TRANSIENT VIBRATION RESPONSE OF A CYLINDRICAL THIN-WALLED VESSEL

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Abstract

In this work, the mathematical analysis of the transient motion of a cylindrical vessel is presented. The vessel is supposed to be axisymmetric and the thin shell theory is used in the derivation of the equations of motion of the shell. The Finite Element Method is used in the solution of the shell, and curved elements of varying meridian's curvature are adopted. A solution of the shell dynamic equations is obtained through displacement functions which depict the several possible circumferential modes in terms of sines and cosines of $n\theta$, where n is an integer and θ is the circumferential angular coordinate. Finally, the Newmark procedure is adopted to solve the vibration problem in the time domain. Typical results of the transient analysis are presented and the natural frequencies of the combined circumferential and meridional modes show good agreement compared to results obtained with the NASTRAN Finite Element software.

Key words: Vibrations, Thin Shells, Finite Elements

1. SHELL EQUATIONS

The thin shell theory used in this work may be classified as a bending theory and assumes the presence of all the internal stress components except for shear stresses normal to the neutral surfaces, which are neglected. In using this theory to obtain the shell equations of motion, some assumptions and fundamental references have to be considered: (a)The material of the shell is homogeneous, isotropic and linearly elastic, following Hookes's law; (b) The geometry of the shell is axisymmetric; (c) The strain-displacement and the stress-strain relationships are based on Novozhilov (1970); (d) The shell element used is a two node axisymmetric element of varying meridional curvature (curved element); (e) The displacement functions follow the approach of Ross *et al.*(1983,1986,1987); (f) The strain-displacement relationships of Novozhilov (1970) are:

$$\varepsilon_{\phi} = \frac{\partial u}{\partial s} + \frac{w}{R_{\phi}} \tag{1}$$

$$\varepsilon_{\theta} = \frac{1}{r}\left(\frac{\partial v}{\partial \theta} + u\cos\phi + w\sin\phi\right) \tag{2}$$

$$\varepsilon_{\phi\theta} = \frac{1}{r}\left(r\frac{\partial v}{\partial s} - v\cos\phi + \frac{\partial u}{\partial \theta}\right) \tag{3}$$

$$k_{\phi}=-\frac{\partial^{2}w}{\partial s^{2}}+u\frac{\partial^{2}\phi}{\partial s^{2}}+\frac{1}{R_{\phi}}\frac{\partial u}{\partial s} \tag{4}$$

$$k_{\theta}=-\frac{1}{r}\left[\frac{1}{r}\frac{\partial^{2}w}{\partial\theta^{2}}-\frac{\sin\phi}{r}\frac{\partial v}{\partial\theta}+\left(\frac{\partial w}{\partial s}-\frac{u}{R_{\phi}}\right)\cos\phi\right] \tag{5}$$

$$k_{\phi\theta}=\frac{2}{r}\left(-\frac{\partial^{2}w}{\partial s\partial\theta}+\frac{\cos\phi}{r}\frac{\partial w}{\partial\theta}+\sin\phi\frac{\partial v}{\partial s}-\frac{\sin\phi\cos\phi}{r}v+\frac{1}{R_{\phi}}\frac{\partial u}{\partial\theta}\right) \tag{6}$$

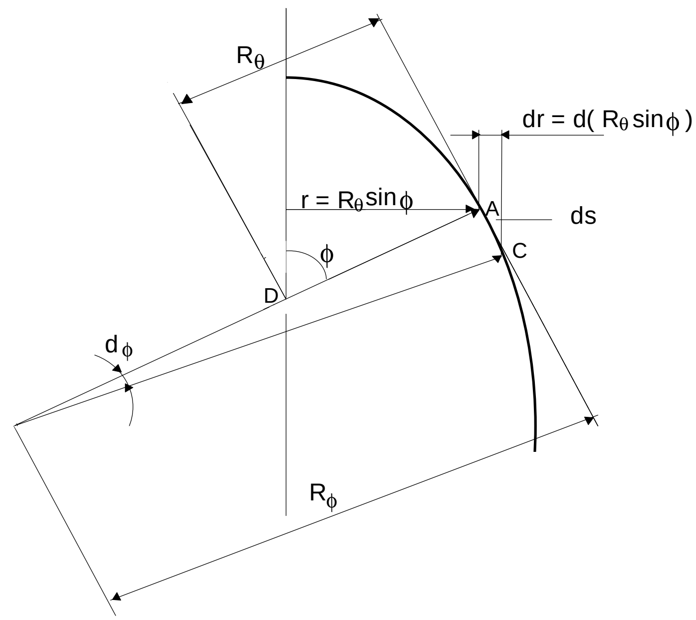

Figure 1. Shell of revolution. meridional profile

where θ is the circumferential angle, ϕ is the meridional angle, r is the circumferential radius of the shell, R_{ϕ} is the meridional radius of the shell, s is a distance along the meridian, ε_{θ} is the circumferential strain, ε_{ϕ} is the meridional strain $\varepsilon_{\phi\theta}$ is the shear strain in the $\phi\theta$ plane, k_{θ} is the circunferential curvature, k_{ϕ} is the meridional curvature, $k_{\phi\theta}$ is the twist in the $\phi\theta$ plane, u is the meridional displacement of the shell, v is the circumferential displacement of the shell, and w is the displacement perpendicular to meridian of the shell.

The assumed displacement functions according to Ross (1983) are:

$$u=\frac{(1-\xi)}{2}u_{i}\cos(n\theta)+\frac{(1+\xi)}{2}u_{j}\cos(n\theta) \tag{7}$$

$$v=\frac{(1-\xi)}{2}v_{i}\sin(n\theta)+\frac{(1+\xi)}{2}v_{j}\sin(n\theta) \tag{8}$$

$$w=\frac{(2-3\xi+\xi^3)}{4}w_i\cos(n\theta)+\frac{(1+\xi)(1-\xi)^2}{4}\ell\beta_i\cos(n\theta)+\frac{(2+3\xi-\xi^3)}{4}w_j\cos(n\theta)+$$

$$\frac{(1-\xi)(1+\xi)^2}{4}\ell\beta_j\cos(n\theta) \qquad (9)$$

where β is the rotational displacement of the shell, n is the circumferential wave number, ℓ is half the shell element meridional length, $\xi=\frac{s}{\ell}$ is the local element coordinate which varies from +1 (node i) to -1 (node j) and u_i, v_i, w_i, β_i, u_j, v_j, w_j, β_j, are nodal displacement values at nodes i and j, respectively.

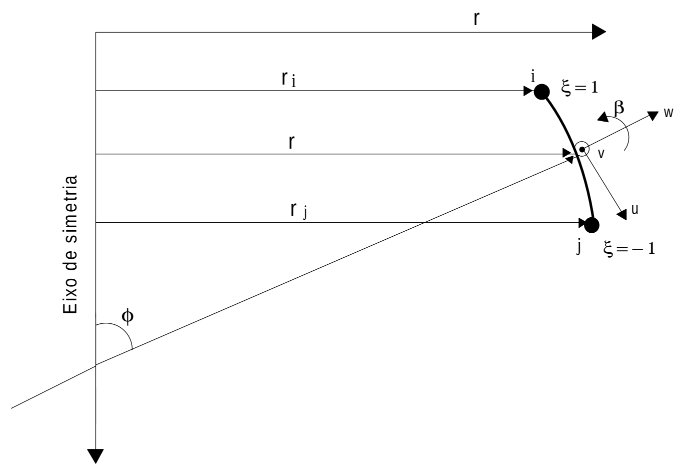

Figure 2. Curved element of varying meridional curvature. Representation of normalised coordinate and node identification. (Component v is orthogonal to w and u and outward from the plane of the figure).

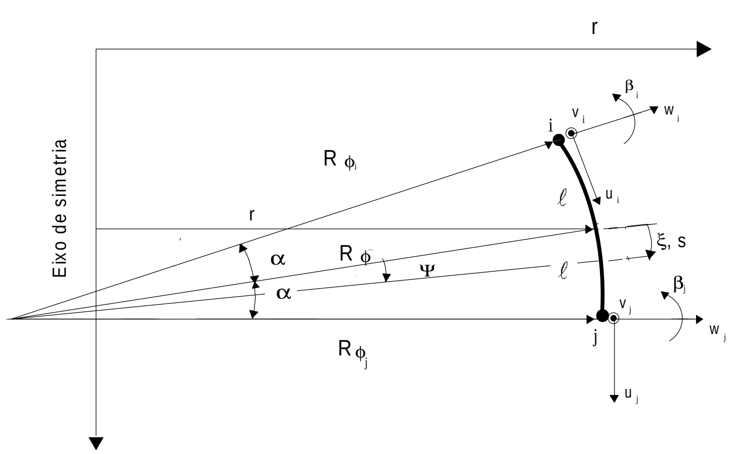

Figura 3. Curved element of varying meridional curvature. Representation of geometrical parameters. (Component v is orthogonal to w and u and outward from the plane of the figure).

The displacements u, v, w and β are assumed to have relative directions as shown in Figure 3. They may be expressed as a matrix product of the shape functions $[N]$ and the nodal displacement vector $\{U_k\}$ as,

$$\{U\} = [N]\{U_k\} \tag{10}$$

where

$$\{U\}^T = [u \quad v \quad w] \tag{11}$$

$$\{U_K\}^T = [u_i \quad v_i \quad w_i \quad \beta_i \quad u_j \quad v_j \quad w_j \quad \beta_j] \tag{12}$$

$$[N] = \begin{bmatrix} n_{11}\ n_{12} \ldots\ldots\ldots\ldots n_{18} \\ n_{21} \\ n_{31} \ldots\ldots\ldots\ldots\ldots\ldots n_{38} \end{bmatrix} \tag{13}$$

The displacement functions given by Equations (7), (8) and (9) may be replaced in Equations (1) to (6) to give,

$$\{\varepsilon\} = [B]\{U_k\} \tag{14}$$

where

$$\{\varepsilon\}^T = \{\varepsilon_\Phi \quad \varepsilon_\theta \quad \varepsilon_{\Phi\theta} \quad k_\Phi \quad k_\theta \quad k_{\Phi\theta}\} \tag{15}$$

$$[B] = \begin{bmatrix} b_{11}\, b_{12} \cdots\cdots b_{18} \\ b_{21} \\ \vdots \\ b_{61}\, b_{62} \cdots\cdots b_{68} \end{bmatrix} \tag{16}$$

The relations between forces, moments and deformations can be expressed for the axisymmetric case in a matrix notation as

$$\{\sigma\} = [D]\{\varepsilon\} \tag{17}$$

$$\{\sigma\}^T = [N_\phi\, N_\theta\, N_{\phi\theta}\, M_\phi\, M_\theta\, M_{\phi\theta}] \tag{18}$$

$$[D] = \begin{bmatrix} d_{11}\, d_{12} \cdots\cdots d_{16} \\ d_{21} \\ \vdots \\ d_{61}\, d_{62} \cdots\cdots d_{66} \end{bmatrix} \tag{19}$$

whrere N_ϕ and N_θ are direct forces per unit length parallel to ϕ (meridional) and θ (circumferential) axes respectively, $N_{\phi\theta}$ is the shear force per unit length parallel to ϕ axis on face with θ as normal, M_ϕ and M_θ are bending moments per unit length about ϕ and θ axes respectively and $M_{\phi\theta}$ is the twisting moment per unit length about ϕ axis on face with θ as normal.

2. SOLUTION OF THE SHELL MATRIX DIFFERENTIAL EQUATIONS

The shell matrix differential equations may be represented as

$$[M]\{\ddot{c}\}+[K]\{c\}=\{R\} \tag{20}$$

where $\{c\}$, $\{\dot{c}\}$ and $\{\ddot{c}\}$ are a generalised definition of displacement, velocity and accelerations vectors respectively. To solve the matrix differential equations above, the Newmark scheme may be employed (Wood, 1990). In Newmark scheme, the first derivative $\{\dot{c}\}$ and the function $\{c\}$ itself are aproximated at the (n+1)th time step by the following expressions

$$\{\dot{c}\}^{n+1}=\{c\}^{n}+[(1-\alpha)\{\ddot{c}\}^{n}+\alpha\{\ddot{c}\}^{n+1}]\Delta t \tag{21}$$

$$\{c\}^{n+1}=\{c\}^{n}+\{\dot{c}\}^{n}\Delta t+[(\frac{1}{2}-\beta)\{\ddot{c}\}^{n}+\beta\{\ddot{c}\}^{n+1}](\Delta t)^{2} \tag{22}$$

where α e β are parameters that control the accuracy and atability of the scheme. The choice $\alpha=\frac{1}{2}$ and $\beta=\frac{1}{4}$ is known as the "constant-average-accelerations method [7].

Rearranging Equation (48) and replacing $\{c\}^{n+1}$ and $\{\ddot{c}\}^{n+1}$ in Equation (46) one arrives at

$$[\overline{K}]\{c\}^{n+1}=[\overline{R}] \tag{23}$$

where

$$[\overline{K}]=[K]+a_0[M] \tag{24}$$

$$\{\overline{R}\}=\{R\}^{n+1}+[M](a_0\{c\}^{n}+a_1\{\dot{c}\}^{n}+a_2\{\ddot{c}\}^{n}) \tag{25}$$

where

$$a_0=[\beta(\Delta t)^2]^{-1} \quad ; \quad a_1=a_0\Delta t \quad ; \quad a_2=\frac{1}{2\beta}-1 \tag{26}$$

3. SIMULATIONS OF AN INITIAL DISPLACEMENT

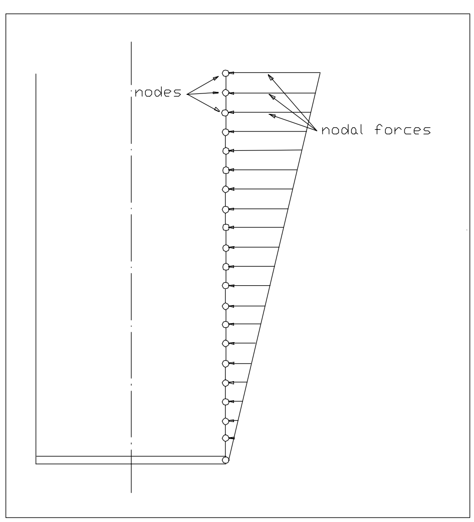

Figure 4. Linearly varied loads applied on the cylindrical vessel along the meridional direction.

Preliminary tests with the shell program have shown that the meridional mode shape is very much dependent on the type of excitation. If one applies an initial concentrated load at the top of the wall, all meridional modes will be excited simultaneously. Consequently, the identification of a natural frequency becomes more difficult and a decoupling procedure had to be considered. In order to stimulate a "pure" beam-type mode, several forms of external load were tested and the one which best reproduces the shape of this mode is a triangularly distributed force. The load is linearly varied from zero at the button, to a maximum value at the top of the shell. These forces are applied on the wall for a certain number of iterations to create an initial displacement. Then velocities and accelerations are set equal to zero and with a stored strain energy, the shell is released to vibrate.

4. NUMERICAL RESULTS

A certain model of a thin walled cylindrical vessel was select for the purpose of comparison. The cylinder was fixed at the button end and kept free at the top end. The adopted geometrical and physical properties of the material were: (a) Circumferential radius of the shell = 0.1 m; (b) Longitudinal length of the cylinder = 0.4m; (c) Shell thickness = 0.0005 m; (d) Elasticity modulus = $2.07x10^{11}\,N/m^2$; (e) Density = $7800 kg/m^3$; (f) Poison ratio = 0.3; (g) Number of elements = 40.

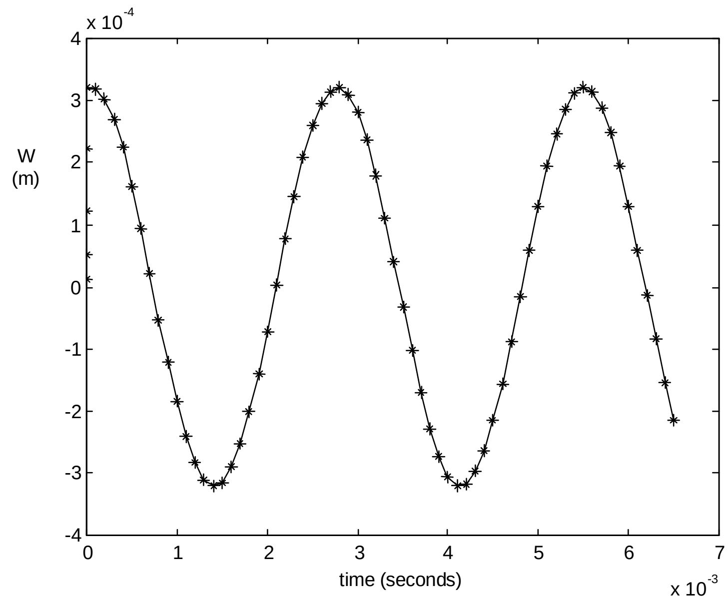

Figure 5. Displacement w versus time for wave number n = 2.

A typical result of the transient analysis is shown in Figure 5 which shows the time history of the normal displacement w at the top of the cylinder. The figure corresponds to a case where n = 2 , where a predictable undamped response is obtained. From this kind of analysis, estimates of the natural frequencies of the shell were made. The time when the first cross-over of the curve with the time axis occurs is taken as a reference. When the first lower half cycle is completed, the time of the second cross-over is recorded. Assuming that the characteristic period of the oscillatory motion remains unchanged, the period of the first lower half cycle can be doubled and inverted for the calculation of the frequency.

A model, with the same geometrical and physical properties cited above, was created with help of NASTRAN software. NASTRAN mesh was built considering 12 rectangular four noded elements in the meridional direction and 18 elements in the circumferential direction . In order to compare results, 6 cases were select and simulated with the present theory and the NASTRAN software. The cases chosen were the circumferential numbers 1, 2, 3, 4, 5 and 6, coupled with the first meridional mode. The results of both methods are presented in Table 1 and Figure 6.

Table 1. Natural frequencies obtained by the present theory (Myshell) and NASTRAN

Wave number n	Natural frequency (Hz) – Myshell	Natural Frequency (Hz) – NASTRAN
1	919,41	921,36
2	366,95	369,05
3	205,61	207,88
4	209,96	215,32
5	301,53	307,36
6	440,09	434,21

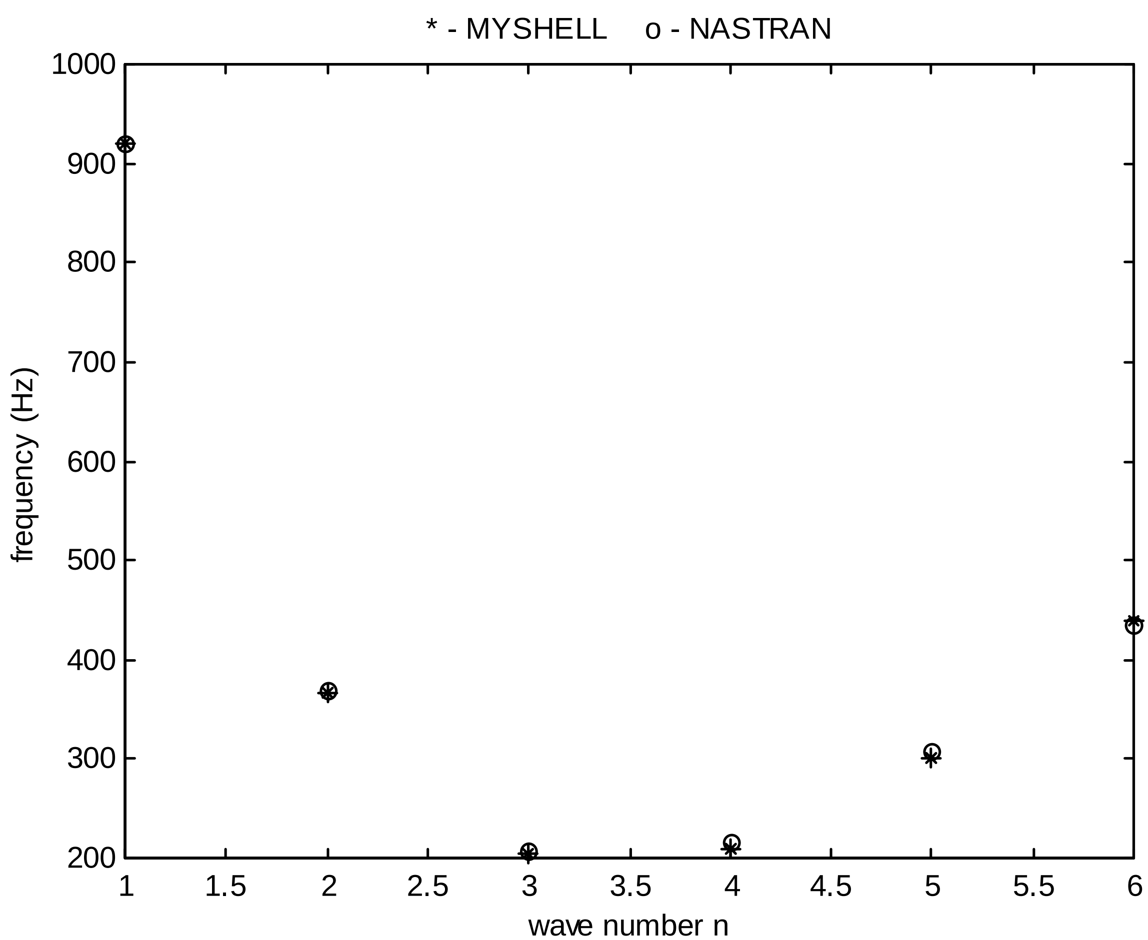

Figure 6. Natural frequencies of obtained with the present theory and NASTRAN

5. COMMENTS AND CONCLUSIONS

The purpose of this work was to present further details and make a more precise comparison of the proposed theory with a widely used finite elements software. Details of the Newmark method applied to the finite elements matrix equations of motion were presented. A method for obtaining a free undamped vibration motion through an initial and appropriate excitation was depicted.

As one can see in Figure 5 the cylinder response to an initial triangular distributed load reveals a consistent succession of constant periods of the vibration cycles with no amplitude reduction, as predicted by the theory. Furthermore, the natural frequencies obtained through the present theory and the NASTRAN finite element software were compared and showed a good degree of correlation.

Although one can use a known finite element software, as NASTRAN, to solve a shell vibration problem, it should be emphasised that in coupled problem, as in fluid-structure interaction problems (Menezes *et al.*, 1993, 1995, 1997) such softwares still present limitations.

6. REFERENCES

- Agrassar, A.D. e Menezes, J.C., “Vibração de Tubos de Parede Fina Conduzindo Fluido Viscoso em Fluxo Axial” - Congresso Ibero Americano sobre Métodos Computacionais para Engenharia - XVI CILANCE, Curitiba, pp. 994-953, Nov.,Dez. 1995.
- Aguiar, D. S. e Menezes, J. C., “Dinâmica de cascas cilíndricas contendo fluido com fluxo circular: Solução do problema em regime permanente” – XIV Congresso Brasileiro de Engenharia Mecânica – XIV COBEM, Bauru, SP, dezembro de 1997.
- Menezes, J.C. e Mistry,J., "Vibration Tests of Liquid Storage Thin Cylindrical Vessels", Revista Brasileira de Ciências Mecânicas, Vol. - XV, No 1, 1993.
- Mistry, J. and Menezes, J.C., "Vibration of Cylinders Partially-Filled With Liquids" - Journal of Vibration and Acoustics - ASME , Vol. 117, pp. 87-93, 1995.
- Novozhilov, V.V., “Thin Shell Theory”, Wolters-Noordhoff Publishing, Groningen, 1970.
- Reddy, J. N., “Applied Functional Analysis and Variation Methods in Engineering”, McGraw- Hill Book Co., 1986.
- Ross, C.T.F. and Jonhs , T ., , “Vibration of Submerged Hemi-Ellipsoidal Domes”, J. Sound Vib.,Vol.91, N°3 , pp.363-373, 1983.
- Ross, C.T.F. and Jonhs , T., “Vibration of Hemi-Ellipsoidal Axisymetric Domes Submerged in Water”, Proc.Inst. Mech.Engrs.,Vol.200, N°C6 , pp.389-398, 1986.
- Ross, C. T. F. and Jonhs ,E. and Jonhs, T., “Vibration of Thin-Walled Domes Under External Water ”, J. Sound Vib.,Vol.114, N°3 , pp.453-463, 1987.
- Ross, C. T. F. and Mackney, M. D. A., “Deformation and Stability Studies of Thin-Walled Domes Under Uniform External Pressure”, J. of Strain Analysis, Vo. 18, NO 3, 1983.
- Wood, W. L., “Practical Time-Stepping Schemes”, Oxford University Press, 1990.