPFD of the IMSL Library (1989), which is appropriate to solve stiff boundary value problems. The integral ODE system coefficients, which appear after the integral transformation procedure, are numerically evaluated, since the internal products that form the integrands involve Bessel functions, and most of them do not allow for analytical integration. Details of the solution and computational algorithms are found in Pereira (2000).

All the results were here obtained within a precision of 10^{-4} for the transformed streamfunction and temperature potentials. The aspect ratio ϖ=2.6 was considered a representative value for reporting numerical results since it is the most utilized in the literature. Table 1 illustrates the convergence behavior of the streamfunction and temperature profiles for three radial positions within the annular space and $\theta=90^o$, for Rayleigh number, (Ra_L), equal to $5\text{x}10^4$ and Prandtl number (*Pr)* of 0.7. It can be noticed that a reasonable convergence was already attained with fairly small truncation orders (NF=NT=16) in the expansions. For all considered positions, a maximum number of terms (NF=NT=30) in the streamfunction and temperature inversion formulae were required to fully warrant convergence to all four significant digits. For smaller values of Rayleigh number, the convergence rates of the eigenfunction expansions, for both the streamfunction and the temperature fields, are somehow improved, due to the increased importance of diffusive effects that are well represented within the adopted eigenvalue problems, the basis for the proposed expansions.

The local Nusselt numbers, at both internal and external channel walls, were evaluated using the following definitions:

$$Nu_1 = -\ln(\varpi) \left[r \frac{\partial \Theta}{\partial r} \right]_{r=r_1} \,; \qquad Nu_2 = \ln(\varpi) \left[r \frac{\partial \Theta}{\partial r} \right]_{r=r_2} \qquad (26.a,b)$$

The results for the local Nusselt numbers at the internal and external cylinder walls are presented in Fig. (2) and a comparison with experimental results obtained by Kuehn and Goldstein (1976) is performed. It can be noticed that a very good agreement between the present hybrid solution and the experimental data was achieved, for all positions along the channel walls.

Table 1. Convergence of streamfunction and temperature fields (ϖ=2.6, Ra_L=5x10^4, Pr=0.7)

$\Psi(r,\ \theta)$		$\theta = 90^o$	
NF=NT	***r* =0.725**	**1.125**	**1.525**
8	6.784E+00	2.157E+01	4.674E+00
16	7.233E-01	4.678E-01	3.339E-01
24	7.233E-01	4.677E-01	3.340E-01
28	7.233E-01	4.677E-01	3.340E-01
30	7.233E-01	4.677E-01	3.340E-01
$\Theta(r,z)$		$\theta = 90^o$	
NF=NT	***r* =0.725**	**1.125**	**1.525**
8	5.284E-01	3.158E-01	1.527E-01
16	5.287E-01	3.160E-01	1.528E-01
24	5.287E-01	3.161E-01	1.528E-01
28	5.287E-01	3.161E-01	1.528E-01
30	5.287E-01	3.161E-01	1.528E-01

To illustrate the flow patterns in the annular space of the channel, Figs. (3a-c) show the isolines for streamfunction (at the left hand side half-plane) and temperature (at the right hand side half-plane), considering a variation of the Rayleigh number between 10^3 and $5\text{x}10^4$. As expected, almost purely diffusive effects are observed for the lower Rayleigh number ($Ra_L=10^3$), as pointed out by Fig. (3a). In this case, the isolines are essentially parallel in both streamfunction and temperature contours.

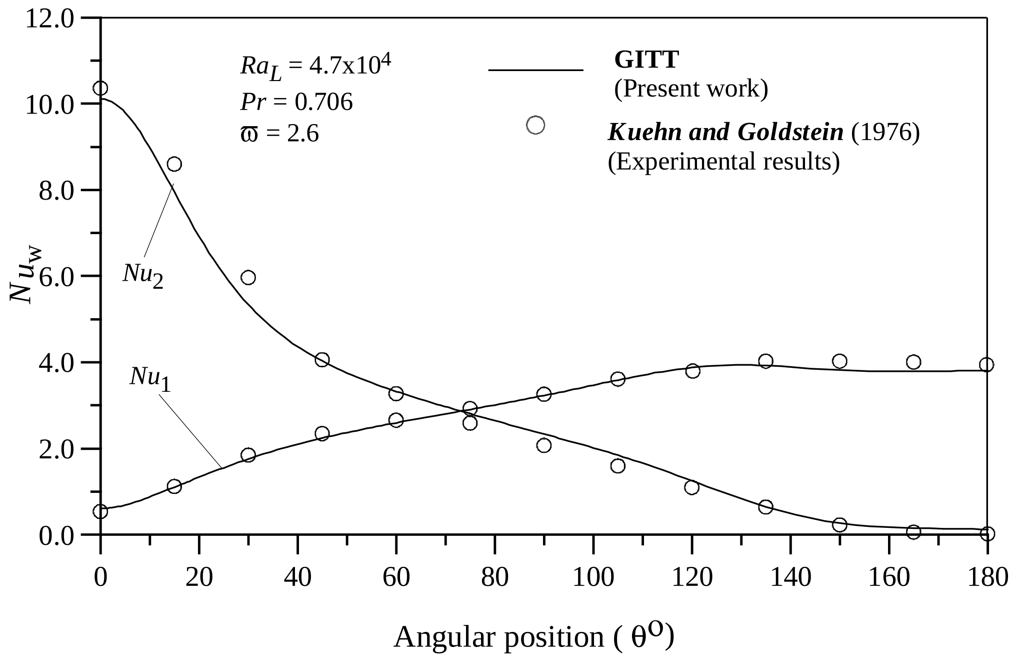
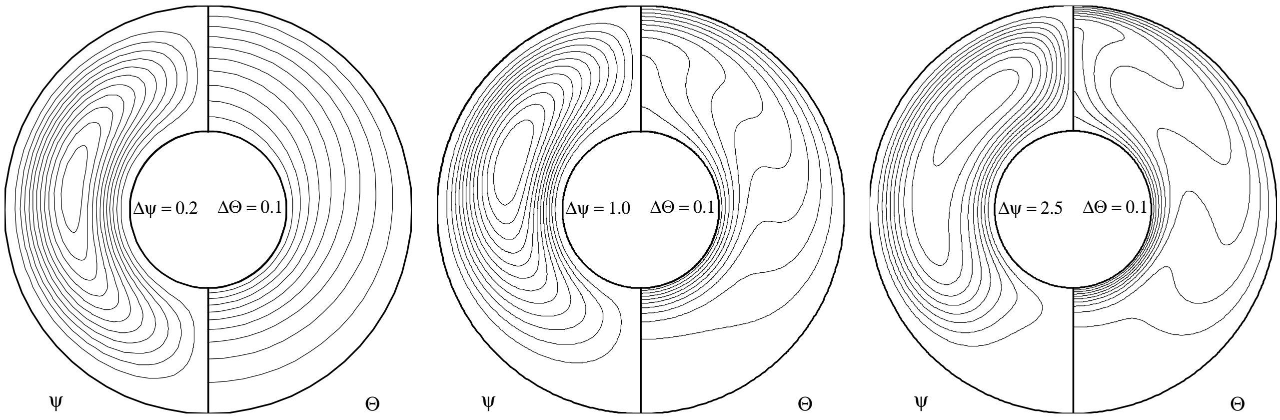


Figure 2. Comparison of the local Nusselt numbers from GITT against experimental results.

When Rayleigh number increases, the convective effects are more pronounced as becomes evident in Figs. (3b and 3c). It is then clearly noticed the increasing presence of the thermal boundary layer at both the lower region of the inner wall and at the upper region of the outer channel wall. In any such situations, convergence was inspected for and achieved to within the requested precision target.



(a) (b) (c)

Figure 3. Streamfunction and temperature isolines for: (a) (ϖ=2.6, Ra_L=10^3, Pr=0.7); (b) (ϖ=2.6, Ra_L=10^4, Pr=0.7); (c) (ϖ=2.6, Ra_L=5x10^4, Pr=0.7)

4. CONCLUSIONS

The Generalized Integral Transform Technique is successfully implemented for the hybrid solution of natural convection within horizontal concentric annular channels, under laminar and steady flow conditions. A set of reference results with global error control is provided, in both tabular and graphical forms, and previously reported experimental results are employed for validation purposes. These encouraging results allow now for the extension of the present analysis towards more involved situations, including non-concentric channels, rotating heat pipes flow analysis and variable thermophysical fluid properties.

5. ACKNOWLEDGEMENTS

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