IDENTIFICATION OF EXCITATION FORCES USING AN INVERSE STRUCTURAL MODEL

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Abstract

This work focuses the indirect identification of excitation forces from dynamic time responses of mechanical systems. A methodology is studied based on the use of a discrete-time inverse state-space model. The inverse model is obtained starting from the direct model by interchanging input and output vectors. Once the inverse model is built, the excitation forces are estimated by recursive resolution of the state and output difference equations. The paper is organized as follows: after preliminary remarks concerning the methods which have been used for force identification, the basic formulation leading to the inverse structural model is presented. Some inherent characteristics concerning the stability of the inverse system are discussed. The results of some numerical simulations are finally presented to evaluate the operational features, robustness and accuracy of the method.

Key-words: Indirect force identification, Inverse problems, Difference equations

1.INTRODUCTION

The estimation of dynamic forces acting on a mechanical system is an important problem that has been treated with only partial success. Methods for such estimation fall into two categories: direct methods and indirect methods. The direct methods are based on the placement of force transducers into the load paths at the point where the force is applied. However, there are many situations in which it is not easy to obtain an accurate description of the excitation conditions using such methods. For example, it can be difficult to measure directly the forces exerted during a shock load. Furthermore, internal forces generated within machinery, or those that are transmitted from machinery to foundation, are generally difficult to be characterized from direct measurements. Under these circumstances it would be beneficial if the forces could be computed indirectly using measured response data together with some form of mathematical model of the structural system. These computed forces could then be used in subsequent studies involving similar types of excitation conditions. In general, the indirect approach to force estimation, characterized as an inverse problem, possesses some inherent difficulties, such ill-posedness and numerical ill-conditioning (Starkey & Merril, 1989). This has made work in this area slow and the gains modest.

Several force identification techniques, operating either in time domain or in frequency domain, have been proposed and are documented in the literature (Stevens, 1987). Most of frequency domain methods are based on the inversion of the frequency response function (FRF) matrix for each frequency line in the band of interest. This methodology demonstrated to suffer from severe ill-conditioning, mainly at frequencies associated with the natural frequencies of the structure (Starkey & Merril, 1989). These techniques also prohibit real-time or near real-time force estimation.

Time domain techniques are more recent developments. The *Sum of Weighted Accelerations Technique* (*SWAT*) (Bateman *et al.*, 1992) has been successfully applied to a variety of different real world impact and collision problems. This method is based on the modal equilibrium equations written for the rigid body modes of system. Due to its features, *SWAT* can only be applied to unconstrained structures and is only capable of providing the resulting forces and moments about the center of mass of the structure, while the actual spatial distribution of the forces remains unknown. A modal approach has also been focused by (Genaro, 1997), enabling to circumvent those drawbacks. Time domain deconvolution has been used by Kammer (1996), Genaro (1997) and Silva & Rade (1999). According to this procedure, the excitation forces are identified by solving a linear system of equations obtained by inverting the discrete-time multi-input-multi-output convolution integral, which has the matrix of input response functions (IRFs) as its kernel.

In this paper a method operating in the time domain is proposed and evaluated. According to this method, an inverse structural model is obtained from the direct model by exchanging the roles of the input and output vectors. Once the inverse system is built, the excitation forces are estimated by recursive resolution of the state and output difference equations. An application to a simple numerically simulated structure is presented to illustrate the main features and capabilities of the identification method.

2. FORMULATION OF THE METHOD

Consider a linear self-adjoint viscously damped mechanical system of *N* degrees-of-freedom, described by the equations of motion in the standard matrix form:

$$[M]\{\ddot{x}(t)\} + [C]\{\dot{x}(t)\} + [K]\{x(t)\} = \{f(t)\} \tag{1}$$

Pre-multiplying equation (1) by $[M]^{-1}$ and introducing the relation $\{f(t)\} = [B_2]\{u(t)\}$, one has:

$$\{\ddot{x}(t)\} = -[M]^{-1}[C]\{\dot{x}(t)\} - [M]^{-1}[K]\{x(t)\} + [M]^{-1}[B_2]\{u(t)\} \tag{2}$$

where $[B_2]_{(N\times f)}$ is the matrix defining the locations of the *f* excitation forces contained in vector $\{u(t)\} \in R^f$. A continuous time state-space representation of system (2) is given by the following equations:

$$\{\dot{w}(t)\} = [A]\{w(t)\} + [B]\{u(t)\} \tag{3}$$

$$\{y(t)\} = [C]\{w(t)\} + [D]\{u(t)\} \tag{4}$$

where:

$\{w(t)\} = \begin{Bmatrix} \{x(t)\} \\ \{\dot{x}(t)\} \end{Bmatrix} \in R^{2N}$ is the state vector ; $\{y(t)\} \in R^s$ is the vector of system outputs

$$[A]=\begin{bmatrix} [0] & [I] \\ -[M]^{-1}[K] & -[M]^{-1}[C] \end{bmatrix} \in R^{2N\times 2N}$$ is the system matrix

$$[B]=\begin{bmatrix} [0] \\ [M]^{-1}[B_2] \end{bmatrix} \in R^{2N\times f}$$ is the input influence matrix

$$[C]=\big[[C_d]-[C_a][M]^{-1}[K] \quad [C_v]-[C_a][M]^{-1}[K]\big] \in R^{s\times 2N}$$ is the outputs influence matrix

$$[D]=\big[[C_a][M]^{-1}[B_2]\big] \in R^{s\times f}$$ is a direct transmission matrix

$[C_a],[C_v]$ and $[C_d]$ are matrices that supply the positions of sensors for acceleration, velocity and displacement respectively.

All real systems operate in continuous time. However, they are sampled discretely in time resulting in a discrete time representation which is governed by the difference equations (Kwakernaak & Sivan, 1976):

$$\{w(k+1)\}=[A_{dis}]\{w(k)\}+[B_{dis}]\{u(k)\} \tag{5}$$

$$\{y(k)\}=[C_{dis}]\{w(k)\}+[D_{dis}]\{u(k)\} \tag{6}$$

where k indicates the appropriate time step.

The system given by equations (3) to (6), containing *N* states, *f* inputs and *s* outputs is named *direct system* in discrete time. Integration of the state equations or recursive resolution of the difference equations allow to obtain the response of the system to a set of excitation forces.

From here on, the discrete-time model will be used. Equations (5) and (6) can be manipulated to exchange the roles of input and output vectors, yielding the equations of the *inverse structural system* in the form (Horta & Sandridge, 1992):

$$\{w(k+1)\}=[\tilde{A}]\ \{w(k)\}+[\tilde{B}]\ \{y(k)\} \tag{7}$$

$$\{u(k)\}=[\tilde{C}]\ \{w(k)\}+[\tilde{D}]\ \{y(k)\} \tag{8}$$

in which the inverse system plant, input influence, output influence, and direct throughput matrices are defined as:

$$[\tilde{A}]\ =[A_{dis}]-[B_{dis}][D_{dis}]^{+}[C_{dis}] \quad ; \quad [\tilde{B}]=[B_{dis}][D_{dis}]^{+}$$

$$[\tilde{C}]\ =-[D_{dis}]^{+}[C_{dis}] \quad ; \quad [\tilde{D}]=[D_{dis}]^{+} \quad ; \quad [D_{dis}]^{+}=\left([D_{ds}]^{T}[D_{dis}]\right)^{-1}[D_{dis}]^{T}$$

The vector of excitation forces $\{u(k)\}$ is obtained from equation (8), given the measured output vector $\{y(k)\}$ and the state vector $\{w(k)\}$, obtained through the recursive resolution of the equation (7).

In the equations above it can be observed that for the inverse system to exist, the Moore-Penrose pseudo-inverse, $[D_{dis}]^{+}$, must be computed. This requires that the number of sensors *s* be greater than the number of force inputs *f*. Moreover, $[D_{dis}]$ must be full column rank. As a result, taking into account the nature of the matrix $[D_{dis}]$, equations (7) and (8) can be used for calculating the excitation forces, provided that the position of the sensors and the locations of the inputs are coincident (collocated input and outputs).

For structural systems with sensors positioned at different locations than those where the forces are applied (called *non-collocated* systems) it can be shown (Hashemi & Hammond, 1996) that the direct system possesses non-minimum phase zeros. In this form, the inverse system matrix $[\tilde{A}]$ can be unstable. This means that some of the transmission zeros of the discrete direct system are outside the unit circle, or for the continuous representation, they are located in the right half complex plane. These zeros can be directly related to the eigenvalues of the inverse system. According to Williams (1989) the transmission zeros are defined as the values of the Laplace variable for which it is possible to apply a nonzero input and get an identically zero output at the sensor locations, for suitable set of initial conditions $w(0)$. Therefore, if the input is assumed in the form $u = \mu e^{\psi t}$, then ψ is said to be a transmission zero of the direct system (5) and (6) if:

$$\begin{bmatrix} [A_{dis}] - \psi[I] & [B_{dis}] \\ -[C_{dis}] & -[D_{dis}] \end{bmatrix} \begin{Bmatrix} \{w(0)\} \\ \mu \end{Bmatrix} = [Q] \begin{Bmatrix} \{w(0)\} \\ \mu \end{Bmatrix} = \{0\} \tag{9}$$

For a non trivial solution to exist, the determinant of Q must vanish. In the case where $[D_{dis}]$ is full rank, $|Q|$ can be written as:

$$|Q| = |[D_{dis}]| \left| [A_{dis}] - [B_{dis}][D_{dis}]^{-1}[C_{dis}] - \psi[I] \right| = |[D_{dis}]| \left| [\tilde{A}] - \psi[I] \right| \tag{10}$$

which produces the condition:

$$\left| [\tilde{A}] - \psi[I] \right| = 0 \tag{11}$$

This last equation corresponds to the characteristic equation of the inverse system, implying that ψ is also an eigenvalue of the inverse system, as previously stated. Williams (1989) can be consulted for more details on transmission zeros of structures.

In the case of non-collocated inputs and outputs, matrix $[D_{dis}]$ drops rank and the Moore-Penrose pseudo-inverse, $[D_{dis}]^{+}$ can not be computed. To circumvent this problem, Kammer and Stelzner (1999) suggested the use of a non-causal inverse system, where the estimate of the input force at time *k* is expressed as a function of the response at a future time *k*+1. Consider a system for which all the sensors and force inputs are non-collocated. For such a system, the direct throughput matrix is the zero matrix, producing:

$$\{w(k+1)\} = [A_{dis}]\{w(k)\} + [B_{dis}]\{u(k)\} \tag{12}$$

$$\{y(k)\} = [C_{dis}]\{w(k)\} \tag{13}$$

Output equation (13) must be stepped forward in time before inversion can take place:

$$\{y(k+1)\} = [C_{dis}][A_{dis}]\{w(k)\} + [C_{dis}][B_{dis}]\{u(k)\} \tag{14}$$

The direct system given by equations (12) and (13) is non-causal. The associated inverse system takes the form:

$$\{w(k+1)\} = \left[[A_{dis}] - [B_{dis}]([C_{dis}][B_{dis}])^{+}[C_{dis}][A_{dis}]\right]\{w(k)\} + [B_{dis}]([C_{dis}][B_{dis}])^{+}\{y(k+1)\} \tag{15}$$

$$\{u(k)\} = -([C_{dis}][B_{dis}])^{+}[C_{dis}][A_{dis}]\{w(k)\} + ([C_{dis}][B_{dis}])^{+}\{y(k+1)\} \tag{16}$$

Equations (12) to (16) can be generalized for a number ℓ of forward time steps, yielding:

$$\text{Direct system}\begin{cases} \{w(k+1)\} = [A_{dis}]\{w(k)\} + [B_{dis}]\{u(k)\} & (17) \\ \{y(k+\ell)\} = [C_{dis}][A_{dis}]^{\ell}\{w(k)\} + [C_{dis}][A_{dis}]^{\ell-1}[B_{dis}]\{u(k)\} & (18) \end{cases}$$

$$\text{Inverse system}\begin{cases} \{w(k+1)\} = [\tilde{A}]\{w(k)\} + [\tilde{B}]\{y(k+\ell)\} & (19) \\ \{u(k)\} = [\tilde{C}]\{w(k)\} + [\tilde{D}]\{y(k+\ell)\} & (20) \end{cases}$$

where:

$$[\tilde{A}] = \left[[A_{dis}] - [B_{dis}]\left([C_{dis}][A_{dis}]^{\ell-1}[B_{dis}]\right)^{+}\left([C_{dis}][A_{dis}]^{\ell}\right)\right]$$

$$[\tilde{B}] = [B_{dis}]\left([C_{dis}][A_{dis}]^{\ell-1}[B_{dis}]\right)^{+}$$

$$[\tilde{C}] = -\left([C_{dis}][A_{dis}]^{\ell-1}[B_{dis}]\right)^{+}[C_{dis}][A_{dis}]^{\ell}$$

$$[\tilde{D}] = \left([C_{dis}][A_{dis}]^{\ell-1}[B_{dis}]\right)^{+}$$

The vector of excitation forces $\{u(k)\}$ can be obtained from equation (20), given the measured output vector $\{y(k+\ell)\}$ and the state vector $\{w(k+1)\}$, obtained through the recursive resolution of the equation (19).

3. NUMERICAL EXAMPLES

3.1 Force identification with non-collocated inputs and outputs

In this first example, a test-system with 11 d.o.f. (Figure 1) was used to identify the excitation forces under the assumption of non-collocated inputs and outputs. The excitation forces given by equation (21) and (22) was assumed to be applied at masses numbers 1 and 6. The acceleration time responses were assumed to have been measured at masses numbers 8 and 11.

$$f_1(t) = 500\cos(2\pi 150t) + 250\cos(2\pi 75t) \tag{21}$$

$$f_6(t) = 100\sin(2\pi 150t) + 80\cos(2\pi 300t) \tag{22}$$

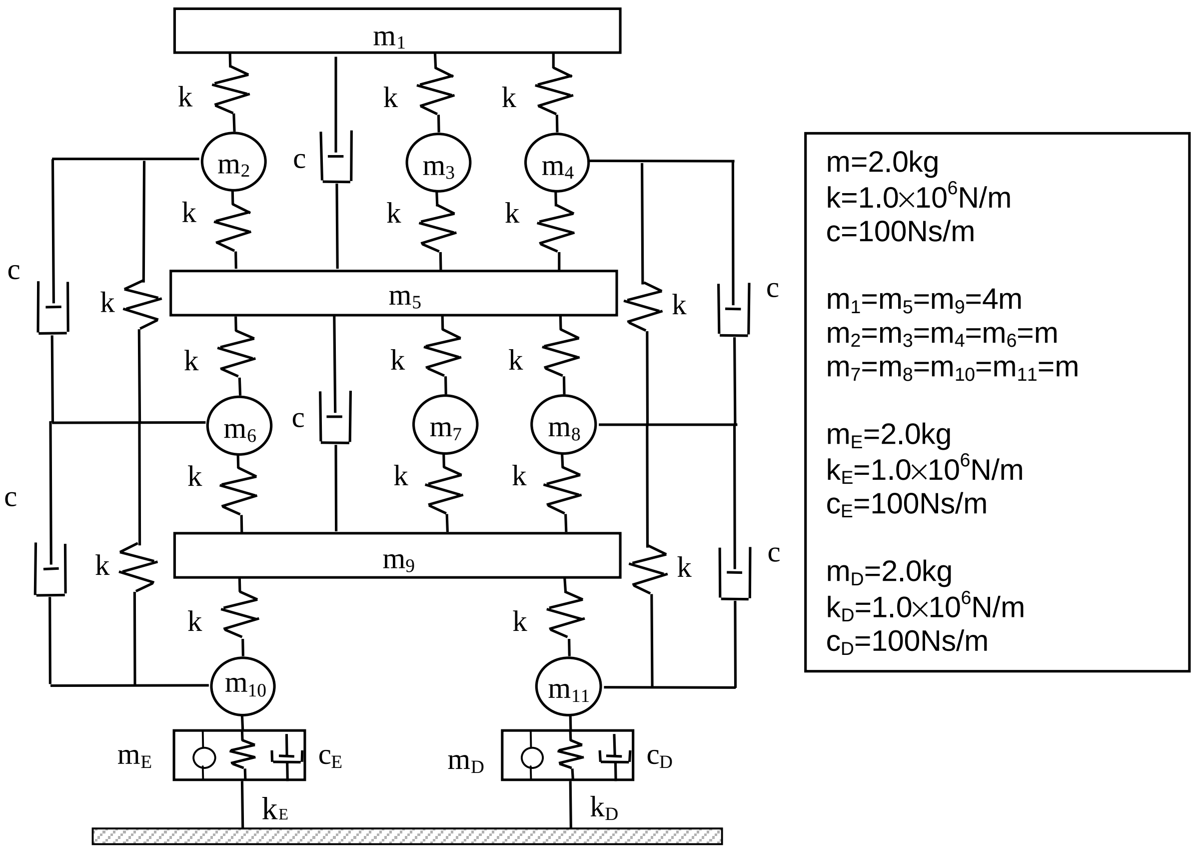


Figure 1. Characteristics of the 11 d.o.f. test-system

Null initial conditions were assumed and accelerations were observed in the interval [0-0.05s], with a time step $\Delta t = 5.00\times10^{-5}\,s$. The acceleration response was first simulated obtained by using equations (3) and (4), corresponding to the continuous-time system. The vector of excitation forces was then obtained from equation (20), using the acceleration response and the state vector calculated from equation (21), with progress $\ell = 3$. Figure 2 depicts the identified forcing functions as compared to the exact ones. As can be seen, fairly accurate identification could be achieved.

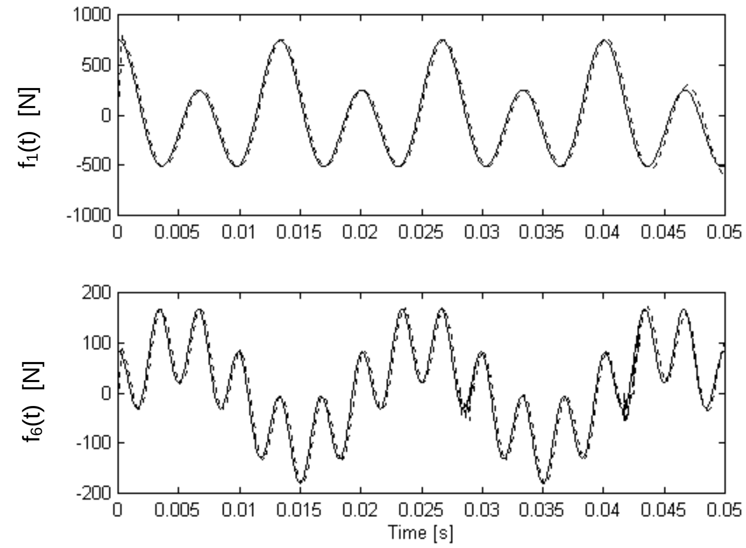
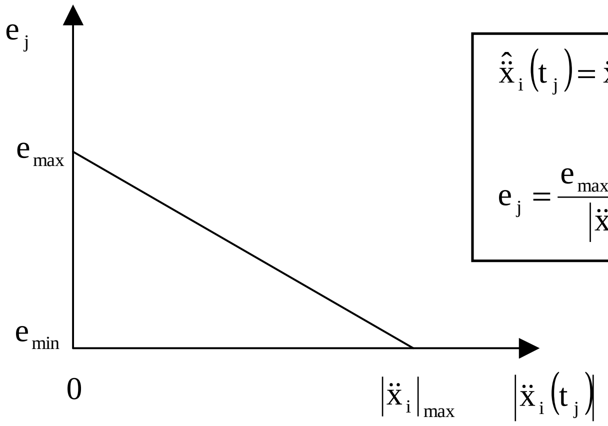


Figure 2. Exact (solid line) and reconstructed (dotted line) input forces

3.2 Evaluation of the sensitivity of the identification method with respect to modeling errors and measurement noise.

Clearly, the success of the force identification procedure depends on the accuracy of the model employed and the level of measurement noise corrupting the output signals. Thus, this second example aims at analyzing the effect of uncertainties introduced in the mass and stiffness distributions of the model and random disturbances introduced in the acceleration time responses.

Consider a single excitation force given by, $f_1(t) = 200\cos(2\pi 360t) + 100\sin(2\pi 450t)$, applied at mass number 1 of the test-system shows in Figure 1. The acceleration time response was computed from the exact continuous-time model given by (3) and (4). For the purpose of input identification, it was assumed that the response was measured only at mass number 1 (collocated input and output). This response was then polluted by random perturbations simulating experimental noise. The model used for noise is such that, for a given sensor, the maximum random error in a given time instant is inversely proportional to the amplitude of the accelerations at that time. This model is described in Figure 3, where $\hat{\ddot{x}}_i(t_j)$ indicates the corrupted acceleration of the i-th sensor at time t_j , $\ddot{x}_i(t_j)$ is the corresponding noise-free acceleration, r_j is a real number from a uniformly distributed random sequence in the range [1 ;1]. Moreover, $|\ddot{x}_i|_{max}$ indicates the maximum amplitude of the acceleration in the whole time history and e_{max} and e_{min} denote the maximum and minimum values assumed for the random errors, respectively. The values adopted for these two parameters are: e_{max} = 10% , e_{min} = 2%. The responses are measured at mass number 1 too.



$$\hat{\ddot{x}}_i\left(t_j\right) = \ddot{x}_i\left(t_j\right)\left[1 + r_j e_j\right] \quad i = 1,2,\cdots,c\,;\ j = 1,2,\cdots,p$$

$$e_j = \frac{e_{max} - e_{min}}{\left|\ddot{x}_i\right|_{max}}\left(\left|\ddot{x}_i\right|_{max} - \left|\ddot{x}_i\left(t_j\right)\right|\right) + e_{min}$$

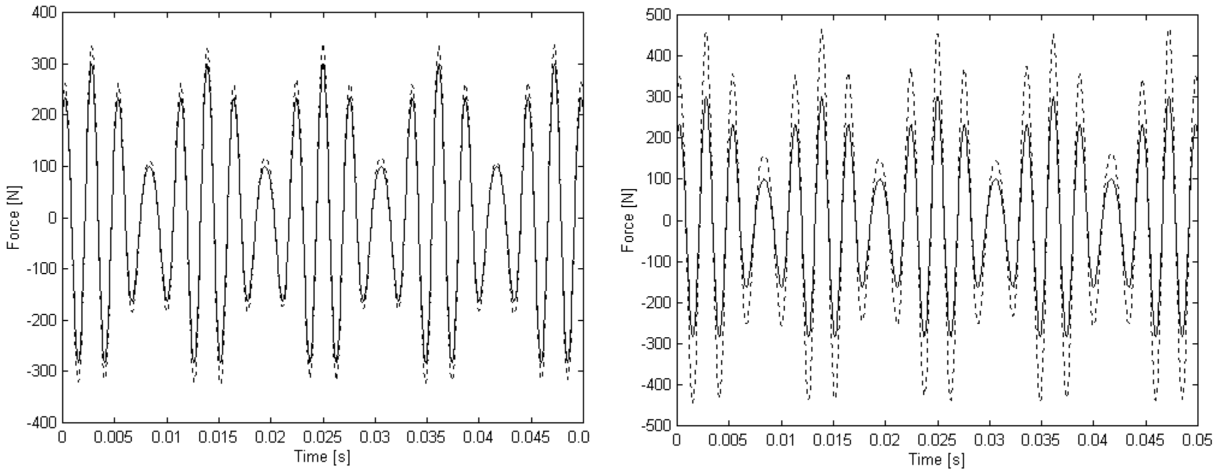
Figure 3. Model of the noise affecting the accelerations responses.

Modeling errors were simulated as random disturbances in some of the elements (arbitrarily chosen) of the exact mass and stiffness matrix of the system. These errors were introduced using uniformly distributed random sequences, generated in the range [1-ε_{max}; 1+ε_{max}]. Two different levels of disturbances were considered, corresponding the ε_{max} =20% and 50%. The resulting effect of the introduction of these disturbances can be evaluated with the aide of Table 1. To simulate a practical situation were the damping matrix is not available, the model used for force identification was assumed to be undamped ([C]=0).

As can be seen in Figure 4, force estimates tend to be less accurate for higher levels of modeling errors. For moderate levels of measurement and model uncertainties, fairly accurate results could be obtained, indicating that the estimates are reasonably robust with respect to the combined uncertainties in the case of collocated inputs and outputs. Some numerical tests have shown that the method seems to be less robust in the case of non-collocated inputs and outputs.

Table 1. Values and locations of the modifications introduced in the model.

Perturbed element	Exact value	Perturbed value (ε_{max} =20%)	Perturbed value (ε_{max} =50%)
m(1,1)	8 kg	8.90 kg	11.97 kg
m(5,5)	8 kg	8.54 kg	6.52 kg
m(9,9)	8 kg	9.46 kg	4.99 kg
k(1,3)	1×10^6 N/m	1.05×10^6 N/m	7.83×10^5 N/m
k(2,5)	1×10^6 N/m	1.18×10^6 N/m	1.16×10^6 N/m
k(4,5)	1×10^6 N/m	1.04×10^6 N/m	1.24×10^6 N/m
k(6,9)	1×10^6 N/m	1.12×10^6 N/m	1.11×10^6 N/m
k(8,9)	1×10^6 N/m	9.54×10^5 N/m	9.41×10^5 N/m



(a) (b)

Figure 4. Exact (solid line) and reconstructed (dotted line) input forces.
(a) 20% error ; (b) 50% error

4. CONCLUSIONS

A method intended for the identification of excitation forces from the time domain responses and a analytical model of the mechanical system has been presented. The basic formulation of the method has been adapted to circumvent the difficulty in constructing an inverse system in the case of non-collocated inputs and outputs. The results obtained from simple numerical simulations demonstrated that the method has the potentiality of providing reasonably accurate force estimates in the presence of moderate levels of measurement noise and modeling uncertainties. The authors are currently evaluating the performance of the method when applied to experimentally tested structures.

5. ACKNOWLEDGMENTS

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