FAILURE ASSESSMENT OF A PIPE WITH A CIRCUMFERENTIAL CRACK IN BENDING

Julio Ricardo Barreto Cruz
CNEN/CDTN, Caixa Postal 941, 30123-970, Belo Horizonte, MG, Brasil. jrbc@urano.cdtn.br
John D. Landes
University of Tennessee, MAES Department, Knoxville, TN, USA. John-Landes@utk.edu

Abstract

A ductile fracture methodology (DFM) has been developed which can take the load *versus* displacement record from a laboratory test specimen containing a crack-like defect and predict the same for a structural component containing a defect. This paper presents some recent developments incorporated to the DFM framework in order to simplify the predicting procedure. The methodology is then applied to predict the structural behavior of a circumferentially through cracked pipe in four point bend loading. The pipe behavior is obtained in terms of a load *versus* displacement curve and shows a good agreement with laboratory tests results from the literature.

Keywords: Fracture mechanics, Structural integrity, Failure assessment, Cracked pipe

1. INTRODUCTION

Among the different assessment tools used for the assessment of cracked components fabricated with ductile materials and subjected to loadings that can produce a significant plasticity is the so-called Ductile Fracture Method (DFM) proposed by Landes *et al.* (1993). With the DFM, the behavior of a cracked component can be predicted directly from the load *versus* displacement record of a laboratory fracture toughness test. The method uses the concept of separable deformation and fracture behaviors to divide the test result into a deformation curve and a J-R fracture toughness curve. These two behaviors refer to the test specimen geometry, but can be related to the structural component of interest by means of a transfer process that provides the deformation and J-R curves for the geometry of the component. With these curves, the load *versus* displacement behavior of the structural component can then be predicted.

This paper describes some recent developments incorporated to the DFM methodology which allows one to use the same deformation curve obtained from the test specimen geometry to predict the behavior of a structural component with no need of a geometry transfer procedure. In fact, the geometry transformation is implicitly considered in the new formulation. The example of a pipe with a circumferential crack in bending is used to illustrate the new formulation appended to the DFM framework. The structural behavior of the cracked pipe is predicted in terms of a load *versus* displacement curve. A comparison with laboratory test results obtained from the literature demonstrates the effectiveness of the predicting procedure. The determination of the parameters that characterize the cracked pipe geometry in the formulation proposed is presented in detail. The same steps can then be applied to find the corresponding parameters for other geometries.

2. THE COMMON FORMAT EQUATION APPROACH

The first proposal of the DFM used a graphical procedure to make the transfer in deformation curves from the specimen to the structural component model. Since that time, additional work has been done on the determination of the deformation behavior for the structural component.

Donoso & Landes (1994) proposed a common format equation (CFE) to represent the behavior of different cracked configurations. Their study was based on the EPRI-GE Handbook solutions (Kumar *et al.*, 1981) for five different test specimen configurations, namely, CT, CCT, SENB, SENT and DENT. According to the CFE approach, the behavior of each fracture test configuration can be expressed as the product of three terms which describe the load, P, as a function of both plastic displacement, v_{pl}, and uncracked ligament, b

$$P = (\Omega / \kappa) \cdot G(b / W) \cdot H(v_{pl} / W) \quad (1)$$

The first term reflects the type of thickness constraint, Ω/κ, to which the test specimen is subjected. The second term is a crack-geometry dependent function, G, which considers the effects due to differences in planar geometry and mode of loading. And the third term is a material-dependent hardening function, H, which represents the deformation behavior of the specimen. In the CFE, the function G is generalized for all two-dimensional configurations as a power law of the normalized ligament, b/W, and has the following form

$$G = B \cdot W \cdot \zeta \cdot (b/W)^m \quad (2)$$

where B is the specimen thickness, W is the specimen width, ζ is a constant term, and m equals the plastic η-factor (η_{pl}). Thus, each different test specimen geometry, for any given material, will be characterized by the parameters ζ and m of Eq. 2.

According to the CFE concept, the relation between normalized load and normalized plastic displacement (the H function)

$$P_N = \frac{P}{(\Omega/\kappa)G(b/W)} = H(v_N) \quad (3)$$

is unique, regardless of the geometry of the cracked specimen (in Eq. 3, $v_N = v_{pl}/W$). In other words, the G function characterizes each particular geometry and H depends only on the material properties. Since H is unique, the behavior of a certain cracked configuration (e.g., a structure) could be predicted from the behavior of another cracked configuration made from the same material (e.g., a laboratory test specimen) if their G functions and thickness constraints (Ω/κ) are known.

The uniqueness of the H function was investigated by Cruz & Landes (1997). The load *versus* displacement test records of different A533 steel specimen geometries were selected. All the specimens had the same thickness, 2.5 mm, and exhibited plane stress constraint. Thus, all of them had the same constraint factor, Ω/κ. Equation 3 was then applied to obtain the P_N *versus* v_N curve for each geometry. The P_N *versus* v_N curves should be very close, since H was supposed to be unique. But, contrary to expectation, these curves were different for each specimen geometry.

Cruz & Landes (1997) then reviewed the equations which originated the CFE and proposed a common format equation with a displacement-based normalization parameter. In this case, the plastic displacement is normalized with $(v_{el})_o$, instead of W. $(v_{el})_o$ is the elastic displacement at $P = P_o$, the limit load for the configuration being considered. Taking the same set of experimental data, the procedure used to build the P_N *versus* v_N curves was repeated, now normalizing v_{pl} with $(v_{el})_o$. The resulting P_N *versus* v_N curves were much closer to a unique representation, which means that the *H* function tends to be the same for all configurations and, therefore, depends only on the material hardening properties. With the new normalization parameter, the CFE becomes

$$P = (\Omega / \kappa) \cdot G(b/W) \cdot H(v_{pl} / v_{el_o}) \tag{4}$$

3. APPLYING THE DFM BASED ON THE CFE PARAMETERS

To apply the load *versus* displacement prediction scheme, the *G* functions for the geometries of the test specimen and the structural component must be known. The *H* function, which is assumed to be unique, is obtained from the test specimen load *versus* displacement record, as will be shown ahead.

3.1 Determination of the G function

The *G* function is generally known for common geometries, but it can be obtained for a new cracked configuration from a set of load *versus* displacement curves, each one corresponding to a different stationary crack length. (Fig. 1a). From Eq. 1 (or Eq. 4), a separation parameter, S_{ij}, defined as the ratio $P(a_i)/P(a_j)$, will not be a function of the plastic displacement. This is represented in Fig. 1b, where one of the curves of Fig. 1a was taken as reference. S_{ij} can be written as

$$S_{ij} = \left.\frac{P(a_i)}{P(a_j)}\right|_{v_{pl}} = \left.\frac{(\Omega/\kappa) \cdot G(a_i/W) \cdot H(v_{pl}/W)}{(\Omega/\kappa) \cdot G(a_j/W) \cdot H(v_{pl}/W)}\right|_{v_{pl}} = \left.\frac{G(a_i/W)}{G(a_j/W)}\right|_{v_{pl}} = \text{constant} \tag{5}$$

S_{ij} is constant for practically the whole range of plastic displacement. The fitting curve for the points S_{ij} *versus* a/W (or b/W, where b is the ligament), Fig. 1c, provides the functional form of *G*, since

$$G(a_i / W) = G(a_j / W) \cdot S_{ij}(a_i / W) \tag{6}$$

where $G(a_j/W)$ is a constant corresponding to the value of a_j/W taken as reference. Thus the exponent *m* in Eq. 2 can be obtained from the slope of the linear regression line through the points (b/W, S_{ij}) in a logarithmic scale.

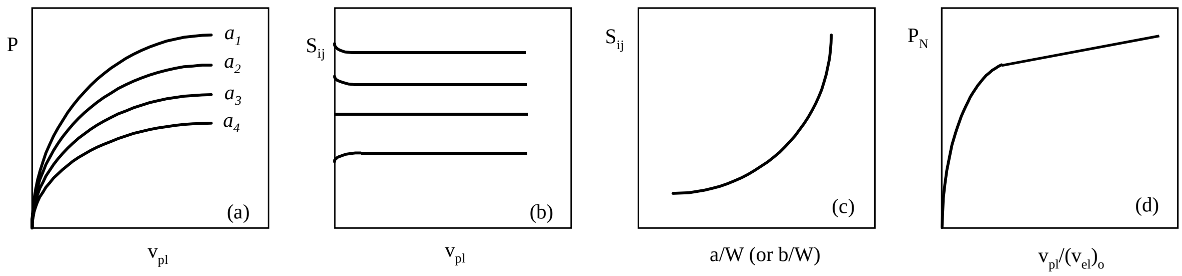

Figure 1. Scheme to obtain the *G* function

A correct representation for the G function is supposed to collapse the curves of Fig. 1a into a single one when the normalized load (Eq. 3) is calculated, Fig. 1d. This curve is the H function. Since in the CFE the H function is assumed to be unique, the parameter ζ of the Eq. 2 can be determined by imposing the following condition: the normalized load *versus* normalized plastic displacement curve for the new geometry, for a given constraint, should match that of any of the fracture specimens for the same material and the same constraint. This will be illustrated later on, when the pipe example is presented.

3.2 Determination of the H function

Knowing the G function for test specimen geometry, the H function is obtained from the specimen load *versus* displacement record by a procedure called normalization (Landes *et al.*, 1991), which also allows one to obtain the J-R curve of the material. A functional form has to be assumed for H. A format that has proven to successfully and accurately describe the normalized load *versus* normalized displacement behavior for most metals is the LMN function (Orange, 1990), which is represented by the following expression

$$H(v_N) = \frac{L + M v_N}{N + v_N} v_N \tag{7}$$

where L, M and N are constants.

3.3 Load *versus* displacement predicting procedure

The procedure to predict the P *versus* v behavior for a structure from the P *versus* v record for a fracture toughness specimen is described in the following steps:

(a) The limit load and compliance for $a=a_o$ are calculated for both the specimen and the structure;

(b) $(v_{el})_o$ is calculated for the specimen and the structure using the compliance and limit load solutions from the previous step;

(c) The method of normalization (Landes *et al.*, 1991) is applied to the specimen P-v record to obtain the J-R curve and the deformation function $H(v_N)$;

(d) The following iterative process is then applied:

d.1- Start with $a=a_o$ and with a small value for $v_N = v_{pl}/(v_{el})_o$;

d.2- Calculate P and J_{app};

d.3- Iterate, adjusting a, until J_{app} matches J_{mat} from the J-R curve equation;

d.4- Calculate the total displacement, v;

d.5- Increment v_{pl} and repeat calculations;

d.6- Continue until P-v range is completed.

The computation of J_{pl} is done in the following way:

$$J_{pl} = \frac{\eta_{pl}}{Bb}\int_0^{v_{pl}} P dv_{pl} = \frac{\eta_{pl} v_{el_o}}{Bb}\int_0^{v_{pl}/v_{el_o}} P d\left(\frac{v_{pl}}{v_{el_o}}\right) = \frac{\eta_{pl} v_{el_o}}{Bb}\left[\frac{(\Omega/\kappa)_{structure}}{(\Omega/\kappa)_{specimen}}\right] G \int_0^{v_N} H(v_N) dv_N \tag{8}$$

The integral of $H(v_N)dv_N$ for H represented as an LMN function (Eq. 7) is:

$$\int_0^{v_N} H(v_N) dv_N = \frac{v_N}{a_1} - \frac{b_1}{a_1^2}\ln(a_1 v_N + b_1) + \frac{(a_2 v_N + b_2)^2}{2a_2^3} - \frac{2b_2(a_2 v_N + b_2)}{a_2^3} + \frac{b_2^2}{a_2^3}\ln(a_2 v_N + b_2) \tag{9}$$

where a_1=1/L, b_1=N/L, a_2=1/M, and b_2=N/M.

4. CIRCUMFERENTIALLY THROUGH CRACKED PIPE IN BENDING

The example of a cracked pipe in bending, taken from the literature (Pan *et al.*, 1984), was used to test the predicting procedure. The pipe contains a circumferential through crack and is loaded in four-point bending (Fig. 2). It is fabricated from a 304 stainless steel. Two cases with different initial crack lengths were analyzed. The geometry properties relative to these two cases are shown in Table 1.

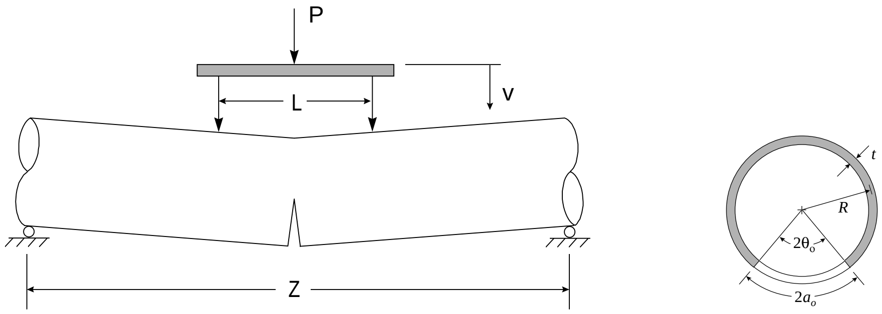

Figure 2. Circunferentially through cracked pipe in four-point bending

Table 1. Geometry properties of the 304 SS pipes

Case	Internal diameter, mm	t, mm	$2a_o$, mm	Z, mm	L, mm
PIPE-1	101.6	8.9	133.1	1520	410
PIPE-2	101.6	8.9	76.1	1520	410

The input for prediction was developed from a compact specimen fracture toughness test. Since the load *versus* displacement record for this was not given by Pan *et al.* (1984), a 304 stainless steel fracture toughness test record was used from a specimen of very similar properties for which a load *versus* displacement curve was available (Landes & McCabe, 1986). Based on this curve and applying the normalization method (Landes et al., 1991), the *H* function and J-R curve for the CT specimen were obtained.

To apply the predicting procedure, the *G* function of the pipe must be known. To keep the *G* representation given by Eq. 2, the pipe was considered to be equivalent to a single edge four-point bending two-dimensional geometry with an effective crack length, a_{eff}, width, *W*, equal to the pipe diameter, and thickness, *B*, equal to 2 times the pipe thickness, *t*, as illustrated in Figure 3.

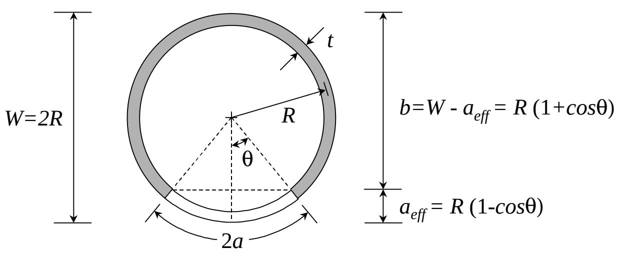

Figure 3. Scheme for pipe equivalence to a two-dimensional geometry

In the CFE, there exist the following relationship between the limit load and the G function for all configurations studied (Donoso & Landes, 1994)

$$P_o = (\Omega / \kappa) G \sigma_o \tag{10}$$

Since G and P_o are related by a constant factor, the exponent m in Eq. 2 can be obtained from the P_o *versus* b/W behavior following the procedure described in Section 3. The limit load expression for the pipe, taken from (Zahoor & Kanninen, 1981), is

$$P_o = \frac{16\sigma_o R^2 t}{Z - L} F(\theta) \tag{11}$$

where $F(\theta) = cos(\theta/2) - 0.5sin\theta$. Therefore, m can be obtained from the slope of the linear regression line through the points $ln(b/W)$, $ln(S_{ij})$ shown in Table 2 and depicted in Fig. 4. The m value found for the pipe (1.99) is in accordance to what was expected since it should be close to 2, the m value for the SENB geometry (Donoso & Landes, 1994), which has a very similar loading mode.

Table 2. Limit load of the pipe for different crack lengths

θ (graus)	F(θ)	P_o (kN)	S_{ij}	ln(S_{ij})	b/W	ln(b/W)
40	0.6183	107.4728	1.4279	0.3562	0.8830	-0.1244
45	0.5703	99.1342	1.3171	0.2754	0.8536	-0.1583
50	0.5233	90.9576	1.2085	0.1894	0.8214	-0.1968
55	0.4774	82.9878	1.1026	0.0977	0.7868	-0.2398
60	0.4330	75.2664	1.0000	0.0000	0.7500	-0.2877
65	0.3902	67.8312	0.9012	-0.1040	0.7113	-0.3406
70	0.3493	60.7164	0.8067	-0.2148	0.6710	-0.3990
75	0.3104	53.9521	0.7168	-0.3329	0.6294	-0.4630
80	0.2736	47.5643	0.6319	-0.4590	0.5868	-0.5330

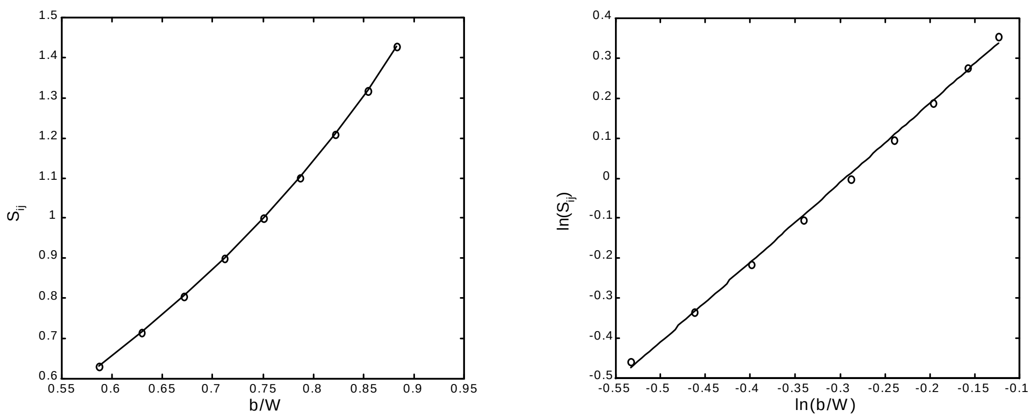

Figure 4. Determination of the exponent m of the G function of the pipe

To find the parameter ζ of the G function, a graphical procedure was used in which the value of ζ was adjusted until the P_N *versus* v_N curve of the pipe matched the P_N *versus* v_N for the CT specimen, that is, demanding the uniqueness of the H function. This is illustrated in Fig. 5, where it can be seen that ζ around 0.67 seems to be appropriate.

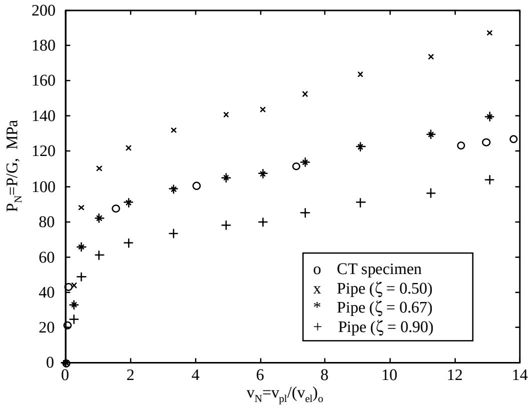

Figure 5. Graphical procedure to obtain ζ

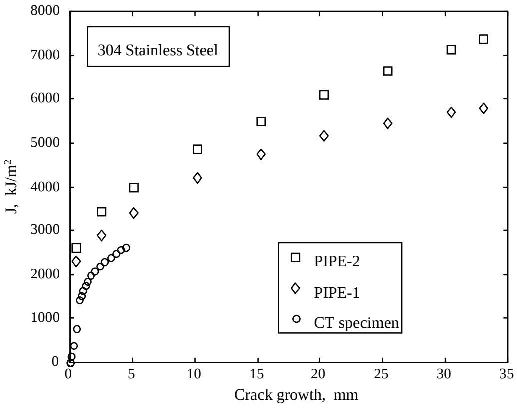

Figure 6. Pipes and CT specimen J-R curves

Once defined all the necessary parameters, the prediction procedure was applied to obtain the *P versus v* curves for the two cases, PIPE-1 and PIPE-2. For each case, two predictions were done, one considering the J-R curve obtained from the CT test record and the other based on the J-R curve of the specific pipe configuration being analyzed. The J-R curves for the pipes were obtained from (Pan *et al.*, 1984). Figure 6 shows a comparison between the J-R curves for the pipes and that for the CT specimen. The predicted load *versus* displacement curves for the two cases are presented in Figs. 7 and 8, respectively. The results show that the maximum load can be reasonably well predicted even considering the J-R curve from the CT specimen. However, using the J-R curve of the own pipes, the predictions are more accurate not only for the maximum load, but also for the behavior beyond the maximum load.

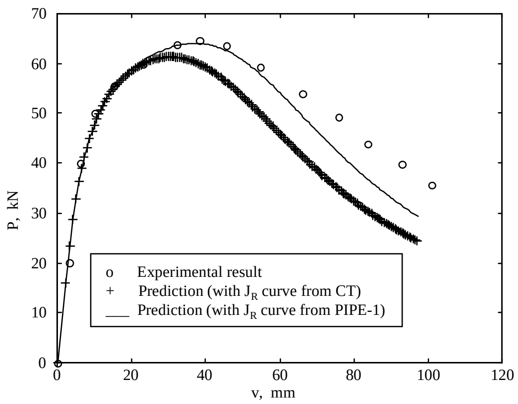

Figure 7. *P versus v* prediction for PIPE-1

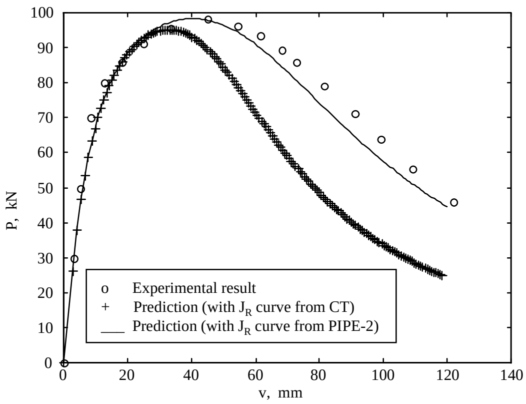

Figure 8. *P versus v* prediction for PIPE-2

5. FINAL REMARKS

The paper presented an analytical procedure for predicting the load *versus* displacement behavior of a structural component from the load *versus* displacement record of a laboratory fracture toughness test. The procedure is based on the fundamentals of a ductile fracture methodology (DFM) in which the fracture and deformation behaviors of the test specimen are first separated, transferred to the structural component geometry and then combined to find the complete load *versus* displacement behavior of the component. But, using a common format principle with a displacement based normalizing parameter, it was shown that the deformation function can be considered the same for both the specimen and the structure. This eliminates the transfer process for this function and, therefore, simplifies the predicting procedure.

On the other hand, it is known that fracture behavior, given in terms of a J-R curve, has a strong geometry dependence and there is still not available a reliable way to make the correlation between the J-R curve of a test specimen and the J-R curve of a structural component. For the cracked pipe example presented here, the predictions were done using J-R curves from the test specimen and from the specific pipe configurations. The results show clearly the influence of J-R curve geometry effects on the complete load *versus* displacement behavior of the pipe. But, even using the J-R curve from the CT specimen, the maximum load for the pipe could be reasonably well predicted. If the J-R curve for the component is known, the procedure offers the additional possibility of doing an accurate prediction of the whole load *versus* displacement curve.

6. REFERENCES

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