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# UNSTEADY COMBINED CONDUCTION-RADIATION HEAT TRANSFER IN MULTI-LAYER PLANAR PARTICIPATING MEDIA

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**Abstract:** *Unsteady combined conduction-radiation heat transfer between parallel flat plates separated by semitransparent media is investigated. The plates are assumed to be opaque, gray and held at constant temperatures. The medium separating the plates emits, absorbs and scatters thermal radiation and is formed by one or more layers of materials. Numerical solutions are obtained by a numerical algorithm for the transient cooldown of the medium due to a discontinuous decrease in temperature at one boundary. Based upon the application of the overall energy and radiative conservation principles it is possible to establish a relationship between the two heat transfer modes, i.e., radiation and conduction. To solve the Radiative Transfer Equation – RTE, the discrete ordinate principle is used together with a Gaussian integration scheme. Effects of conduction-radiation parameter and scattering albedo on the temperature profiles are analyzed.*

**Key words:** *thermal radiation, radiative transfer equation (RTE), conduction-radiation heat transfer*

## 1. INTRODUCTION

Several articles have been written in the last three decades concerning the transient energy transfer by radiation and conduction (Siegel, 1998). It is an important subject because in many practical applications, such as furnaces design, process of steel and glass solidification and thermal control of aerospace vehicles, both heat transfer mechanisms occur simultaneously. The differences among then are the geometry and configuration of the problem and the numerical method used to solve the Radiative Transfer Equation – RTE. A classic paper was published by Doornink and Hering (1972), in which the authors present a relevant contribution to the study of transient conductive-radiative heat transfer, addressing important questions as the situations in which interaction between the two energy transport mechanisms must be considered and the usefulness and accuracy of some common approximations for the radiative flux term. However, their study has some limitations such as the assumption that the surfaces bounding the medium are black and the absence of scattering.

A more complex case was studied by Ho and Özisik (1987), where the interaction of conduction and radiation was investigated under both transient and steady-state conditions for an absorbing, emitting and isotropically scattering two-layer slab having opaque coverings at both boundaries and subjected to an externally applied constant heat flux at one boundary surface. The radiative transfer equation was solved by the generalization of the Galerkin method. By using the method of exponencial integral function to solve the RTE, Tsai and Nixon (1986) had already studied the later problem, however, composed by a multilayer composite wall.

This work considers transient combined conduction-radiation heat transfer between two infinite plane parallel surfaces, separated by a semitransparent medium composed by one or more layers. The application of the overall energy conservation principle provides an ordinary differential equation for temperature, whereas the conservation of radiative energy provides an integro-differential equation for the intensity of radiation, known as the Radiative Transfer Equation (RTE). This set of equations are coupled through the temperature field in the medium. To solve the ordinary differential equation for temperature, the finite difference method is used, whereas a novel procedure proposed by Pessoa-Filho and Thynell (1994) is used to solve the RTE. In the following sections the equations describing the phenomena and the method of solution are presented and the obtained results compared with values published by other authors.

## 2. ANALYSIS

Consider a medium composed by N layers of semitransparent materials that absorb, emit and isotropically scatter radiation, Fig. (1). The interfaces separating the layers are assumed to be transparent. The optical properties of the materials such as scattering-albedo, optical thickness and the conduction-to-radiation parameter can change from one layer to another but they are assumed constant and uniform within each layer. The walls, located at $\tau=0$ and $\tau=\tau_o$, are also assumed gray, diffuse and opaque and held at constant temperature.

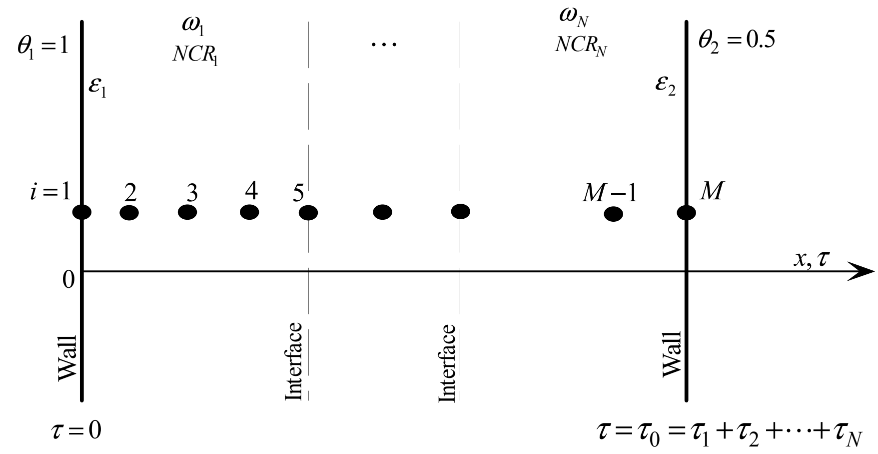


Figure 1. Geometrical configuration

The application of the overall energy conservation principle to an infinitesimal volume element within the medium, under the assumption of one-dimensional heat transfer and constant physical properties, yields Özisik (1973),

$$\frac{d\theta(\tau,\xi)}{d\xi} = \frac{d^2\theta(\tau,\xi)}{d\tau^2} - \frac{1-\omega}{NCR}\left[\theta^4(\tau,\xi) - G^*(\tau,\xi)\right], \qquad 0 < \tau < \tau_o, \tag{1}$$

where $\theta$, $\tau$, $\xi$, $\omega$, $NCR$ and $G^*$ refer to dimensionless temperature, optical thickness, time, scattering albedo, conduction-to-radiation parameter and incident radiation defined, respectively, by

$$\tau \equiv \beta x, \qquad NCR \equiv \frac{k\beta}{4n^2\sigma T_r^3}, \qquad Q^r(\tau,\xi) \equiv \frac{q^r(\tau,\xi)}{n^2\sigma T_r^4}, \tag{2.a,b,c}$$

$$\theta(\tau,\xi) \equiv \frac{T(\tau,\xi)}{T_r}, \qquad G^*(\tau,\xi) \equiv \frac{G(\tau,\xi)}{4n^2\sigma T_r^4}, \qquad \xi = \alpha\beta^2 t\ . \tag{2.d,e,f}$$

Where $x$ is the physical length and $\alpha$, $\beta$, $k$, $T$, $t$ and $n$ are the thermal diffusivity, medium's extinction coefficient, thermal conductivity, temperature, time and refraction index, respectively; $\sigma$ is the Stefan-Boltzmann constant and $T_r$ is a reference temperature; $Q^r(\tau,\xi)$ and $G^*(\tau,\xi)$ are the dimensionless radiative heat flux and incident energy, respectively.

Since the interfaces are transparent and the walls are held at constant and known temperatures, we have:

$$NCR_j \left.\frac{d\theta}{d\tau}\right|_{Layer\ j} = NCR_{j+1} \left.\frac{d\theta}{d\tau}\right|_{Layer\ j+1}, \quad j = 1, N-1, \text{ at the interfaces.} \tag{3}$$

As initial and boundary conditions, we have:

$$\theta(\tau,0) = \theta_2, \qquad 0 \le \tau \le \tau_0, \tag{4.a}$$

$$\theta(0,\xi) = \theta_1, \qquad \xi > 0, \tag{4.b}$$

$$\theta(\tau_0,\xi) = \theta_2, \qquad \xi \ge 0. \tag{4.c}$$

The quantities $q^r$ and $G$ appearing in Eq. (2.c) and (2.e) are the radiative heat flux and the incident radiation, given by:

$$G(\tau,\xi) = 2\pi\int_{-1}^{1} I(\tau,\mu,\xi)d\mu, \tag{5}$$

$$q^r(\tau,\xi) = 2\pi\int_{-1}^{1} I(\tau,\mu,\xi)\mu d\mu. \tag{6}$$

Where $I(\tau,\mu,\xi)$ is the intensity of radiation at a position $\tau$ within the medium, $\mu\,(\equiv \cos\theta)$ gives its direction of propagation and $\xi$ is the dimensionless time, Fig. (2). To evaluate $I(\tau,\mu,\xi)$ we apply the conservation of radiative energy to infinitesimal volume element inside the medium obtaining the so-called Radiative Transfer Equation (RTE), Özisik (1973):

$$\mu\frac{dI(\tau,\mu,\xi)}{d\tau} + I(\tau,\mu,\xi) = S(\tau,\xi), \tag{7.a}$$

where $S(\tau,\xi)$ is the source term given by

$$S(\tau,\xi) = [1-\omega]I_b[\theta(\tau,\xi)] + \frac{\omega}{2}\int_{-1}^{1} I(\tau,\mu,\xi)\,d\mu\,. \tag{7.b}$$

In obtaining Eqs. (7), we assumed a gray, isotropically scattering medium with constant optical properties. It has also been assumed that the intensity of radiation is independent on the azimuth angle $\phi$. Equations (7) are valid for any layer within the medium. $I_b[T(\tau,\xi)]$ is the Planck function which is dependent on the medium temperature, and given by

$$I_b[\theta(\tau,\xi)] = \frac{n^2\,\sigma\,\theta(\tau,\xi)^4}{\pi\,T_r^4}\,. \tag{8}$$

At the walls, we have:

$$I(0,\mu,\xi) = \varepsilon_1\,I_b(\theta_1) + 2(1-\varepsilon_1)\int_0^1 I(0,\mu,\xi')\,\mu'\,d\mu', \tag{9.a}$$

$$I(\tau_o,\mu,\xi) = \varepsilon_2\,I_b(\theta_2) + 2(1-\varepsilon_2)\int_0^1 I(\tau_o,\mu',\xi)\,\mu'\,d\mu'. \tag{9.b}$$

The objective of solving Eqs. (1)-(9) is to find the temperature distribution within the medium and to investigate how it is affected by the physical and optical properties of the medium, as well as its boundary conditions. By solving Eq. (1), it is possible to obtain the temperature distribution within the medium. However, solution of Eq. (1) requires the knowledge of $G^*(\tau,\xi)$, as defined by Eqs. (2.e) and (5). To obtain $G(\tau,\xi)$ it is necessary to solve the RTE which, by its turn, is dependent on the temperature distribution within the medium through the Planck function. As a consequence, we have a coupled radiative-conductive heat transfer problem whose solution is of the iterative type.

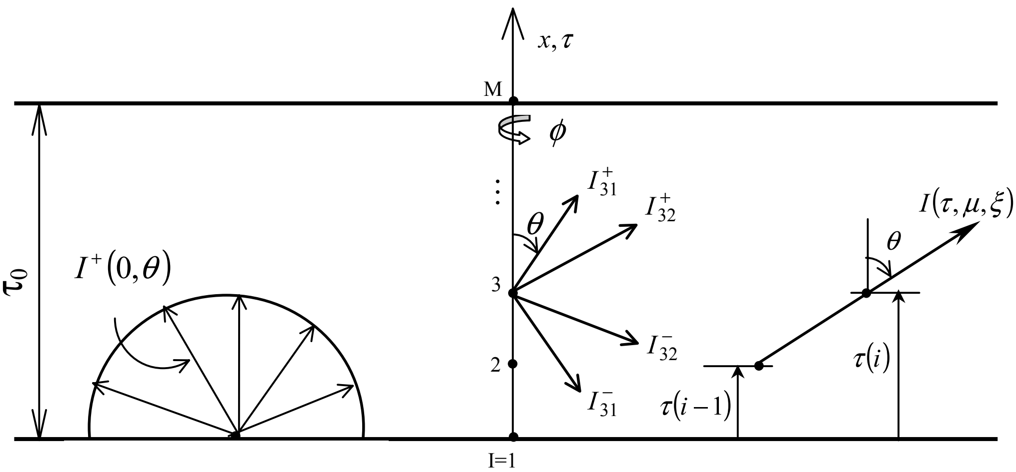


Figure 2. Schematic diagram of physical model, coordinates and discretization of $I(\tau, \mu,\xi)$

## 3. SOLUTION METHOD

The solution method requires solving two sets of equations, i.e., Eq. (1) and Eq. (7), simultaneously. To cluster more nodes near the boundaries and the interfaces a variable grid size is used, according to Eq. (10.d), Fig. (3). Equation (1) is discretized according a second order central difference scheme yielding

$$F(i,n)=\left.\frac{\partial\theta}{\partial\xi}\right|_i^{(n+1)}=\frac{\theta_{i-1}^{(n)}}{BCK(i)}-\left[\frac{1}{FWD(i)}+\frac{1}{BCK(i)}+\frac{(1-\omega_i)}{NCR_j}\theta^3\Big|_i^{(n)}\right]\theta_i^{(n)}+ \\ +\frac{\theta_{i+1}^{(n)}}{FWD(i)}+\frac{(1-\omega_i)}{NCR_j}G_i^* \tag{10.a}$$

where:

$$FWD(i)=(\tau_{i+1}-\tau_i)\left(\frac{\tau_{i+1}-\tau_{i-1}}{2}\right),\quad BCK(i)=(\tau_i-\tau_{i-1})\left(\frac{\tau_{i+1}-\tau_{i-1}}{2}\right), \tag{10.b, c}$$

$$\tau(i+1) = \tau(i) + 1.05\times[\tau(i) - \tau(i-1)]\ ,\quad i= 2, 3, \ldots M-1, \tag{10.d}$$

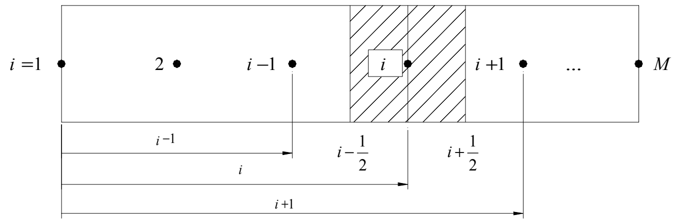


Figure 3. Grid distribution within the medium

with $\tau(1)=0$ and $\tau(2)=\Delta\tau$, a defined value in the numerical procedure. At the interfaces, a first order forward difference scheme is used and Eq. (3) becomes

$$\left(\frac{-NCR_j}{\tau(i)-\tau(i-1)}\right)\theta_{i-1}^{(n+1)}+\left(\frac{NCR_j}{\tau(i)-\tau(i-1)}+\frac{NCR_{j+1}}{\tau(i+1)-\tau(i)}\right)\theta_i^{(n+1)}+ \\ +\left(\frac{-NCRj}{\tau(i+1)-\tau(i)}\right)\theta_{i+1}^{(n+1)}=0 \qquad j=1,2,\ldots,NI, \tag{11}$$

where *NI* is the number of interfaces and the superscript *(n+1)* refers to the actual iterative time step in the numerical procedure. Solution of Eq. (10) and (11) gives the temperature profile witin the medium. Nonetheless, they can only be solved if $G^*(\tau,\xi)$ is known. To obtain $G^*(\tau,\xi)$, the RTE solution is required.

To solve the RTE we integrate Eq. (7), between $\tau(i-1)$ and $\tau(i)$, along a given $\mu$-direction within the medium, Fig. 2, obtaining

$$I(\tau_i,\mu,\xi)=I(\tau_{i-1},\mu,\xi)\,e^{-(\tau_i-\tau_{i-1})/\mu}+\bar{S}(\tau,\xi)\left[1-e^{-(\tau_i-\tau_{i-1})/\mu}\right], \tag{12}$$

where $\bar{S}(\tau,\xi)$ represents the average value of the source term between $\tau_{i-1}$ and $\tau_i$. For sake of simplicity, we define

$$I^-(\tau,\mu,\xi)=I(\tau,\mu,\xi),\quad -1\le\mu<0 \quad \text{and} \quad I^+(\tau,\mu,\xi)=I(\tau,\mu,\xi),\quad 0<\mu\le 1. \tag{13.a, b}$$

To solve Eq. (12), the discrete ordinates concept is used. However, since $I(\tau,\mu,\xi)$ is discontinuous at $\mu = 0$ ($\theta = 90^o$) (Pessoa-Filho and Thynell, 1994), such discontinuity has to be removed prior to the numerical integration, namely

$$G(\tau,\xi) = 2\pi\int_{-1}^{1} \mathrm{I}(\tau,\mu,\xi)d\mu = 2\pi\left(\int_{-1}^{0} I^{-}(\tau,\mu,\xi)d\mu + \int_{0}^{1} \mathrm{I}^{+}(\tau,\mu,\xi)d\mu\right), \tag{14.a}$$

$$q(\tau,\xi) = 2\pi\int_{-1}^{1} \mathrm{I}(\tau,\mu,\xi)d\mu = 2\pi\left(\int_{-1}^{0} \mathrm{I}^{-}(\tau,\mu,\xi)d\mu + \int_{0}^{1} \mathrm{I}^{+}(\tau,\mu,\xi)d\mu\right). \tag{14.b}$$

Since the discontinuity has been considered, a quadrature formulae is now applied to evaluate the integrals appearing on the right hand side of Eqs. (14). The application of a Gaussian quadrature formulae to Eqs. (14), yields

$$G(\tau,\xi) \approx G_i = \pi\sum_{j=1}^{N} \varsigma_j \left(I_{ij}^{-} + I_{ij}^{+}\right), \tag{15.a}$$

$$q(\tau,\xi) \approx q_i = q_i^{+} + q_i^{-} \quad , \qquad q_i^{\pm} = \pi\sum_{j=1}^{N} \varsigma_j \mu_j^{\pm} I_{ij}^{\pm}, \tag{15.b}$$

where $\varsigma_j$ are the weights given by the quadrature formulae and $N$ is the number of quadrature points. In writing Eqs. (15), the following notation was used

$$I_{ij}^{\pm} = I^{\pm}(\tau_i, \mu_j^{\pm}), \tag{15.c}$$

where the subscripts "$i$" and "$j$," refer to the $\tau$-*position* and $\mu$-*direction*, respectively. It should be mentioned that $N$ gives the number of angular directions in which $I(\tau,\mu,\xi)$ is discretized. Therefore, $N=2$ indicates that $I(\tau,\mu,\xi)$ is evaluated along two directions in the "*positive-direction*", $0<\mu\leq 1$ , and two in the "*negative-direction*", $-1\leq\mu<0$, as schematically shown in Fig. 2. Additionally, the time dependence of $I(\tau,\mu,\xi)$ was not shown in the discretized equations to simplify the notation. Since the zeroes of the Gaussian quadrature formulae are defined for a continuous integration interval $[-1,1]$, they have to be shifted for the intervals $[-1,0]$ and $[0,1]$, according to the equation

$$\mu_j^{\pm} = 0.5\left[\psi_j \pm 1\right], \qquad \text{j=1,2, . . ., N} \ , \tag{16}$$

where $\psi_j$ are the zeroes of the Gaussian quadrature formulae. Eq. (12) can now be written as

$$I_{ij}^{\pm} = I_{i-1j}^{\pm}\, e^{-(\tau_i - \tau_{i-1})/\mu_j} + \bar{S}(\tau_{i-1/2})\left[1 - e^{-(\tau_i - \tau_{i-1})/\mu_j}\right], \qquad i=2, . . ., M-1, \quad j=1, . . . , N. \tag{17}$$

It should be pointed out that even if the temperature profile in the medium is known, the RTE solution is an iterative one. To illustrate that, let us consider the calculation of $I^{+}(\tau_2,\mu_1)$ for a given time $\xi$ which, according to Eq. (17), is given by

$$I_{21}^{+} = I_{11}^{+}\, e^{-(\tau_2 - \tau_1)/\mu_1} + \bar{S}_{12}\left[1 - e^{-(\tau_2 - \tau_1)/\mu_1}\right], \tag{18.a}$$

with $\tau_2 = \Delta\tau$, $\tau_1 = 0$ and $\bar{S}_{12} = \frac{1}{2}(S_1 + S_2)$. $S_1$ e $S_2$ are given by

$$S_1 = S(0) = [1-\omega] I_b[\theta(0)] + \frac{\omega}{4} \sum_{j=1}^{N} \varsigma_j \left( I_{1j}^- + I_{1j}^+ \right), \tag{18.b}$$

$$S_2 = S(\Delta\tau) = [1-\omega] I_b[\theta(\Delta\tau)] + \frac{\omega}{4} \sum_{j=1}^{N} \varsigma_j \left( I_{2j}^- + I_{2j}^+ \right). \tag{18.c}$$

Since $I_{1j}^-, I_{2j}^-$ and $I_{2j}^+$ are unknown, $S_1$ and $S_2$ cannot be evaluated at this point. $I_{1j}^+$ is given by the boundary condition, i.e., Eq. (9.a). If the wall is black, $\varepsilon_1=1$ and $I_{1j}^+ = \varepsilon_1 I_b(\theta_1)$. Otherwise, $I_{1j}^+$ cannot be evaluated either. The solution procedure to solve the RTE can be summarized as follows:

***i)*** A temperature distribution within the medium based on the initial condition is assumed;
***ii)*** $I_{ij}^{\pm} = 0,\ i=1, \ldots, M;\quad j=1, \ldots, N;$
***iii)*** $I_{1j}^+ = \varepsilon_1 I_b(\theta_1) + 2(1-\varepsilon_1)\left|q_1^-\right|;\quad I_{Mj}^- = \varepsilon_2 I_b(\theta_M) + 2(1-\varepsilon_2) q_M^+;\ j=1,2, \ldots, N;$
***iv)*** Starting from $i=2$ and $i=M-1$, simultaneously, $I_{ij}^+$ and $I_{ij}^-$ are evaluated according to Eq. (17);
***v)*** By using Eq. (15.a) and (15.b) $G_i$ and $q_i^{\pm}$, $i=1, \ldots, M,$ are evaluated;
***vi)*** Steps ***iii)*** through ***v)*** are repeated until the following convergence criterion is verified

$$\sum_{i=1}^{M} \frac{\left| G_i^{p+1} - G_i^p \right|}{G_i^{p+1}} \leq 10^{-5}\ , \tag{19}$$

where the superscript *(p+1)* denotes results from the last iterative step. The obtained RTE solution was based on a guessed temperature distribution. By using $G^*(\tau,\xi)$ obtained from RTE solution, Eq. (10.a) can be integrated, in time, to obtain a new temperature profile. Since the radiative and conductive heat transfer phenomena are coupled, an iterative type of solution, involving Eqs. (10), (11) and (17), is required, which can be summarized as follows

***i)*** Equations (10) and (11) are solved by taking $G^*(\tau,\xi)$ obtained from the temperature profile given by the initial condition;
***ii)*** $\theta(\tau,\xi)$ obtained from step *i* is used to solve the RTE and a new $G^*(\tau,\xi)$ is calculated;
***iii)*** $G^*(\tau,\xi)$ calculated from step ***ii)*** is used to solve Eqs. (10) and (11);
***iv)*** Steps ***ii)*** and ***iii)*** are repeated until a desired final observation time is reached.

## 4. RESULTS AND DISCUSSION

In a prior investigation, Gino et al. (2001) analysed situations involving steady-state one-dimensional, combined conduction-radiation heat transfer in muti-layer planar participating media. The goal of such an investigation was to validate the solution method proposed to resolve the RTE and the conduction problem. The obtained results were compared against other available in the literature and a very good agreement was observed. Now, focus is directed towards the validation of the proposed method for unstaedy-state situations. Three different physical configurations are investigated.

The first case involves one-dimensional, two-layer medium, bounded by diffuse, opaque, black surfaces kept at constant temperatures. The medium is supposed to be gray and semitransparent. As initial condition, the medium and both surfaces are at the same temperature $\theta(\tau,0) = 1.0$. Suddenly, the wall located at $\tau = 0$ has its temperature lowered to $\theta(0,\xi) = 0.5$, which provokes a transient cooldown in the temperature of the medium until steady-state is reached. The optical thickness of the medium is $\tau_0 = 3.0\ (\tau_1 = \tau_2 = 1.5)$. The conduction-radiation parameters are made $NCR_1 = 3.00$, for $0 < \tau < 1.5$, and $NCR_2 = 0.03$, for $1.5 < \tau < 3.0$. Two different values of scattering albedo are considered: $\omega_1 = 0.0$ and $\omega_2 = 1.0$. As a consequence of choosing $\Delta\tau = \tau(2) - \tau(1) = 0.2$ and a stretchting factor of 1.05, Eq. (10.d), the total optical thickness was divided in M=17 nodes. In the Gaussian integration scheme, N was made equal to 2, wich indicates that $I(\tau, \mu, \xi)$ was evaluated along four directions. To allow convergence the time-step, $\Delta\xi$, was fixed in $1\times10^{-3}$. The average number of iterations to solve the radiation problem at each time-step was $p = 12$. The results were obtained using a PENTIUM 100 MHz microcomputer. The CPU-time required to perform the simulation was about 10[s]. Figure (4.a) shows the temperature distribution within the medium as a function of the dimensionless time, $\xi$, until the steady-state is reached. For sake of validation, the results obtained by Genaro et al. (2001), for steady-state condition, are ploted in the figure. It shows that the obtained results are in very good agreement with those published by Genaro et al. (2001).

The second case involves an one-dimensional, one-layer medium, bounded by diffuse, opaque, black surfaces kept at constant temperatures. As in the first case, the medium is supposed to be gray and semitransparent, subjected to the same initial condition. The optical thickness of the medium is unity. The conduction-radiation parameter is fixed to 0.005 and the scattering albedo is equal to 0.5. In Fig. (4.b), the temperature profiles at dimensionless times of 0.001, 0.01, 0.1 and steady-satate are compared with results presented by Weston and Hauth (1973). Once again, a very good agreement is observed.

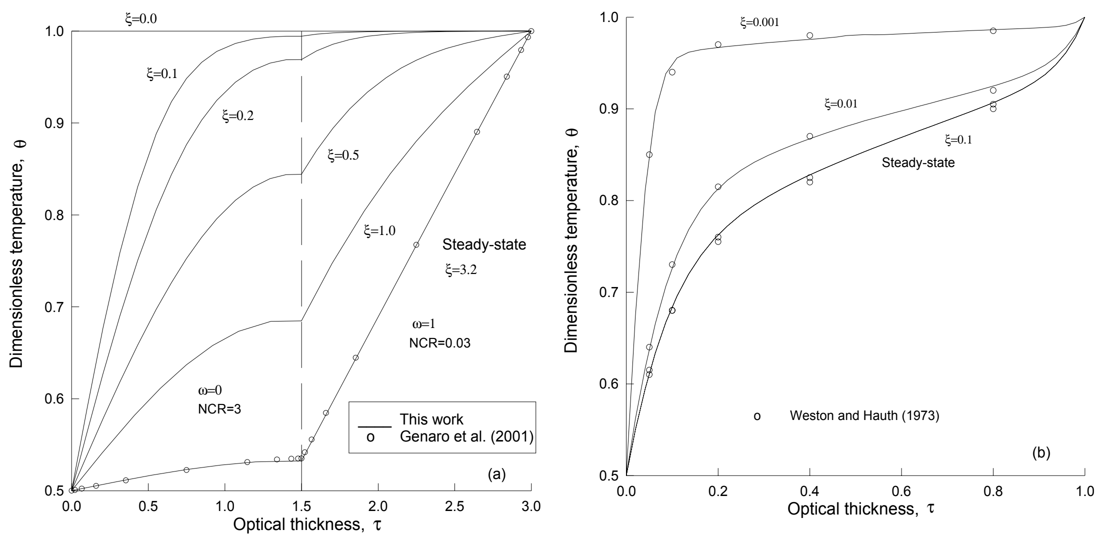


Figure 4 (a,b): Dimensionless temperature and its evolution with time

We next consider an one-layer, nonscattering medium with unity optical thickness $(\tau_0 = 1.0)$ bounded by black isothermal diffuse walls. To ilustrate the effects of the conduction-radiation parameter on the medium's temperature distribution, three different values of NCR are considered, namely, 0.005, 0.05 and 0.5. For this simulation, $\Delta\tau = 0.01$ and $\Delta\xi = 3\times10^{-5}$. As before, the stretching factor was equal to 1.05 and, as consequence, 69 nodes were used to discretize the energy equation. Such a discretization is due to the high temperature gradient next to the cold wall. The obtained results are ploted in the Fig. (5). Figure (5.a) reveals that for early times $(\xi = 0.001)$ and near the cold wall $(\tau < 0.05)$, the effects of the conduction-radiation parameter are negligible. This behavior is caused by the large temperature gradient in that region, making conduction the principal form of energy transfer. For inner regions of the medium, the effects of NCR are more pronounced, showing that the smaller the NCR, the lower the medium's temperature. At $\xi = 0.001$, the main effect of thermal radiation is to accelerate the cooling of medium in regions of higher temperatures.

Figure (5.b) shows that, for $\xi = 0.01$, the effects of NCR on the temperature distribution near the cold wall are small, however, for inner regions, $\tau > 0.1$, they become evident. As expected, radiation heat transfer accelerates the medium's cooling. As time increases, Fig. (5.c), an interesting phenomenon occurs. Near the cold wall, the energy transfer process is composed by conductive cooling and radiative heating. In this region, the medium is absorbing more energy coming from inner regions of the medium, than it is emitting. In regions next to the hot wall, there is more emittion than absortion, causing an decrease in temperature. Under steady-state regime, Fig. (5.d), the smaller NCR, the larger the temperature gradient near the walls. This fact occurs because near the walls, the temperature must rapidly approach the surface temperatures to guarantee the continuity of the conductive transfer process. As NCR increases, the conduction becomes more important and the temperature distribution becomes similar to the pure conduction case (shown in the figure). The figure also shows the dimensionless time, $\xi$, spent by the system to reach steady-state. The smaller NCR, the lower $\xi$, confirming the radiative heat transfer property of accelerates the medium's cooling.

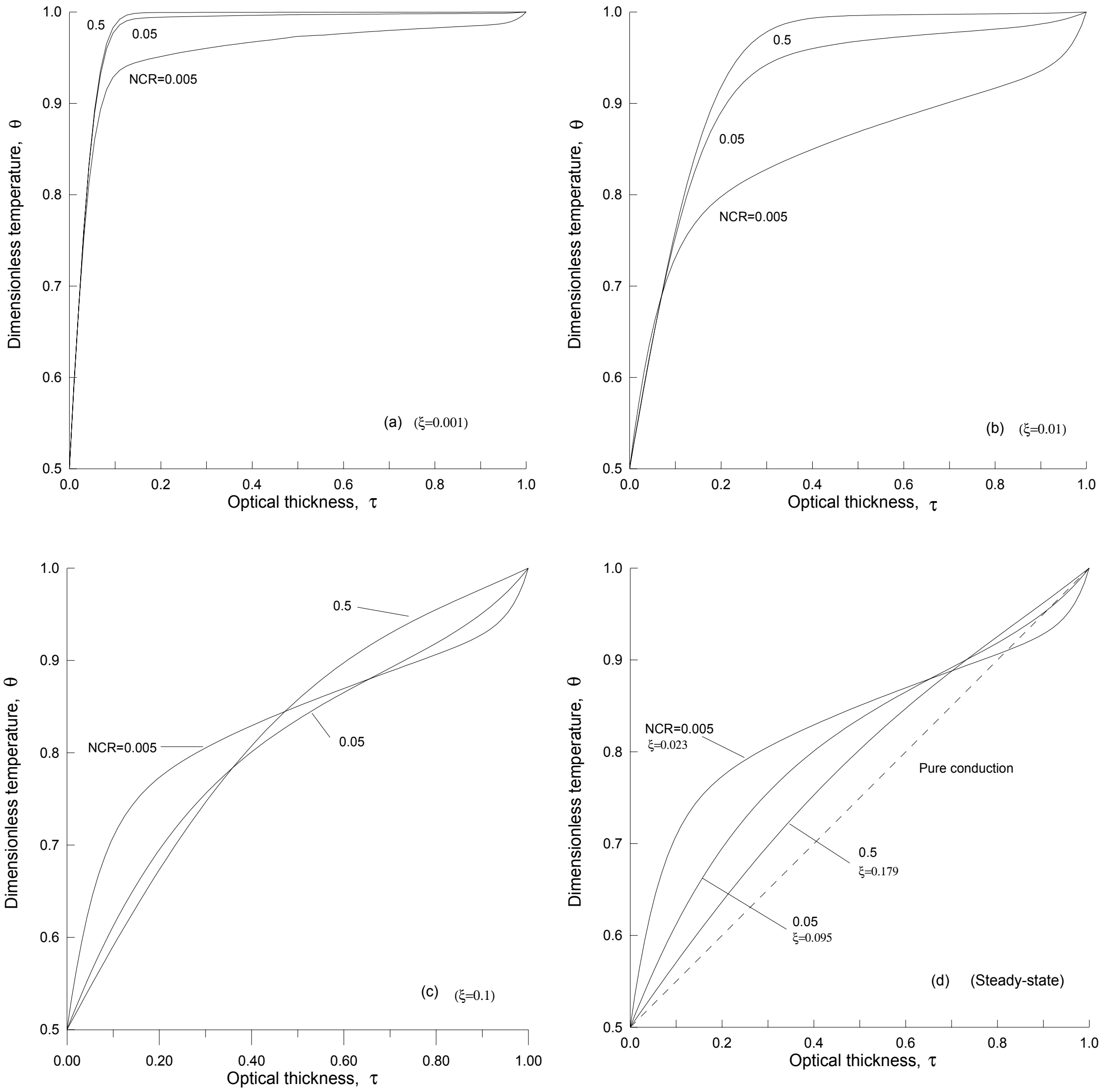


Figure 5(a,b,c,d). Effects of NCR and $\xi$ on the dimensionless temperature

It is interesting to note that, as pointed out by Modest (1993), not always conduction-radiation parameter gives a good estimate of the relative importance of conductive and radiative heat transfer, as shown in this case. For optically thin media $(\tau_0 << 1)$, not shown, we could have an already nearlinear temperature distribution even for NCR=0.01, for example. This behavior may be explained by noting that - for small $\tau_0$ - little emission and absorption takes place inside the medium and radiative heat flux can travel directly from surface to surface.

Another aspect analyzed is the effect of the albedo and conduction-radiation parameter on dimensionless time required by the system to reach steady-state, $\xi^*$. The conduction-radiation parameter assumes the values 0.005, 0.05, 0.5 and 1, while albedo is fixed to 0, 0.5 and 1. Table (1) shows that $\xi^*$ increases with $\omega$ and NCR. For small values of NCR, the dominant form of energy transfer is radiation, that propagates at light speed. As NCR increases, conduction becomes more important and the energy transfer occurs eminently by diffusive effects.

Table 1. Dimensionless time to the steady-state be reached. $(\tau_0 = 1)$

| NCR | $\omega = 0$ | $\omega = 0.5$ | $\omega = 1$ |
|---|---|---|---|
| 0.005 | 0.023 | 0.029 | 0.203 |
| 0.05 | 0.095 | 0.111 | 0.203 |
| 0.5 | 0.179 | 0.186 | 0.203 |
| 1.0 | 0.190 | 0.194 | 0.203 |

By fixing conduction-radiation parameter, the results reveal an increase of $\xi^*$ as $\omega$ rises. The albedo measures the grade of scattering of a medium and varies from zero to one, corresponding to nonscattering and purely scattering media, respectively. When scattering is present, the radiative energy emitted by the hot wall and by the medium has its propagation direction changed, contributing to increase the time required to steady-state is reached. Once scattering is an eminently radiative phenomenon, the albedo's influence on $\xi^*$ is more significant for small values of NCR. While for NCR=0.005 the difference in $\xi^*$ between $\omega = 0$ and $\omega = 0.5$ is about 20%, for NCR=1.0, this difference drops to 2%. It's interesting to note that, for purely scattering media ($\omega = 1.0$), $\xi^*$ reaches its maximum value and, as conduction and radiation are uncoupled by Eq. (1), it does not depend on NCR.

## 5. CONCLUSION

A numerical procedure for solving unsteady, coupled conduction-radiation heat transfer between two flat parallel walls, separated by semitransparent materials was presented in this work. To solve the second order ordinary differential equation resulting from the application of the overall energy conservation equation, a second order finite-difference method was used. The solution of the RTE was obtained by applying a novel solution method which is based on the basic idea of the discrete ordinate method differing from it in the sense that the discontinuous nature of the radiative intensity is properly treated. The obtained results for steady and unsteady-state were compared with other presented in the literature. A very good agreement was observed in both cases.

Also, transient effects of conduction and radiation during cooling have been analyzed. It was observed that, for early times and regions near the cold wall, conduction-radiation parameter does not interfere significantly on the temperature distribution and conduction is the dominant form of energy transfer. For inner regions of the medium and small values of NCR, radiation is dominant, contributing to accelerate the cooling process. As time increases, radiative heating, near the cold wall, and radiative cooling, in regions removed from the cold wall, take place. When steady-state is reached, the smaller NCR, the greater temperature gradient near the walls. As NCR increases, conduction becomes more important and the temperature distribution approaches of that for pure conduction. It has been seen that the dimensionless time necessary to reach steady-state increases with conduction-radiation parameter and albedo, except for purely scattering media, where the time does not depend on conduction-radiation parameter. The next stage of the present study is to apply the developed procedure to a more realistic and complex situation involving unsteady coupled radiative-conductive heat transfer between two flat parallel walls separated by several layers of semitransparent materials with opaque interfaces.

## 6. REFERENCES


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