ATTITUDE SIMULATION AND MANEUVER FOR A SUN-SYNCHRONOUS SPINNING SATELLITE

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Summary

The present work simulates the behavior of a Sun-synchronous spinning satellite, using a digital computer. The results yield an attitude control law, for maintenance of the angular velocity and the sun aspect, by using the Earth's magnetic field, a magnetometer as spin-rate sensor, three magnetic coils as actuators and two sun sensors. The magnetic torque availability is strongly dependent on the relative motion Sun-Earth-satellite and on the Sun-Earth pointing requirements. The first step is the evaluation of the initial orbit and attitude conditions; the second step are define and determine a coefficient, which will be used to evaluate the eddy current effects on the satellite's attitude. Finally, we simulate the nominal attitude behavior considering all determined parameters and the magnetic coils maneuvers.

Guidance, navigation and control, flight dynamic and orbital dynamic.

1 – ANGULAR VELOCITY VECTOR $\vec{\omega}$ DEFINITION

To determine the orientation of the spin axis, for a Sun-synchronous spinning satellite, on the Inertial System, the precession has to be computed. Fonseca (1995) assumed the geomagnetic field to be parallel to spin axis, under this assumption, the satellite system must be rotated three times, as shown in Figure 1.

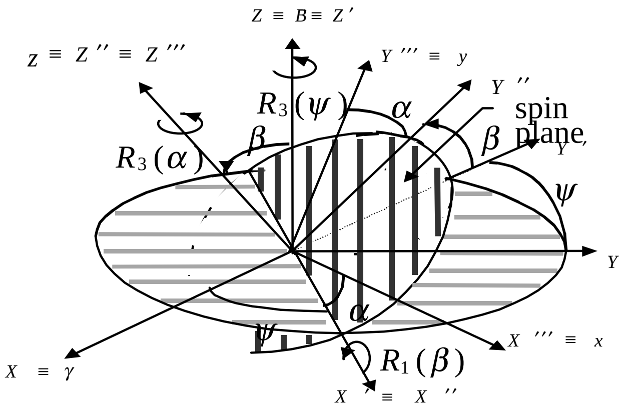

Figure 1 – Relationship between the orientations of the satellite's spin axis and the geomagnetic field, in the Inertial System.

The rotation matrix $R_3(\psi)$, denoting the precession, represents de rotation from the inertial X - axis to the nodal line X'. The rotation matrix $R_1(\beta)$ transforms the (X'Y') plane to the plane (X"Y") = (x y), which is normal to the Z"- axis and parallel to the spin axis, z - axis. The rotation matrix $R_3(\alpha)$, transforms positions X" and Y" to the instantaneous positions X"' and Y"', i. e., x and y. In this way the **X** system is transformed to the **X"'** system, this means, to the **x** system, as follows:

$$\mathbf{X'''} = R_3(\psi)\, R_1(\beta)\, R_3(\alpha)\, \mathbf{X}, \tag{1}$$

in a the matrix notation as

$$\mathbf{X'''} = \begin{vmatrix} c\,\alpha & s\,\alpha & 0 \\ -\,s\,\alpha & c\,\alpha & 0 \\ 0 & 0 & 1 \end{vmatrix} \cdot \begin{vmatrix} 1 & 0 & 0 \\ 0 & c\,\beta & s\,\beta \\ 0 & -\,s\,\beta & c\,\beta \end{vmatrix} \cdot \begin{vmatrix} c\,\psi & s\,\psi & 0 \\ -\,s\,\psi & c\,\psi & 0 \\ 0 & 0 & 1 \end{vmatrix} \mathbf{X}, \tag{2}$$

in which the abbreviation $c\theta = \cos\theta$ and $s\theta = \mathrm{sen}\,\theta$ are used to save space.

Thus, B_x, B_y and B_z, components of the geomagnetic field $\vec{\mathrm{B}}$, can be obtained, as follows:

$$\begin{aligned} B_x &= B_X\left(c\alpha\, c\psi - s\alpha\, c\beta\, s\psi\right) + B_Y\left(c\alpha\, s\psi + s\alpha\, c\beta\, c\psi\right) + B_Z\, s\alpha\, s\beta \\ B_y &= -B_X\left(s\alpha\, c\psi + c\alpha\, c\beta\, s\psi\right) - B_Y\left(s\alpha\, s\psi - c\alpha\, c\beta\, c\psi\right) + B_Z\, c\alpha\, s\beta \\ B_z &= B_X\ s\beta\, s\psi - B_Y\ s\beta\, c\psi + B_Z\ c\beta \end{aligned} \tag{3}$$

Where B_X, B_Y and B_Z are the geomagnetic field components at the Inertial System and will be propagated with the orbital elements. But, if this data does not available at the satellite's onboard computer, it is possible to use $\left|\,\vec{B}\,\right|$ which could be obtained directly from the magnetometer measured components B_x, B_y and B_z, in regions where the geomagnetic field is nearly parallel to the $Z\hat{k}$ inertial direction ($\vec{B} \uparrow\uparrow Z\hat{k}$), through $\left|\,\vec{B}\,\right| = (B_x^2 + B_y^2 + B_z^2)^{1/2}$.

Thus,

$$\begin{vmatrix} B_x \\ B_y \\ B_z \end{vmatrix} = \begin{vmatrix} c\alpha\ c\psi - s\alpha\ c\beta\ s\psi & s\psi\ c\alpha + s\alpha\ c\beta\ c\psi & s\alpha\ s\beta \\ -\,s\alpha\ c\psi - c\alpha\ c\beta s\psi & -\,s\alpha\ s\psi + c\alpha\ c\beta\ c\psi & c\alpha\ s\beta \\ s\beta\ s\ \psi & -\,s\beta\ c\psi & c\beta \end{vmatrix} \cdot \begin{vmatrix} 0 \\ 0 \\ B \end{vmatrix}, \tag{4}$$

which in view of the zeros reduces to

$$B_x = B\,\mathrm{sen}\,\alpha\,\mathrm{sen}\,\beta,\ B_y = B\cos\alpha\,\mathrm{sen}\,\beta,\ B_z = B\cos\beta, \tag{5}$$

where:

$$\alpha = tg^{-1}\left[\frac{B_x}{B_y}\right] \quad \text{and} \quad \beta = \cos^{-1}\left[\frac{B_z}{B}\right] \tag{6}$$

In this way the angles α and β could be obtained directly and this system is almost the same as the system proposed by Fonseca (1995), in which the precession ψ was assumed to be null. However, for this case it is not truth and we assume the precession ψ to be very small, but not null. Therefore, de derivative $\dot{\psi}$ must be kept in account on the evaluation of the angular velocity vector $\vec{\omega}$, so:

$$\begin{vmatrix} \omega_x \\ \omega_y \\ \omega_z \end{vmatrix} = \begin{vmatrix} \operatorname{sen}\beta \operatorname{sen}\alpha & \cos\alpha & 0 \\ \operatorname{sen}\beta\cos\alpha & \operatorname{sen}\alpha & 0 \\ \cos\beta & 0 & 1 \end{vmatrix} \cdot \begin{vmatrix} \dot{\psi} \\ \dot{\beta} \\ \dot{\alpha} \end{vmatrix}. \tag{7}$$

Thus, the angular velocity vector $\vec{\omega}$ components of the satellite, in terms of the nodal system, were obtained from temporal derivatives $\dot{\alpha}$ and $\dot{\beta}$:

$$\omega_x = \dot{\psi}\operatorname{sen}\beta\operatorname{sen}\alpha + \dot{\beta}\cos\alpha\,,\ \omega_y = \dot{\psi}\operatorname{sen}\beta\cos\alpha + \dot{\beta}\operatorname{sen}\alpha\,,\ \omega_z = \dot{\psi}\cos\beta + \dot{\alpha}\,, \tag{8}$$

where $\dot{\alpha}$ and $\dot{\beta}$ are obtained as shown bellow:

$$\dot{\alpha} = \frac{d}{dt}\left\{tg^{-1}\left[\frac{B_x}{B_y}\right]\right\} \Rightarrow \quad \dot{\alpha} = \frac{B_y\dot{B}_x - B_x\dot{B}_y}{B_y^2 + B_x^2} \tag{9}$$

$$\beta = \cos^{-1}\left[\frac{B_z}{B}\right], \quad \Rightarrow \quad \dot{\beta} = \frac{d}{dt}\left\{\cos^{-1}\left[\frac{B_z}{B}\right]\right\} \Rightarrow \quad \dot{\beta} = \frac{B_z\dot{B} - B\dot{B}_z}{B\sqrt{B^2 - B_z^2}}. \tag{10}$$

The derivatives were obtained as follows:

$$\dot{B}_x = \frac{B_{xk} - B_{xk-1}}{\Delta t}, \quad \dot{B}_y = \frac{B_{yk} - B_{yk-1}}{\Delta t}, \quad \text{and} \quad \dot{B} = \frac{B_k - B_{k-1}}{\Delta t}, \tag{11}$$

this means, that for small time intervals one could take

$$\dot{\alpha} \cong \frac{\Delta\alpha}{\Delta t}, \qquad \dot{\beta} \cong \frac{\Delta\beta}{\Delta t}, \tag{12}$$

where $\Delta\alpha$ and $\Delta\beta$, are

$$\Delta\alpha = \alpha_k - \alpha_{k-1} \quad \text{and} \quad \Delta\beta = \beta_k - \beta_{k-1}, \tag{13}$$

instead of

$$\frac{d\alpha}{dt} = \dot{\alpha}, \qquad \frac{d\beta}{dt} = \dot{\beta}, \tag{14}$$

Although this method is consistent, for the full orbit, only if the geomagnetic field $\vec{B}$ could be propagated. If $\vec{B}$ couldn't be propagated the control will be consistent only on latitudes near to the equator, this means, where the geomagnetic field $\vec{B}$ is nearly parallel to the $Z\hat{k}$ inertial direction ($\vec{B}\uparrow\uparrow Z\hat{k}$).

Thus, we suggest to use a well known cinematic relation (Goldstein, 1973) for evaluate the temporal derivatives of the geomagnetic flux density vector at satellite coordinate system, i.e.,

$$\left(\frac{d\vec{B}}{dt}\right)_s = \left| \dot{B}_x \dot{B}_y \dot{B}_z \right|^T + (\vec{\omega})_s \times \left(\vec{B}\right)_s . \tag{15}$$

Assuming that the Earth's orbital and the angular velocities are very small compared with the satellite's angular velocity, this implies that we could assume the derivative $\dot{B}_s$, as null, or:

$$\left(\frac{d\vec{B}}{dt}\right)_s = \left| \dot{B}_x \dot{B}_y \dot{B}_z \right|^T + (\vec{\omega})_s \times \left(\vec{B}\right)_s = \vec{0} , \tag{16}$$

where

$$-(\vec{\omega})_s \times \left(\vec{B}\right)_s = \left| \dot{B}_x \dot{B}_y \dot{B}_z \right| . \tag{17}$$

Consequently the satellite's angular velocity vector $\vec{\omega}$ could be obtained, as follows:

$$(\vec{\omega})_s = \frac{\left|\dot{B}_x \ \dot{B}_y \ \dot{B}_z\right|^T \times \left(\vec{B}\right)_s}{\left(\vec{B}\right)_s \cdot \left(\vec{B}\right)_s} + \frac{k\left(\vec{B}\right)_s}{\left(\vec{B}\right)_s \cdot \left(\vec{B}\right)_s} , \tag{18}$$

where $k = \hat{\omega} \cdot \hat{B}$, and $\left|\dot{B}_x \ \dot{B}_y \ \dot{B}_z\right|^T$ are given by:

$$\dot{B}_x = \frac{B_{xk} - B_{xk-1}}{\Delta t},$$

$$\dot{B}_y = \frac{B_{yk} - B_{yk-1}}{\Delta t}, \tag{19}$$

$$\dot{B}_z = \frac{B_{zk} - B_{zk-1}}{\Delta t}$$

and

$$\dot{B} = \frac{B_k - B_{k-1}}{\Delta t} \qquad \text{for} \qquad B = (B_x^2 + B_y^2 + B_z^2)^{1/2} , \tag{20}$$

where B_X , B_Y and B_Z could be obtained, as follows:

$$
\begin{aligned}
B_x &= -B_X sin\alpha_s + B_Y \cos\alpha_s \\
B_y &= -B_X \cos\alpha_s sin\delta_s - B_Y sin\alpha_s sin\delta_s + B_Z \cos\delta_s \\
B_z &= B_X \cos\alpha_s sin\delta_s + B_Y sin\alpha_s sin\delta_s + B_Z sin\delta .
\end{aligned}
\tag{21}
$$

A better way to obtain the satellite's attitude is to determine it at the Inertial System, and afterwards transform it to the nodal system, like shown in Figure 2.

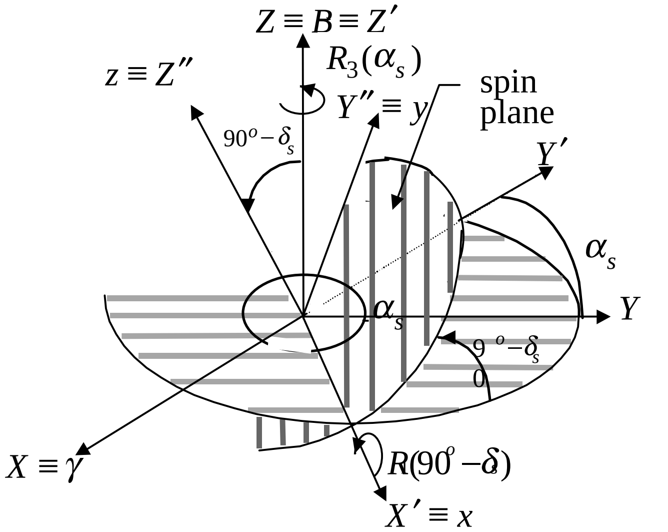

Figure 2 – Relationship between the orientations of the satellite's spin axis and the nodal coordinate system α_s and δ_s in the Inertial System.

Because of the slow orbital and the angular velocities of the Earth, compared with the satellite's angular velocity, the temporal derivatives $\dot{B}_X$, $\dot{B}_Y$ and $\dot{B}_Z$ could be considered constants. Where B_X , B_Y and B_Z could be obtained in a Geomagnetic Dipole Model like the IGRF85 or that one published by Mead and Fairfield (American Geophysical Union, 1972) (Ferreira, *et al* 1987). The GEOMAG model (Lopes, *et al* 1983), could be found at INPE 's software library as function of α_s e δ_s, where α_s is the right ascension and δ_s declination. Thus

$$\dot{B}_x = -B_X c\alpha_s \frac{d\alpha_s}{dt} - B_Y s\alpha_s \frac{d\alpha_s}{dt} , \tag{22}$$

$$\dot{B}_y = \left(B_X c\alpha_s c\delta_s - B_Y s\alpha_s c\delta_s - B_Z s\delta_s\right)\frac{d\delta_s}{dt} - \left(B_X c\alpha_s s\delta_s + B_Y c\alpha_s s\delta_s\right)\frac{d\alpha_s}{dt} \tag{23}$$

$$\dot{B}_z = -\left(B_X c\alpha_s s\delta_s + B_Y s\alpha_s s\delta_s - B_Z c\delta_s\right)\frac{d\delta}{dt} - \left(B_X s\alpha_s c\delta_s - B_Y c\alpha_s c\delta_s\right)\frac{d\alpha_s}{dt} . \tag{24}$$

Where the derivatives $\dot{\alpha}_s$ and $\dot{\delta}_s$, are given by:

$$\frac{d\alpha_s}{dt} = \left(B_X \cos\alpha_s + B_Y \operatorname{sen}\alpha_s\right) tg\delta_s - B_Z \ ,$$

$$\frac{d\delta_s}{dt} = -B_X \operatorname{sen}\alpha_s + B_Y \cos\alpha_s \, . \tag{25}$$

Finally, we have the new control variables $\dot{B}_X$, $\dot{B}_Y$ and $\dot{B}_Z$.

2 – INITIAL ATTITUDE ACQUISITION

The initial attitude acquisition could be made through Sun sensors, where unique problem consists in how to make the initial Sun acquisition. The initial Sun acquisition is a function of the threshold definition, the sun sensor quantity and it's location on the satellite. Prudêncio, (1997) suppose two analog Sun sensors, with a 60 mV threshold, each one. This means two sensors with a 141^o field view (141^o visibility angle in respect to the sensor's reference axis) each one. Assuming that the threshold's maximum error is around 10%, the field view will be reduced to 116^o. Otherwise, Prudêncio (1997) proposed a configuration where "sensor 1" is aligned with the satellite's spin axis, "sensor 2" made a 135^o angle with "sensor 1" and simultaneously a 45^o angle with the spin axis. Making testes for this Sun sensor configuration, we obtain the initial Sun acquisition possibility illustrated in Figure 3.

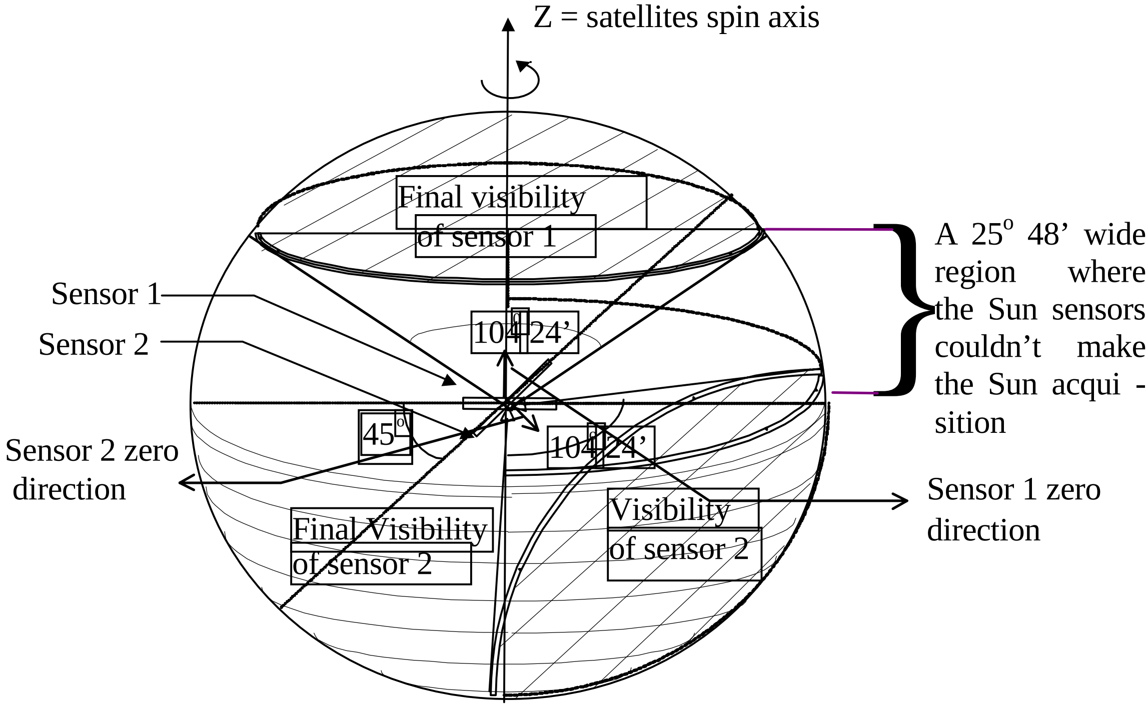

Figure 3 - The Sun's acquisition region for two Sun sensors with a 60 mV threshold, each one, and 10% maximum error, that means a 116^o visibility angle with a 104^{o}24' wide visibility angle projection onto the Earth.

Thus we conclude that there will be a 25^o 48' wide region where any one of the two Sun sensors acquire the Sun.

After some testes, we could define a new positioning for the two analog Sun sensors, with same threshold definition. Where we assume that both sensors made a 45^o angle with the satellite's spin axis and a 180^o angle between them. Thus, we have the initial Sun acquisition possibility, as illustrated in Figure 4.

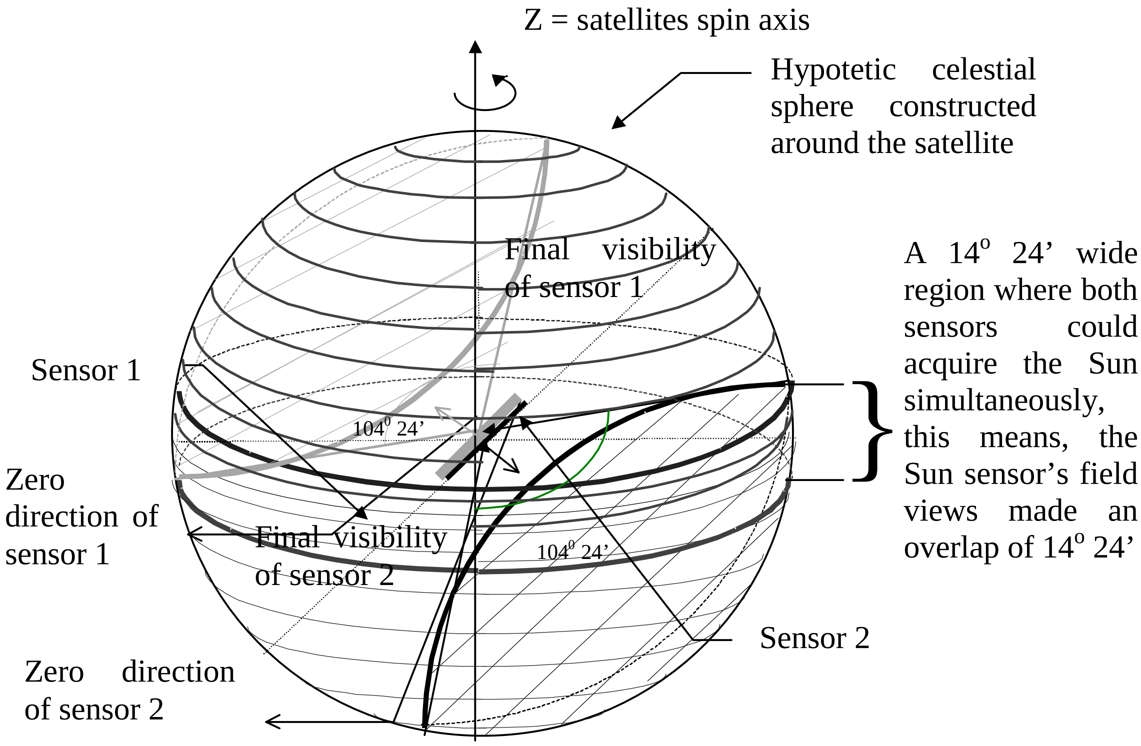

Figure 4 - The Sun's acquisition region, for two Sun sensors with a 116^o visibility angle, where the visibility angle projection onto the Earth is 104^{o}24' wide. Both sensors made a 45^o angle with the spin axis and a 180^o angle between them. The Sun sensor's field views made an overlap of 14^o 24'.

Thus we conclude that there will be a 14^o 24' wide overlap region, where both Sun sensors could acquire the Sun simultaneously, and in this way the initial Sun acquisition is warranted.

3 - CONCLUSION

We conclude that Shigehara's (1972) method for equatorial spinning satellites could be used also for polar spinning satellites, as proposed by Prudêncio (1997). For autonomous control is it necessary to take $Z\hat{k}$ approximately parallel to $\vec{B}$, but this is truth only near the equatorial region. Thus, for this kind of control is it necessary to propagate at least the orbital elements on the onboard computer, and to fix previously the regions where the autonomous control system should be turned on or of. Otherwise, if the computer has sufficient memory for propagate the geomagnetic density field $\vec{B}$, together with the orbital elements, all decisions could be made, in real time, by the onboard computer as function of the actual angle between $\vec{B}$ and $Z\hat{k}$.

4 - REFERENCES

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