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MODELING AND POSITION CONTROL OF A MULTI-LINK FLEXIBLE STRUCTURE USING LQR DESIGN

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Abstract. *The objective of this work is to describe the positional control of an unconstrained multi-link flexible structure. The experimental apparatus was designed to be representative of a flexible space structure such as a satellite with multiple flexible appendages. In this work we describe the analytical modeling and position control using a Linear Quadratic Regulator.*

Keywords: Flexible structures, Modal analysis, Identification, Control of structures.

1. INTRODUCTION

This paper presents the analytical modeling of a multibody flexible structure prototype and its position control using LQR design with reduced-order estimator states. The experimental setup, shown in the figure 1, was assembled with the aim to investigate the dynamics and the position control of flexible structures representative of aerospace structures such as a satellite with flexible appendages. The experimental setup is composed of two flexible aluminum beams coupled to a central rigid hub. The hub is mounted on a steel disc supported on a gas bearing in order to minimize the static friction and to simulate the structure's slew motion in space conditions. The steel disc is linked to a brushless DC motor, which gives the necessary excitation to the structure. The direct-drive torque actuation avoids the introduction of spurious non-linear effects such as dry friction and backlash in the gear transmission system.

The instrumentation and measurement subsystems consist of collocated and non-collocated sensors and their respective signal conditioning systems. An accelerometer is used to monitor the vibration displacement of the beam tip. Two full strain-gage bridges are used to measure the elastic deformation at two known positions along the arms. The collocated sensors consist of a tachometer and a potentiometer both fixed to the motor axis.

A schematic view of the experimental set up is shown in Fig. 1.

Figure 1- Experimental Setup

2. THE ANALYTICAL MODEL

The generalized Lagrangean approach is used to derive the analytical model of the unconstrained multi-link flexible structure, where the unconstrained characteristic results from the natural motion without external influences, i.e., all the structure is allowed to vibrate and its solution involves both the inertia of the rigid and the flexible parts (Barbieri & Özgüner, 1988). In this study we assume that the elastic deformation of the beams are symmetric with respect to the hub, consequently it is necessary to model only the elastic displacement of one of the arms (Junkins and Kim, 1993). The position of a generic point on the beam is written on a local body fixed coordinate system, as shown in the Fig. 2.

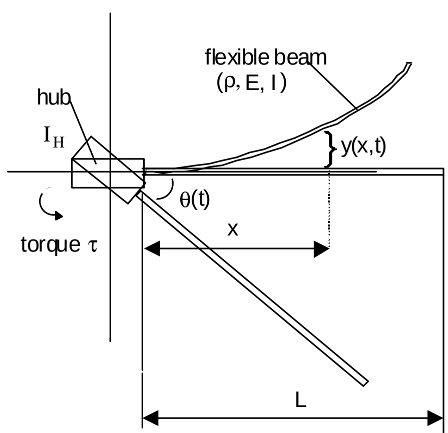


Figure 2. Coordinate system

The kinetic energy of the system is the sum of the kinetic energy of the hub, the arms and the tip mass (boundary elements).

$$T = T_{hub} + T_{beam} + T_{boundary\backslash} \tag{1}$$

with

$$T_{hub} = \frac{1}{2} I_{hub} \dot{\theta}^2 \tag{2}$$

$$T_{beam} = \int_0^L \rho \dot{\mathbf{R}}^2 dx \tag{3}$$

$$T_{boundary} = \frac{1}{2} m_t \dot{\mathbf{R}}^2(L) \tag{4}$$

where I_{hub} is the hub inertia, ρ is the linear mass density of the beam, L is the appendages length, m_t is the mass of the accelerometer located at the tip of the beam and R is a position of a generic point on the beam.

The potential energy of the distributed parameter system does not take into account the shear deformation and the rotary inertia of the beam and is given by the following expression:

$$V = \int_0^L EI \left[\frac{\partial^2 y(x,t)}{\partial x^2} \right]^2 dx \tag{5}$$

The Lagrangian of the system, is written as the total kinetic energy minus the potential energy of the structures and the nonconservative work done by the applied torque are respectively:

$$L = T - V \quad ; \quad dW_{nc} = t dq \tag{6}$$

Rearranging the terms in the kinetic and potential energy equations and the equations of motion is derived by applying the Lagrange equation for the hybrid (flexible plus rigid body) system which is expressed in a linear form in terms of a mass, M, and stiffness, K, matrices for the first three assumed modes (Negrão and Góes (2001).

$$M\ddot{q} + Kq = F \tag{7}$$

$$M = \begin{bmatrix} I_T & 0 & 0 & 0 \\ I_j & 1 & 0 & 0 \\ I_j & 0 & 1 & 0 \\ I_j & 0 & 0 & 1 \end{bmatrix} \text{ and } K = \begin{bmatrix} 0 & 0 \\ 0 & diag[w_1^2 \dots w_3^2] \end{bmatrix} \tag{8}$$

where: $I_T = \mathbf{I_{Hub}} + I_{beam} + \mathbf{m_t\, l^2}$; $I_j = \left(-\left(\mathbf{I_{Hub}} + I_{beam} + \mathbf{m_t\, l^2}\right)\theta_j\right) / \left(\int_0^l \rho\phi_j^2 \mathbf{dx} + m_t\phi_j^2\mathbf{(l)} + \mathbf{I_{Hub}}\theta_j^2\right)$; $j = 1,2,3$ (9)

$$q = [\Theta \quad h_1 \quad h_2 \quad h_3]^T \; ; \quad F = [t_m \quad f_1'(0)t_m \quad f_2'(0)t_m \quad f_3'(0)t_m]^T \tag{10}$$

$$I_{beam} = \int_0^L \rho x^2 \, dx \quad ; \quad \theta_i = -\left(\rho \int_0^L (x)\phi_i(x)dx + m_t(L)\phi_i(L)\right) / I_{hub} \quad ; \quad i,j=1,2,3... \tag{11}$$

$\phi_j(x)$ are the eigenfunctions of the hub-beam system.

The state-space representation of the system is in the form:

$$\dot{\mathbf{x}} = \mathbf{A}\,\mathbf{x} + \mathbf{B}\,\mathbf{u} \tag{12}$$

where the $\mathbf{A} = \begin{bmatrix} \mathbf{0} & \mathbf{I} \\ \mathbf{M}^{-1}\mathbf{K} & \mathbf{0} \end{bmatrix}$; $\mathbf{B} = \begin{bmatrix} \mathbf{0} \\ \mathbf{M}^{-1}\mathbf{F} \end{bmatrix}$ (13)

We define the observation matrix, **C**, that describes the measured signals in terms of the state variables. This matrix is obtained from the model of the available sensors. The accelerometer is located at the free tip of the beam and, its signal is conditioned by a pre-amplifier and a double integrator filter with a global coefficient of sensitivity given by G_a, in V/cm units. Thus, we can write:

$$e_{ac} = Ga(L\theta + y(L,t)) \tag{14}$$

Rewriting the integrated accelerometer equation, as in (Negrão, 1998):

$$e_{ac} = Ga[L \quad \phi_1(L) \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0][\theta(t) \quad \eta_1(t) \quad \eta_2(t) \quad \eta_3(t) \quad \dot{\theta}(t) \quad \dot{\eta}_1(t) \quad \dot{\eta}_2(t) \quad \dot{\eta}_3(t)]^T \tag{15}$$

The potentiometer provides a voltage proportional to the angular position of the hub, $e_p = G_p\theta(t)$. The full strain-gage bridge gives a signal proportional to the axial strain of the beam (ε_s), which can be related with the elastic deformation y(x, t), at the point were it is located by the eq. (16),

$$\varepsilon_s\big|_x = [e/2]\left(\partial^2 y/dx^2\right)\Big|_x \tag{16}$$

where *e* is the thickness of the beam. The strain-gage sensor is rewritten as:

$$\varepsilon_s = [e/2]\left[0 \quad d^2\phi_1(x_1)/dx^2 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0\right][\theta(t) \quad \eta_1(t) \quad \eta_2(t) \quad \eta_3(t) \quad \dot{\theta}(t) \quad \dot{\eta}_1(t) \quad \dot{\eta}_2(t) \quad \dot{\eta}_3(t)]^T \tag{17}$$

where x_1 is the position where the sensor is located on the beam. The tachometer gives a signal proportional to the angular velocity of the hub, $e_t = \dot{q}(t)$, which combined with the other sensor equations, gives the observation vector $\mathbf{y} = \mathbf{C}.\mathbf{x}$,

where $\mathbf{y} = [e_{ac} \quad e_p \quad e_s \quad e_t]^T$ (18)

and,

$$\mathbf{C} = \begin{bmatrix} L & \phi_1(L) & \phi_2(L) & \phi_3(L) & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{e}{2}\left[\frac{d^2\phi_1(x_1)}{dx^2}\right] & \frac{e}{2}\left[\frac{d^2\phi_2(x_1)}{dx^2}\right] & \frac{e}{2}\left[\frac{d^2\phi_3(x_1)}{dx^2}\right] & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \end{bmatrix} \tag{19}$$

To obtain the numeric state space and the analytical transfer functions, we used the physical parameters listed in table 1, for the unconstrained multi-link flexible system.

Table 1. Model parameter of the unconstrained flexible beams

Aluminum density	ρ	$2.7950\ 10^{3}$	Kg/m³
Aluminum Young's modulus	E	$6.8900\ 10^{10}$	N/m²
Beams width	Eb	$4.1200\ 10^{-3}$	M
Beams height	Hb	$8.0780\ 10^{-2}$	M
Beams length	L	$9.7150\ 10^{-1}$	M
Beams cross-section area	A	$3.3281\ 10^{-4}$	m^2
Beams moment of inertia	I	$4.7070\ 10^{-10}$	m^4
Beams mass moment of inertia	I_b	$2.8430\ 10^{-1}$	Kg m^2
Hub mass moment of inertia	I_{hub}	$7.6749\ 10^{-1}$	Kg m^2
Hub radius	R	$9.0000\ 10^{-2}$	M

3. POSITION CONTROL

The positional loop control is implemented to control the angular position of the hub in real time using collocated and non-collocated sensor feedback and is implemented using the MATLAB real-time program environment. The control strategy consists a rapid positioning of the tip at the end of the beam so that the tip vibration should be minimum. We show experimental position control using LQR design with two modes.

Consider the system:

$$\begin{aligned}\mathbf{x} &= \mathbf{Ax} + \mathbf{Bu} \\ \mathbf{y} &= \mathbf{Cx}\end{aligned} \tag{20}$$

and the quadratic cost functional $J = \frac{1}{2}\int_0^T (\underline{x}^T \underline{\underline{Q}}\underline{x} + \underline{u}^T \underline{\underline{R}}\underline{u})dt$

where R and Q are constant, symmetric matrices with R positive definite and Q nonnegative definite.

The optimal control law , which minimizes the cost functional, is:

$$\mathbf{u} = -k\mathbf{x} \,, \tag{21}$$

with $k = R^{-1}B^TP$ and P is the symmetric, positive definite matrix solution of the Riccati equation :

$$A^TP + PA + Q - PBR^{-1}B^TP = 0 \tag{22}$$

The implementation of the state feedback law requires state vector $\underline{x}$ available for measurement and feedback. Instead a reduced-order observer will be used for the modal coordinates of the system. From Chen(1984) the reduced-order observer is:

$$\begin{aligned}\dot{w} &= Fw + Hy + Gu \\ \hat{x} &= Mw + Ny\end{aligned} \tag{23}$$

where

$$F = A_{22} - LA_{12}\,; \quad H = FL + A_{21} - LA_{11}\,; \quad G = B_2 - LB_1\,; \quad N = P + ML\,; \tag{24}$$

L is the observer gain;

$$\left[\begin{array}{c:c} A_{11} & A_{12} \\ \hdashline A_{21} & A_{22} \end{array}\right] = \begin{bmatrix} CAP & CAM \\ TAP & TAM \end{bmatrix}; \tag{25}$$

N,M,P,T are from matrix transformation as defined in Chen(1984);

and the control law with an external reference *r* is:

$$u = -k\hat{\mathbf{x}} + r\,; \tag{26}$$

The closed loop can be obtained directly combining the closed loop system and observer equations using an external reference. Thus, we can write:

$$\begin{bmatrix} \dot{x} \\ \dot{w} \end{bmatrix} = \begin{bmatrix} A - BkNC & -BkM \\ HC - GkNC & F - GkM \end{bmatrix} \begin{bmatrix} x \\ w \end{bmatrix} + \begin{bmatrix} B \\ G \end{bmatrix} r\,; \tag{27}$$

A scheme of the LQR design using a reduced-order observer is shown in the Fig. 3.

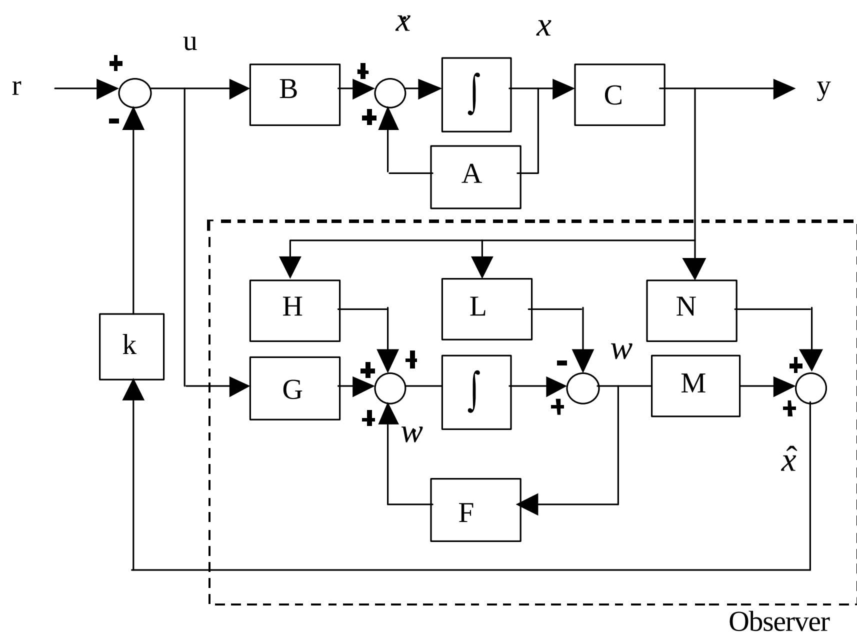


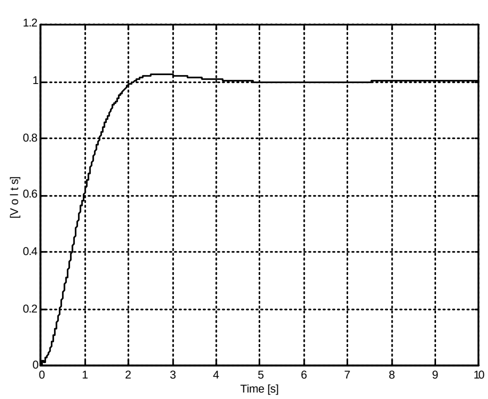
Figure 3. LQR Control Scheme

The eigenvalues of the estimator is chosen arbitrarily which are shown in table 2. The gain L is determined such that the eigenvalues of $A_{22} - LA_{12}$ are the eigenvalues of the estimator.

Table 2. Eigenvalues of the estimator

-14.7575 +/- 9.8400i
-6.3481 +/- 4.6586i

Using a step reference of 1 [Volts], the results of the position control using LQR design with reduced-order observer are illustrated in Fig. 4-7.



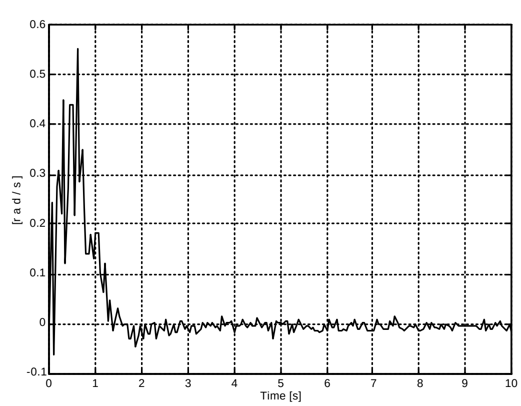
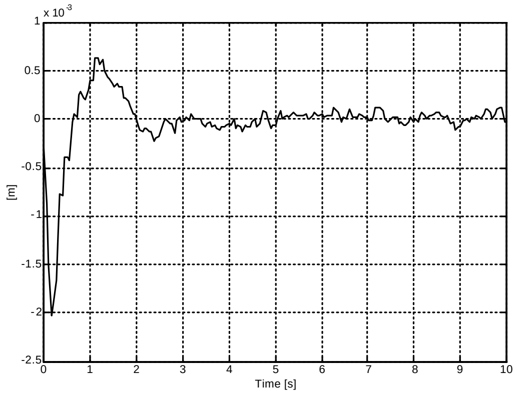


Figure 4. Angular position for a step reference Figure 5. Angular velocity for a step reference



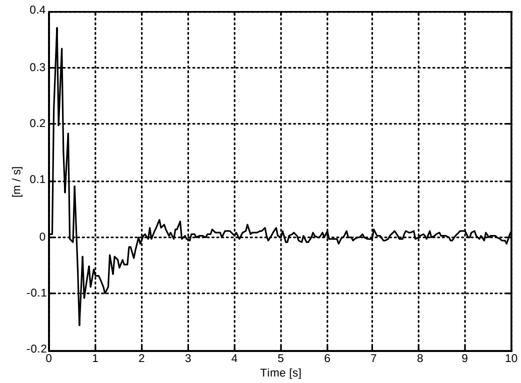


Figure 6. Transversal deformation for a step reference

Figure 7. Tip acceleration for a step reference

As one can see in the Figures (7)-(8), that the positional control is efficient in terms of overshoot and settling time. The response of the final position was reached in 2 seconds and without excitation of the higher vibrations modes of the beam.

4. CONCLUSIONS

This paper reports results obtained with an experimental apparatus with multiple flexible bodies. The model was derived using the Lagrangian approach and its discretization was done with the Assumed Modes Method. The results in control position using LQR design have shown effective control and reached the final reference position in 2 seconds and the tip with minimum vibration after settling time. This work is still in progress and using MATLAB to implement control we intend to implement other control strategy, such as robust control

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