II CONGRESSO NACIONAL DE ENGENHARIA MECÂNICA
II NATIONAL CONGRESS OF MECHANICAL ENGINEERING
12 a 16 de Agosto de 2002 - João Pessoa – PB

ANALYTICAL APPROXIMATIONS TO STUDY THE GRAVITATIONAL CAPTURE IN THE FOUR-BODY PROBLEM

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Abstract. The objective of the present paper is to study in some detail the ballistic gravitational capture in a dynamical model that has the presence of four bodies. In particular, the Sun-Earth-Moon-Spacecraft system is studied in detail. This phenomenon is explained in terms of the integration of the perturbing forces with respect to time. Analytical equations for those forces are derived to estimate their magnitude and to show the best directions of approaching for the spacecraft. Using those equations, an analytical estimate of the fourth body effect is derived.

Keywords: Astrodynamics, Four-Body Problem, Orbital Maneuvers, Gravitational Capture.

1. INTRODUCTION

The ballistic gravitational capture is a characteristic of some dynamical systems in celestial mechanics, as in the restricted four-body problem that is considered in this paper. The basic idea is that a spacecraft (or any particle with negligible mass) can change from a hyperbolic orbit with a small positive energy around a celestial body into an elliptic orbit with a small negative energy without the use of any propulsive system. The force responsible for this modification in the orbit of the spacecraft is the gravitational force of the third and the fourth bodies involved in the dynamics. In this way, this force is used as a zero cost control, equivalent to a continuous thrust applied in the spacecraft. One of the most important applications of this property is the construction of trajectories to the Moon.

The application of this phenomenon in spacecraft trajectories is recent in the literature. The first demonstration of this was in Belbruno, 1987. Further studies include Belbruno (1990 and 1992); Krish (1991); Krish, Belbruno and Hollister (1992); Miller and Belbruno (1991); Belbruno and Miller (1990 and 1993). They all studied missions in the Earth-Moon system that use this technique to save fuel during the insertion of the spacecraft in its final orbit around the Moon. Another set of papers that made fundamental contributions in this field, also with the main objective of constructing real trajectories in the Earth-Moon system, are those of Yamakawa, Kawaguchi, Ishii and Matsuo (1992 and 1993), Yamakawa (1992) and Kawaguchi (2000). The first real application of a ballistic capture transfer was made during an emergency in a Japanese spacecraft (Belbruno and Miller, 1990). After that, some studies that consider the time required for this transfer appeared in the literature. Examples of this approach can be find in the papers by Vieira-Neto and Prado (1995 and 1998). An extension of the dynamical model to consider the effects of the eccentricity of the primaries is also available in the literature (Vieira-Neto and Prado, 1996; Vieira-Neto, 1999). A study of this problem, from the perspective of invariant manifolds, was developed by Belbruno (1994). An application for a mission to Europa is shown in Sweetser (1997).

Looking in the literature related to the weak stability boundaries, it is possible to see that there are several definitions of ballistic gravitational capture, depending on the dynamical system considered.

Those differences exist to account for the different behavior of the systems. In the restricted four-body problem, the system considered in the present paper, ballistic gravitational capture is assumed to occur when the massless particle stays close to the second primary (the Moon) of the system for some time. A permanent capture is not required, because an impulsive maneuver will be performed to complete the maneuver.

For the practical purposes of studying spacecraft trajectories, the majority of the papers available in the literature study this problem looking in the behavior of the two-body energy of the spacecraft with respect to the Moon. A quantity called C_3 (that is twice the total energy of a two-body system) is defined, with respect to the closer primary, by

$$C_3 = V^2 - 2\mu/r \quad (1)$$

where V is the velocity of the spacecraft relative to the closest primary, r is the distance of the spacecraft from this primary and μ is the dimensionless gravitational parameter of the primary considered. From the value of C_3 it is possible to know if the orbit is elliptical ($C_3 < 0$), parabolic ($C_3 = 0$) or hyperbolic ($C_3 > 0$) with respect to the Moon. Based upon this definition, it is possible to see that the value of C_3 is related to the velocity variation (ΔV) needed to insert the spacecraft in its final orbit around the Moon. In the case of a spacecraft approaching the Moon, it is possible to use the gravitational force of the Earth to lower the value of C_3 with respect to the Moon, so the fuel consumption required to complete this maneuver is reduced. In that way, the search for trajectories that arrives at the Moon with the maximum possible value for the reduction of C_3 is very important. In the majority of the studies relative to this problem, a numerical approach of verifying the values of C_3 during the trajectory is used to identify useful trajectories. If there is a change of sign in C_3 from negative (closed trajectory) to positive (open trajectory) when leaving the Moon, it means that a ballistic gravitational capture occurs in the positive sense of time and this particular trajectory can be used to reduce the amount of fuel in an Earth-Moon transfer. The present paper has the goal of developing analytical equations to estimate the effects of the fourth body in the reduction of C_3.

2. MATHEMATICAL MODEL

Next, the formulation of the restricted four-body problem is shown. In the model shown here, we have (Fig. 1): i) The Earth and the Moon constitute the two main primaries of the system, both in circular orbits around the common center of mass; ii) The Sun is the third body, assumed to be in a circular orbit around the center of mass of the Earth-Moon system and its orbit is coplanar with the orbit of the Moon around the Earth; iii) The motion a massless particle that has its motion governed by the three primaries will be studied. The center of mass of the Earth-Moon system is the origin. The positions of the bodies are: $(-\mu_2,0,0)$ for the Earth and $(\mu_1,0,0)$ for the Moon. The values of the masses are given by $\mu_1 = 0.9878493317$ for the Earth, $\mu_2 = 0.0121506683$ for the Moon and $\mu_{sun} = 328900.48$ for the Sun (canonical units). The equations of motion are (Yamakawa, 1992)

$$\frac{d^2x}{dt^2} - 2\frac{dy}{dt} - x - \frac{\mu_{sun}}{R_{sun}^2}\cos\psi_{moon} = -\frac{\delta U_n}{\delta x}$$
$$\frac{d^2y}{dt^2} + 2\frac{dx}{dt} - y + \frac{\mu_{sun}}{R_{sun}^2}\sin\psi_{moon} = -\frac{\delta U_n}{\delta y} \quad (2)$$
$$\frac{d^2z}{dt^2} = -\frac{\delta U_n}{\delta z}$$

where

$$U_n = -\frac{\mu_1}{r_1} - \frac{\mu_2}{r_2} - \frac{\mu_{sun}}{r_{sun}} \quad (3)$$

$$\frac{\delta U_n}{\delta x} = \frac{\mu_1}{r_1^3}(x+\mu_2) + \frac{\mu_2}{r_2^3}(x-\mu_1) + \frac{\mu_{sun}}{r_3^3}(x + R_{sun}\cos\psi_{moon})$$
$$\frac{\delta U_n}{\delta y} = \frac{\mu_1}{r_1^3}(y) + \frac{\mu_2}{r_2^3}(y) + \frac{\mu_{sun}}{r_{sun}^3}(y - R_{sun}\cos\psi_{moon}) \qquad (4)$$
$$\frac{\delta U_n}{\delta z} = \frac{\mu_1}{r_1^3}(z) + \frac{\mu_2}{r_2^3}(z) + \frac{\mu_{sun}}{r_{sun}^3}(z)$$

r_1, r_2, and r_{sun} represents the distances between the spacecraft and the Earth, the Moon and the Sun, respectively. R_{sun} = 389.1723985 is the distance between the Sun and the origin of the coordinate system. (x_s, y_s, z_s) represents the position of the Sun with respect to the rotating system. Those variables are given by:

$$r_1^2 = (x+\mu_2)^2 + y^2 + z^2 \qquad (5)$$
$$r_2^2 = (x-\mu_1)^2 + y^2 + z^2 \qquad (6)$$
$$r_{sun}^2 = (x-x_s)^2 + (y-y_s)^2 + (z-z_s)^2 = (x + R_{sun}\cos\psi_{moon})^2 + (y - R_{sun}\sin\psi_{moon})^2 + z^2 \qquad (7)$$
$$\Psi_{moon} = \overline{\alpha} + (\omega_{E-M} - \Omega_{sun})t \qquad (8)$$

Where $\overline{\alpha}$ is the phase angle of the Sun, given by (anti-Sun direction)-(origin)-(Moon), that is the initial position of the Moon in a system that has the Earth and the Sun in fixed positions; ω_{E-M} is the angular velocity of the Earth-Moon system (= 1.00); Ω_{sun} is the angular velocity of the Sun with respect to the system of reference (= 0.07480133).

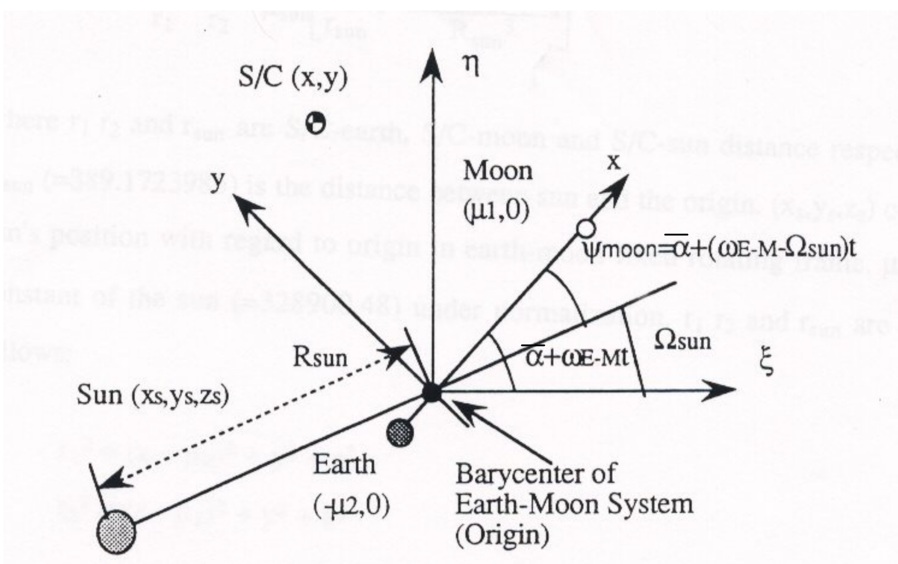

Figure 1 – The restricted four-body problem (Yamakawa, 1992).

The standard canonical system of units is used, in which: the unit of distance is the distance between M_1 (the Earth) and M_2 (Moon); the angular velocity (ω_{E-M}) of the motion of M_1 and M_2 is set to unity; the mass of the smaller primary (M_2) is given by $\mu = m_2/(m_1+m_2)$ (where m_1 and m_2 are the real masses of the Earth and the Moon, respectively) and the mass of M_2 is $(1-\mu)$; the unit of time is defined such that the period of the motion of the two primaries is 2π and the gravitational constant is unity.

A trajectory is considered a ballistic gravitational capture when the distance from the Moon reaches 100,000 km in a time smaller than 50 days without the use of any propulsive system (Yamakawa, 1992). This distance will be called here the gravitational sphere of capture of the Moon. The main parameters of the trajectory are: r_p, that is the periapsis distance (assumed to be 1838 km in the calculations performed in the present paper); α, that is the periapsis position angle that specifies the point of closest approach with the Moon and β, that is the entry position angle, the angle that specifies the point where the spacecraft reaches the sphere of capture of the Moon.

3. FORCES INVOLVED IN THE DYNAMICS

To understand better the physical reasons of this phenomenon, it is useful to calculate the forces acting over the massless particle. Figure 2 shows the gravitational force $\vec{F}_g$ of the Earth acting in a spacecraft M_3 that is approaching the Moon and Fig. 3 shows the gravitational force of the Sun and the centrifugal force acting in the same situation. There is also the Coriolis force, given by $-2\vec{\omega}_{E-M} \times \vec{v}$, where $\vec{v}$ is the velocity of the spacecraft. This force is not analyzed in detail because the main idea of this paper is to explain the ballistic gravitational capture as a result of perturbative forces acting in this direction and the Coriolis force acts perpendicular to the direction of motion of the spacecraft all the time. In this way, it does not contribute to the phenomenon studied here. The direction $\vec{r}$ points directly to the center of the Moon and the direction $\vec{p}$ is perpendicular to $\vec{r}$, pointing in the counter-clockwise direction. The distance between the spacecraft and the Earth is d, the angle formed by the line connecting the Earth to the spacecraft and the direction $\vec{r}$ is γ. The angle ϕ is used to define instantaneously the direction $\vec{r}$. From geometrical considerations shown in more detail in Prado (2002), it is possible to write for the gravitational force:

$$\vec{F}_g = \frac{(1-\mu)(r+\cos\phi)}{d^3}\vec{r} + \frac{(1-\mu)\sin\phi}{d}\vec{p} = \frac{(1-\mu)(r+\cos\phi)}{(1+r^2+2r\cos\phi)^{3/2}}\vec{r} + \frac{(1-\mu)\sin\phi}{(1+r^2+2r\cos\phi)^{1/2}}\vec{p} \qquad (8)$$

For the centrifugal force the expression is (Prado, 2002):

$$\vec{F}_{ce} = -[r+(1-\mu)\cos\phi]\vec{r} + (\mu-1)\sin\phi\,\vec{p} \qquad (9)$$

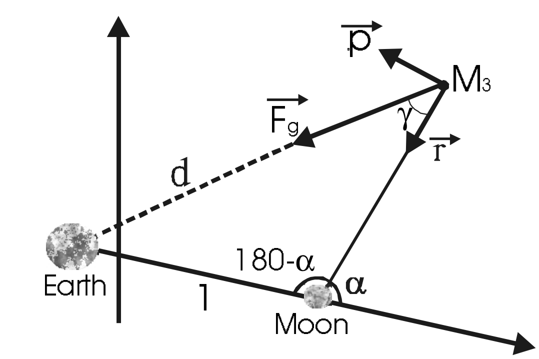

Fig. 2 – Gravitational force of the Earth.

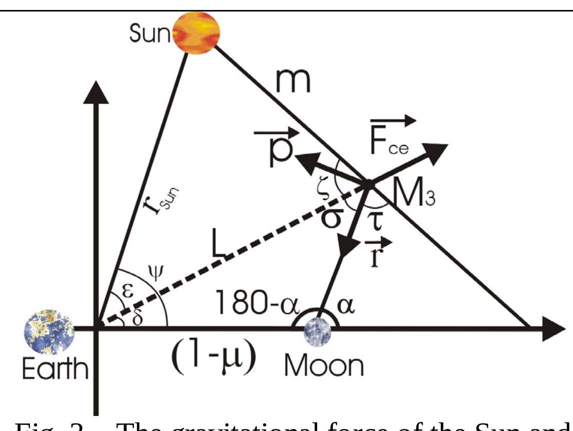

Fig. 3 – The gravitational force of the Sun and the centrifugal force.

Now, it is necessary to develop an equivalent equation for the gravitational force of the Sun ($\vec{F}_S$). From Fig. 3, it is possible to find the following relations, where $\vec{F}_{Sr}$ stands for the radial component and $\vec{F}_{Sp}$ stands for the perpendicular component:

$$\vec{F}_{Sr} = -\frac{\mu_{sun}\cos\tau}{m^2}\vec{r}\,, \qquad \vec{F}_{Sp} = -\frac{\mu_{sun}\sin\tau}{m^2}\vec{p} \tag{10}$$

$$R_{sun}^2 = L^2 + m^2 - 2Lm\cos\zeta \Rightarrow \cos\zeta = \frac{R_{sun}^2 - L^2 - m^2}{-2Lm} \tag{11}$$

$$\tau = \pi - \sigma - \zeta \tag{12}$$

$$\delta = \alpha - \sigma \tag{13}$$

$$\varepsilon = \psi - \delta = \psi - \alpha + \sigma \tag{14}$$

$$m^2 = L^2 + R_{sun}^2 - 2LR_{sun}\cos\varepsilon \tag{15}$$

Those equantions can be used to find analytical equations for the radial and perpendicular components of the gravitational force of the Sun.

During the approach phase, when the spacecraft is close to the Moon, the force that dominates the dynamics is due to the central body (the Moon). All others forces are perturbations on the motion of the massless particle. In the model considered here, the perturbations are due to the gravitational force of the Earth and the Sun and the centrifugal force due to the rotation of the system. In that way, an approach to understand the behavior of the perturbing forces is to study the components of each force during the approach phase. This study is performed in Prado (2002), that shows an equation that relates the reduction of C_3 with the integral of the forces over the time.

4. ANALYTICAL ANALYSES OF THE FORCES

The next step of this research is to use the analytical expressions derived in Prado (2002) for the effects of the gravitational force of the Earth and the centrifugal force and to derive an equivalent equation for the gravitational force of the Sun, in order to obtain an estimate of the effects of the forces studied. The main idea is to estimate the potential of the field around the Moon to reduce the value of the C_3 due to the Earth and the Sun and not to make predictions for a single trajectory. The analytical equations to measure the effects of this perturbation are derived under the assumption that the trajectory followed by the spacecraft is an idealized trajectory that does not deviate from the radial direction. The real trajectories are not radial, as can be seen in the references shown in this paper, but the equations derived under this assumption can be used to: i) estimate the values of the possible reductions in the value of C_3; ii) show the existence of directions of motion that results in larger reductions of C_3, so mapping analytically the decelerating field that exists in the neighborhood of the Moon; iii) estimate the effects of the periapsis distance and the size of the sphere of capture, since the equations derived are explicitly functions of those parameters; iv) to study the effect of the fourth body in the savings obtained in the gravitational capture. Another justification for the radial trajectories used to derive the equations is that the reduction of C_3 is a result of the effects of the forces in time during the whole trajectory and, even for trajectories that shows several loops before arriving at the periapsis, during most of the time the trajectory can be seen as composed by a set of trajectories close to radial.

For the derivation performed here, the component measured is the radial, because this is the direction of motion under the assumption used here. Then, assuming that the spacecraft is in free-fall (subject only to the gravitational and centrifugal forces) traveling with zero energy (parabolic trajectory) and that the trajectories do not deviate from a straight line, the result is:

$$\text{Total energy} = E = 0 = \frac{1}{2}V^2 - \frac{\mu}{r} \Rightarrow V = \sqrt{\frac{2\mu}{r}} = \frac{ds}{dt} \tag{16}$$

Where ds is the space traveled by the particle during the time dt. To obtain the integral of the effect of the perturbing forces with respect to time, it is possible to perform the calculations in terms of the radial distance, by making the substitution:

$$\int_{t_0}^{t_f} F\,dt = \int_{S_0}^{S_f} (F/V)ds = \int_{r_{min}}^{r_{max}} (F/V)dr \tag{17}$$

The extreme points of the integration changes position (S_0 by r_{min} and S_f by r_{max}) here and in all the following integrations to take into account that the positive sense of the radial direction points towards the Moon. Since the spacecraft is assumed to approach the Moon on a radial trajectory the result $\phi = \alpha = \beta$ is valid, and the variable α is used as the independent parameter. Then, for the radial component of the Earth's gravity, up to the first order, the integral is (Prado, 2002):

$$F_1(\alpha) = \left[\frac{(1-\mu)(q+\cos\alpha)}{(2\mu/q)^{1/2}(1+q^2+2q\cos\alpha)^{3/2}} r + (1-\mu)\left(-\frac{3\,(q+\cos\alpha)^2}{(2\mu/q)^{1/2}(1+q^2+2q\cos\alpha)^{5/2}} + \right.\right.$$
$$\left.\left. + \frac{\mu(3q+\cos\alpha)}{q^2(2\mu/q)^{3/2}(1+q^2+2q\cos\alpha)^{3/2}} \right)\left(\frac{r^2}{2} - qr\right) \right]_{r_{min}}^{r_{max}} \tag{18}$$

Using the values r_{min} = 1838/384400 (100 km above the lunar surface), r_{max} = 100000/384400 (100000 km above the lunar surface, the usual value for the sphere of capture of the Moon in the ballistic gravitational capture studies), μ = 0.0121 (Earth-Moon system) and $q = (r_{min} + r_{max})/2$ (the medium point of the trajectory) the first order equation obtained is (Prado, 2002):

$$F_1^1(\alpha) = (0.0782 + 0.5902\cos(\alpha))(1.0176 + 0.2649\cos(\alpha))^{-1.5} \tag{19}$$

The equivalent equation for the second order expansion is shown below, since in this form it is not too large (Prado, 2002):

$$F_1(\alpha) = (0.2836(0.0170 - 0.0730(1.0175 + 0.2649\cos\alpha) + 0.3076(1.0175 + 0.2649\cos\alpha)^2 +$$
$$+ 0.0680\cos\alpha - 0.0847(1.0175 + 0.2649\cos\alpha)\cos\alpha + 2(1.0175 + 0.2649\cos\alpha)^2\cos\alpha +$$
$$+ 0.0168\cos 2\alpha - 0.0640(1.0175 + 0.2649\cos\alpha)\cos 2\alpha + 0.0212\cos 3\alpha))$$
$$(1.0175 + 0.2649\cos\alpha)^{-7/2} \tag{20}$$

For the radial component of the centrifugal force, the integral is:

$$\int_{r_{min}}^{r_{max}} (F_{ce}/V)ds = \int_{r_{min}}^{r_{max}} ((\mu-1)\cos\alpha + r)(2\mu/r)^{-1/2}dr = \left[\left(-0.4r^2 + \frac{2}{3}r(\mu-1)\cos\alpha\right)(2\mu/r)^{-1/2}\right]_{r_{min}}^{r_{max}} \tag{21}$$

Using the same values used in the above situation for the variables, this last equation can be reduced to:

$$F_2(\alpha) = -0.0887 - 0.5603\cos\alpha \tag{22}$$

Repeating the process for the gravitational force due to the Sun, we have:

$$F_3^1(\alpha)=\left[\left(\frac{\mu_{sun}\left(q+\cos\alpha-\mu\cos\alpha-r_{sun}\cos(\alpha-\psi)\right)}{\sqrt{2\mu/q}\left(1+q^2+r_{sun}^2-2\mu+\mu^2+2q(1-\mu)\cos\alpha-2qr_{sun}\cos(\alpha-\psi)-2r_{sun}\cos\psi+2r_{sun}\mu\cos\psi\right)^{3/2}}\right)r\right]_{r_{min}}^{r_{max}}$$
$$+\left[\left(-\frac{3\left(2q+2(1-\mu)\cos\alpha-2r_{sun}\cos(\alpha-\psi)\right)\left(q+\cos\alpha-\mu\cos\alpha-r_{sun}\cos(\alpha-\psi)\right)}{\sqrt{2\mu/q}\left(1+q^2+r_{sun}^2-2\mu+\mu^2+2q(1-\mu)\cos\alpha-2qr_{sun}\cos(\alpha-\psi)-2r_{sun}\cos\psi+2r_{sun}\mu\cos\psi\right)^{5/2}}+\right.\right.$$
$$\left.\left.+\frac{\sqrt{q/\mu}+\sqrt{q/\mu}\left(\frac{\left(q+\cos\alpha-\mu\cos\alpha-r_{sun}\cos(\alpha-\psi)\right)}{2q}\right)}{\left(1+q^2+r_{sun}^2-2\mu+\mu^2+2q(1-\mu)\cos\alpha-2qr_{sun}\cos(\alpha-\psi)-2r_{sun}\cos\psi+2r_{sun}\mu\cos\psi\right)^{3/2}}\right)\left(\frac{\mu_{sun}}{\sqrt{2}}\right)\left(\frac{r^2}{2}-qr\right)\right]_{r_{min}}^{r_{max}} \quad (23)$$

and, using the numerical values as done before:

$$F_3^1(\alpha)=\frac{A}{B/C} \quad (24)$$

where:

$$A=0.0377(151454+0.2617\cos\alpha-103.1020\cos(\alpha-\psi)-768.9220\cos\psi)^{1/2}(-82.3068-268.9200\cos\alpha$$
$$-65.7260\cos(2\alpha)-4.2907\cos(3\alpha)-119.2110\cos(\alpha-2\psi)+6.5904\cos(3\alpha-2\psi)+0.5753\cos(4\alpha-2\psi)$$
$$+50403.4000\cos(\alpha-\psi)+1.7363\cos(2\alpha-2\psi)+12832.6000\cos(2\alpha-\psi)+845.1080\cos(3\alpha-\psi)$$
$$-0.0029\cos(4\alpha-\psi)+12832.9000\cos(\psi)-32.0000\cos(2\psi)+845.5000\cos(\alpha+\psi)+0.0842\cos(2\alpha+\psi)$$
$$+0.0054\cos(3\alpha+\psi)-2.1454\cos(\alpha+2\psi)) \quad (25)$$

$$B=\left[(3.7960+\cos\alpha)^2(-196.9700-0.0003\cos\alpha+0.1341\cos(\alpha-\psi)+\cos\psi)^3\right]$$
$$-0.2845(151454+0.2617\cos\alpha-103.1020\cos(\alpha-\psi)-768.9220\cos\psi)^{1/2}(15.4477+37.9366\cos\alpha$$
$$+8{,}8588\cos(2\alpha)+0.5684\cos(3\alpha)+19.2626\cos(\alpha-2\psi)+2.5985\cos(3\alpha-2\psi)+0.1524\cos(4\alpha-2\psi)$$
$$-6676.6500\cos(\alpha-\psi)+13.4051\cos(2\alpha-2\psi)-1699.9300\cos(2\alpha-\psi)-111.9640\cos(3\alpha-\psi)$$
$$-0.0008\cos(4\alpha-\psi)-1699.9200\cos(\psi)+4.4675\cos(2\psi)-112.0040\cos(\alpha+\psi)-0.0115\cos(2\alpha+\psi)$$
$$-0.0007\cos(3\alpha+\psi)+0.2842\cos(\alpha+2\psi)) \quad (26)$$

$$C=\left[(3.7960+\cos\alpha)^2(-196.9700-0.0003\cos\alpha+0.1341\cos(\alpha-\psi)+\cos\psi)^3\right] \quad (27)$$

The second order equations are too large to be shown and it also has a small contribution compared with the other forces. It is clear that the best result, regarding capture, occurs for $\alpha = \psi$, that means that the spacecraft is aligned with the Sun, what is an expected result. Figure 4 shows the effects of the gravitational force of the Sun as a function of α and ψ.

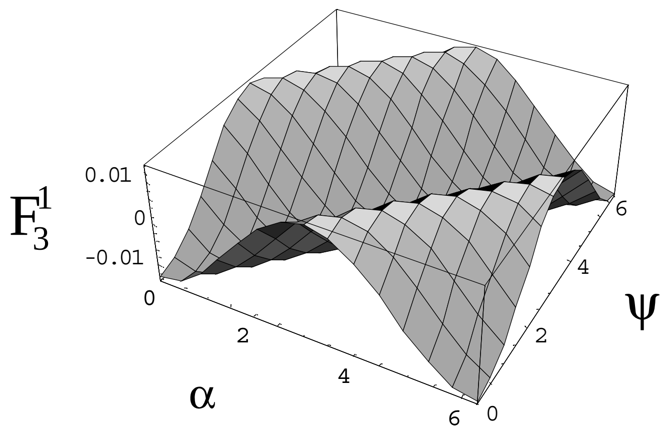

Fig. 4 – Effects of the gravitational force of the Sun as a function of α and ψ.

Adding the radial effects of all the forces, the equation for the resultant force in the radial direction is obtained. This force will be called $F_r(\alpha)$. All the forces are plotted as a function of α in Fig. 5, for the case where ψ = 0. The numbers represents: 1 for the gravitational force due to the Earth; 2 for the centrifugal force; 3 for the gravitational force due to the Sun; 4 for the resultant force. From those results, it is clear that the integral of the total effect is always negative, which means that the spacecraft always has its velocity reduced by the perturbation. It is never increased. There are two points where the integral of the effect is null, which means that the two perturbing forces acting on the spacecraft cancel each other and it travels as if there were no perturbations at all. In this figure it is also possible to obtain the best point to perform the ballistic gravitational capture. This point is at α = 180°, which has the strongest accumulated effect for the resultant force. Figure 6 shows the perturbation of the fourth body $F_{4b}(\alpha)$ in more detail, for the case where ψ = 0. Figure 7 shows the resultant forces acting in the motion of the spacecraft including and excluding the Sun. It is clear that the Sun helps to increase the effect of slowing down the spacecraft in a amount of the order of 3%.

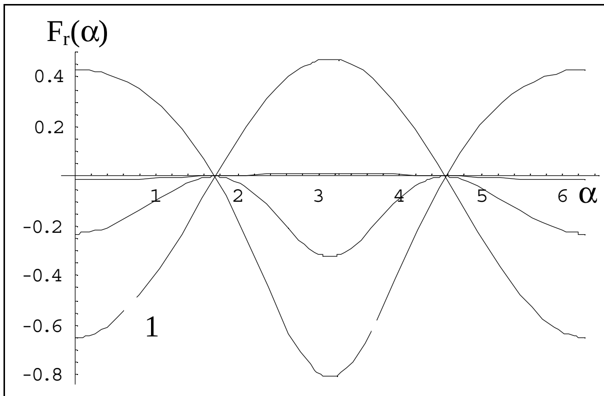

Fig. 5 – Integral of the disturbing radial forces (canonical units) vs. α (rad) for ψ = 0.

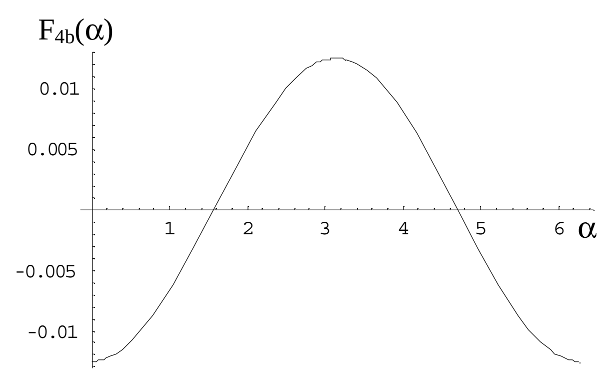

Fig. 6 – Effects of the gravitational force of the Sun as a function of α for ψ = 0.

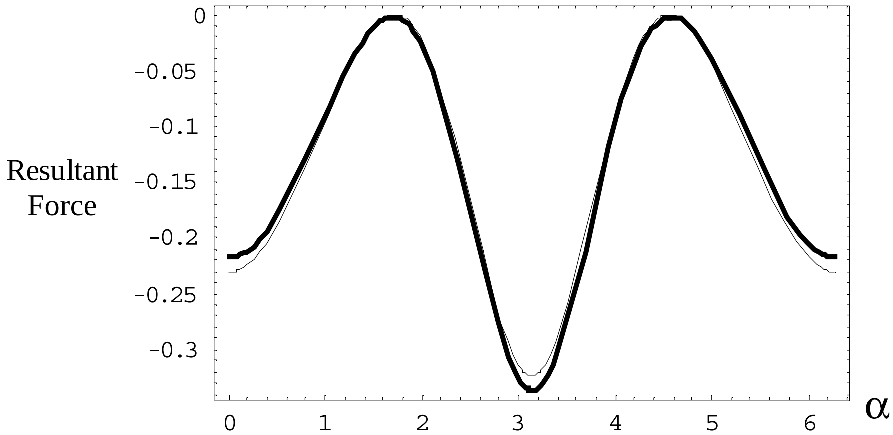

Fig. 7 – Integral of the resultant force including the Sun (dark line) and excluding the Sun.

5. CONCLUSIONS

This paper had the main goal of studying the ballistic gravitational capture problem under the model given by the restricted four-body problem. It showed an explanation of the phenomenon based in the calculation of the forces involved in the dynamics as a function of time and in its integration with respect to time. It also derived analytical equations to study the effect of the fourth body, under the assumption of radial motion. There are three forces that act as disturbing forces in the direction of motion: the gravitational forces due to the Earth and the Sun and the centrifugal force. These forces can slow down the motion of the spacecraft, working opposite to its motion. This is equivalent to applying a continuous propulsion force against the motion of the spacecraft. In the radial direction the gravitational force due to the Earth and the centrifugal force work in opposite directions, but the resultant force always works against the motion of the spacecraft, with the exception of two points where they cancel each other. Understanding these behaviors explains why a particle with a velocity slower than the escape velocity can escape from the Moon. The results also showed that the inclusion of the Sun in the dynamics could increase by about 3% the effects of the forces.

6. ACKNOWLEDGEMENTS

The author is grateful to CNPq (National Council for Scientific and Technological Development) - Brazil for the contract 300221/95-9 and to FAPESP (Foundation to Support Research in São Paulo State) for the contract 2000/14769-4.

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