# APPLICATION OF A LOCALLY-COUPLED NUMERICAL METHOD TO BUOYANT SWIRLING FLOW IN A VERTICAL CILINDRICAL CHAMBER 

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#### Abstract

This work reports numerical results for the case of incompressible laminar heated flow with a swirl in a vertical cylindrical chamber. Computations are obtained with a point-wise block-implicit scheme. Flow governing equations are written in terms of the so-called primitive variables and are recast into a general form. The discretized momentum equations are applied to each cell face and then, together with the mass-continuity, tangential velocity and energy equations, are solved directly in each computational node. The effects of Rayleigh, Reynolds and Swirl numbers on the temperature field are discussed upon. Flow pattern and scalar residual history are reported. Further, it is expected that more advanced parallel computer architectures can benefit from the error smoothing operator here described.


Keywords: Model Furnace, Numerical Methods, Implicit Solution, Coupled Solution, Buoyancy

## 1. INTRODUCTION

Today, new technologies for efficient energy production are based on the so-called lean and low-NOx combustion. Accordingly, an ascending stream with an induced swirling motion characterizes most flow fields in such systems. Swirling induces flame stabilization allowing peak temperature reduction, ultimately reducing pollutant formation rates.

In such cases, the buoyancy term together with the centripetal and Coriollis accelerations make the system of governing equations of a high degree of coupling. Linearization of governing equations followed by the use of iterative solvers is the common route found in the literature for solving such nonlinear problems.

Segregated methods, in which one individual flow variable is relaxed while holding the others still, are known to be rather sensitive when handling strong physical coupling. For that, the so-called coupled solvers, where all dependent variables are relaxed in the same domain location, have received much attention lately.

For buoyancy-driven laminar flows, benchmark solutions for the field in a square cavity have been presented [1]. Multi-grid solution for this problem has also been published [2]. In the great majority of those works a segregated method is generally employed with the repetitive solution of a pressure or pressure-correction equation, followed by subsequent updates of the velocity and scalar fields. This strategy forms the basis of the SIMPLE family of algorithms [3]. Coupled line solvers for the temperature and velocity fields have shown improvements in computer time requirements for natural convection flows with large Rayleigh numbers [4]. The work in [4] is an indication of the advantage of coupled schemes for solving algebraic equations set with a high degree of interlinkage among the variables. Recently, the block implicit technique has also been applied to calculation of buoyant flows in a partially-coupled manner [5].

For swirling flows, most solutions found in the literature are also based on segregated relaxation procedures [6-8]. In the present context, a fully-implicit treatment is associated with the idea of simultaneously updating flow and scalar fields at each step within the error smoothing operator. To the best of the author's knowledge, in all published work, neither temperature nor tangential velocity fields, seen here as scalars, are treated in a fully implicit manner.

Following the aforementioned and based on Vanka's SGCS method [9, 10], simulated liddriven cavity fluid motion through a cylindrical tank using a block-implicit numerical scheme was presented in [11]. Later, the technique was extended to buoyancy-driven streams [12], including vertical [13] and inclined cavities [14] in addition to calculation of swirling flows in model combustor [15]. In those papers, a fully-implicit treatment for the scalar (temperature or tangential velocity) was made use of.

The objective of this paper is to further extend the technique presented in [12, 13] for temperature and in [15] for the azimuthal velocity, combining now the solution of both scalars into a single fullyimplicit numerical treatment. To this end, computations are presented for a model furnace comprising incompressible laminar flow simultaneously heated and subjected to an incoming flow with swirl. Effects of Reynolds, Rayleigh number and swirling strength on temperature patterns and convergence rates are reported.

## 2. GOVERNING EQUATIONS AND NUMERICAL METHOD

Geometry. The geometry here considered is schematically shown in Figure 1. A typical furnace combustion zone is approximated by a model consisting of a circular chamber of constant radius $R$ and height $H$. At inlet, the mixture air+fuel enters through a circular slot of clearance $r_{1}-r_{2}$. At one diameter downstream the entrance, combustion gases are able to exit through an annulus of thickness $r_{3}-r_{4}$. The temperature level is prescribed over the entire lateral wall and on the bottom and top lids, except at the exit area where a null temperature gradient is assumed to be established by the outward motion of the fluid.

Although it is recognized that the geometry of Figure 1 might be an oversimplification of state-of-the-art industrial furnaces, essential elements, namely swirl, buoyancy and recirculating zones provide a good test case for the numerical method here discussed.

Compact notation. The conservation equations for mass, momentum and energy here analyzed can be written in a compact form if the existing analogies among the processes of accumulation, transport, convection and generation/destruction of those quantities are observed. This generic equation is commonly known in the literature as the general transport equation and can be written in its conservative two-dimensional laminar for axi-symmetric cases as:

$$
\begin{equation*}
\frac{\partial}{\partial z}\left[\rho W \phi-\Gamma_{\phi} \frac{\partial \phi}{\partial z}\right]+\frac{1}{r} \frac{\partial}{\partial r}\left[r\left(\rho U \phi-\Gamma_{\phi} \frac{\partial \phi}{\partial r}\right)\right]=S_{\phi} \tag{1}
\end{equation*}
$$

In equation (1) $\phi$ can represent any quantity of vectorial or scalar nature (velocity or temperature), $\rho$ is the fluid density, $U$ and $W$ are the velocity components in the $r$ - and $z$ directions, respectively, $\Gamma_{\phi}$ is the transport coefficient for diffusion and $S_{\phi}$ is the source term.
Table 1 identifies correspondent terms for the different equations represented by (1). In both Table 1 and equation (1) gravity acts in the z-direction, $\mu$ is the fluid viscosity, $\operatorname{Pr}$ the Prandtl number, $T$ the temperature and $V$ the tangential velocity component.

Computational grid and finite-difference formulation. In this work, the set of equations for mass, momentum and energy above is differentiated by means of the widely-used control-volume approach of Patankar, 1980 [3]. The differential equations are integrated over each volume yielding a set of algebraic equations. Internodal variation for the dependent variables can be of different kind corresponding to different finite-difference formulations. In the present work, for simplicity, the Upwind Differencing Scheme is used to model convective fluxes across volume faces. However, the formulation below is presented in such way that no difficulties arise if another differencing scheme is employed.

Discretized Equations. The block-implicit arrangement below for the flow and continuity equations, as mentioned, was first presented by Vanka [9,10]. For the sake of completeness when extending it to buoyant and swirling problems, the flow equations are here also included. Integrating then the continuity equation around point ( $i j$ ) (see notation in Figure 1) following standard practices in numerical differentiation, one has [3]:

$$
\begin{equation*}
F_{i}^{1} U_{i+1 / 2, j}-F_{i}^{2} U_{i-1 / 2, j}+F_{j}^{l} W_{i, j+1 / 2}-F_{j}^{2} W_{i, j-1 / 2}=0 \tag{2}
\end{equation*}
$$

where the geometric coefficients $F$ 's make computations convenient and efficient and can be interpreted as (area of flow)/(volume of computational node). For the radial momentum equation the final form for the $U_{i-1 / 2, j}$ component contains coefficients representing influences by convection and diffusion mechanisms in addition to all sources and pressure gradient terms. For application in the numerical algorithm below, the equation can be written in such a way that [15]:

$$
\begin{equation*}
U_{i-\frac{1}{2}, j}=\hat{U}_{i-\frac{1}{2}, j}+\hat{d}_{i-\frac{1}{2}}\left[P_{i-1, j}-P_{i, j}\right]+\hat{e}_{i-\frac{1}{2}} V_{i, j} \tag{3}
\end{equation*}
$$

In (3), $P$ is the pressure and the last term on the right hand side represents the influence of $V$ on the radial velocity $U$ and entails a linearization of the centripetal acceleration [15]. All sources term, except the pressure gradient and the contribution due to the tangential velocity, are compacted in the first term on the right hand side. For the coupled treatment here presented, the explicit contribution of $V$ in the source term of $U$ is necessary, as it will be seen below.

A similar equation for the axial velocity component $W_{i, j-1 / 2}$ is given by:

$$
\begin{equation*}
W_{i, j-\frac{1}{2}}=\hat{W}_{i, j-\frac{1}{2}}+\hat{d}_{j-\frac{1}{2}}\left[P_{i, j-1}-P_{i, j}\right]+\hat{g}_{j-\frac{1}{2}} \Theta_{i, j} \tag{4}
\end{equation*}
$$

For natural convection flows oriented as in Figure 1, the non-dimensional temperature $\Theta$ appearing in (4) is defined as $\Theta=\left(T-T_{0}\right) /\left(T_{1}-T_{0}\right)$ and is based on the maximum temperature drop across the computational domain $\Delta T=\left(T_{1}-T_{0}\right)$. Accordingly, here again it is important to notice that the source term in (4) explicitly shows the contribution of $T$ (or $\Theta$ ) on $W$. For the coupled treatment here presented, this explicit arrangement is also as shown later.

Following then a similar procedure for the $\Theta$ and $V$ equations, final finite-difference equations can be assembled in the following form:

$$
\begin{align*}
& a_{i j}^{\Theta} \Theta_{i, j}=b_{i j}^{\Theta} \Theta_{i+1, j}+c_{i j}^{\Theta} \Theta_{i-1, j}+d_{i j}^{\Theta} \Theta_{i, j+1}+e_{i j}^{\Theta} \Theta_{i, j-1}  \tag{5}\\
& \quad a_{i j}^{V} V_{i, j}=b_{i j}^{V} V_{i+1, j}+c_{i j}^{V} V_{i-1, j}+d_{i j}^{V} V_{i, j+1}+e_{i j}^{V} V_{i, j-1}+f_{i j}^{V}+g_{i j}^{V} U_{i, j} \tag{6}
\end{align*}
$$

It is interesting to observe that the last term in (6) comes from discretization of the Coriollis acceleration in the $V$-equation and represents the feedback effect of the cross-flow on the tangential velocity (see Table 1). In this work, however, this term is not treated implicitly and, when solving for $V$, it is compacted in the explicitly-treated source term. The centripetal acceleration, however (see Table 1 and equation 3), is here implicitly handled. The explicit treatment is also employed when discretizing the convection terms in the $T$ and $V$ equations since no particular terms with $U^{\prime} s$ or $W^{\prime} s$ are shown in (5)-(6).

For simplicity, equations (5)-(6) can be rearranged such that

$$
\begin{equation*}
\Theta_{i, j}=\widehat{\Theta}_{i, j} ; \quad V_{i, j}=\hat{V}_{i, j} \tag{7}
\end{equation*}
$$

where,

$$
\hat{\Theta}_{i, j}=\left\{b_{i, j}^{\Theta} \Theta_{i+1, j}+c_{i, j}^{\Theta} \Theta_{i-1, j}+d_{i, j}^{\Theta} \Theta_{i, j+1}+e_{i, j}^{\Theta} \Theta_{i, j-1}\right\} / a_{i, j}^{\Theta}
$$

$$
\begin{equation*}
\hat{V}_{i, j}=\left\{b_{i, j}^{V} V_{i+1, j}+c_{i, j}^{V} V_{i-1, j}+d_{i, j}^{V} V_{i, j+1}+e_{i, j}^{V} V_{i, j-1}+f_{i, j}^{V}+g_{i, j}^{V} U_{i, j}\right\} / a_{i, j}^{V} \tag{8}
\end{equation*}
$$

Numerical Strategy. In order smooth out errors due to initial guessed fields, corrections are defined as differences between exact and not-yet-converged variables. Residuals for momentum transport at each control volume face, continuity of mass and $\phi$ equations are obtained by applying the just defined approximate values into (3)-(4)-(7). After some manipulation (details in 12,15) a system connecting the residuals and corrections can be written into matrix form as,

$$
\left[\begin{array}{ccccccc}
1 & 0 & 0 & 0 & \hat{d}_{i-\frac{1}{2}, j} & 0 & \hat{e}_{i-\frac{1}{2}}  \tag{9}\\
0 & 1 & 0 & 0 & -\hat{d}_{i-\frac{1}{2}, j} & 0 & \hat{e}_{i-\frac{1}{2}} \\
0 & 0 & 1 & 0 & \hat{d}_{i, j-\frac{1}{2}} & \hat{g}_{j-\frac{1}{2}} & 0 \\
0 & 0 & 0 & 1 & -\hat{d}_{i-\frac{1}{2}, j} & \hat{g}_{j+\frac{1}{2}} & 0 \\
-F_{i}^{1} & F_{i}^{2} & -F_{j}^{1} & F_{j}^{2} & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1
\end{array}\right]\left[\begin{array}{l}
U_{i-\frac{1}{2}, j}^{\prime} \\
U_{i+\frac{1}{2}, j}^{\prime} \\
W_{i, j-\frac{1}{2}}^{\prime} \\
W_{i, j+\frac{1}{2}}^{\prime} \\
P_{i, j}^{\prime} \\
\Theta_{i, j}^{\prime} \\
V_{i, j}^{\prime}
\end{array}\right]=\left[\begin{array}{c}
R_{i-\frac{1}{2}, j} \\
R_{i+\frac{1}{2}, j} \\
R_{i, j-\frac{1}{2}} \\
R_{i, j+\frac{1}{2}} \\
R_{i, j} \\
R_{i, j}^{\Theta} \\
R_{i, j}^{V}
\end{array}\right]
$$

where the subscript identify locations in the grid, the superscript ' distinguishes corrections and the l.h.s. represents the residue vector calculated at previous iteration.

In (9) the influence of $\Theta$ on the flow field is directly accounted for by the e-terms. Similarly, the influence of $V$ on $U$ is implicitly considered by the $g$-terms. For the radial and axial directions, the $g$ - and $e$-terms are of null value, respectively. As mentioned before, the reverse effect, or say the crossflow influence on the $\Theta$ and $V$ fields is here not treated implicitly. The solution of system (9) is then easily obtained by finding first corrections for $\Theta$ and $V$, calculating later the pressure $P$ and velocity components $U$ and $W$. Essentially, the method consists of finding the corrective values for $U, V$ and $P$, such that the balance equations are correctly satisfied.

Boundary conditions. Boundary conditions used for all velocity components were given value at the flow inlet and non-slip condition at chamber walls. For cells facing the outlet plane, overall mass-conservation balance at each computational cell was used to calculate the control-volume outgoing axial velocity (at the top lid). Initial null values were set for all velocities.

For temperature, a linear profile along the vertical direction was assumed to prevail over the lateral wall (see Figure 1). Except in the opened areas, at the bottom and at the top, the nondimensional temperature took the values +1 and -1 , respectively. Through the inlet and outlet areas, the applied boundary conditions for the temperature were $\Theta=0$ and $\partial \Theta / \partial z=0$, respectively.

Numerical implementation of boundary conditions was achieved by maintaining the constant initial values at the boundaries, where applicable, or by updating them at each iteration, as in the cases of outlet surfaces or symmetry line.

All computations below used a $18 \times 36$ single grid equally distributed in the domain of calculation. An essential characteristic of Vanka's work, the multigrid artifice, has not been used in the present work due to the relatively modest grid here analyzed. Multigrid techniques are known to perform well with mid-size to large grids, but are rather ineffective when applied to small size problems. For this reason, no multigrid or any other large grid accelerating scheme was implemented.

Computational Parameters. The same relaxation parameters $(\alpha=0.55$ for $U, W, P, V$ and $\Theta)$ were used in all calculations. The swirling strength, $S$, Reynolds number, Re, and Rayleigh number, $R a$, are defined as,

$$
\begin{equation*}
\left.S=\frac{V}{W}\right)_{i n} ; \operatorname{Re}=\frac{W_{i n} 2 R}{\mu} ; \operatorname{Ra}=\frac{\operatorname{Pr}^{2} \beta g_{z} \Delta \mathrm{~T} R^{3}}{\mu^{2}} \tag{10}
\end{equation*}
$$

These three parameters were varied in the range $1<S<10^{3}, 2<R e<10^{3}$ and $10^{2}<R a$ $<10^{5}$. The incoming axial velocity at inlet, $W_{i n}$, was such that the Reynolds number, in most of the cases run, took the value in the range 2-200. This relatively small input value for $W_{i n}$ indicates that although the flow comes inside the chamber with appreciable rotation ( $S$ up to $10^{3}$ ), it carries almost no momentum in the axial direction. This incoming velocity level was found to be consistent with the weak currents driven in a thermally-driven flow. With that, cases with balanced natural and forced convection mechanisms could be analyzed.

Partially Segregated Scheme. The algebraic equations for the velocity field were solved, in addition to the fully-coupled scheme here described, by performing outer iterations for the components $\Theta$ and $V$ while keeping $U-W-P$ from the previous iteration. A line-by-line smoothing operator, fully described elsewhere (e.g. [3]), was used to relax $\Theta$ and $V$, being the secondary flow field ( $U, W$ ) calculated by the locally-coupled method seen above. This partially segregated solution was set in such a way that the same number of sweeps throughout the scalar $(\Theta, V)$ and cross-flow fields ( $U, W, P$ ) was obtained. Since in the coupled scheme every sweep for $U-W-P$ also implies in smoothing out $\Theta-V$ errors, this procedure was found to be a reasonable way to fairly compare the two methods. In all partially segregated computations, a total of four sweeps per scalar per outer iteration was performed.

The reason for recalling this second procedure a partial rather than a full segregated scheme lies in the fact that in full segregated methods all variables, including $U, W$ and $P$, are solved independently and in sequence along the entire algorithm. In the case here presented for comparison, only $\Theta$ and $V$ are excluded from the implicit treatment implied by eqn. (9).

## 3. RESULTS AND DISCUSSION

Temperature Field. Figure 2 shows results for the temperature field when subjected to an increase in the incoming mass flow rate (increase in $R e$, see eqn. 10). The Figure indicates that the core of the flow becomes homogenized as more fluid comes into the chamber due to higher recirculating motion in the $r-z$ plane. Outlet temperatures are correctly increased when the higher axial mass flow rate sweeps hot fluid from bottom layers through the exit (see Figure 1 for geometry details). Increase of the central recirculating bubble is also clearly detected by the downward wash of isolines at the centerline.

Figure 3 shows calculations for the temperature field done with different values for Ra spanning from $10^{2}$ to $10^{4}$. Distortion of the temperature profiles also indicates strength of convective ascending currents close to the wall with corresponding downward motion at the central region. Interesting to note is the increase in temperature gradients close to the center, at the bottom lid, due to the just mentioned downward stream. When analyzing real equipment, not subjected to the imposed b.c.'s here used, steep gradients of temperature close to walls might be an indication of possible temperature raise at some particular locations. Design engineers may then use this sort of information to overcome potential material damage when performing preliminary thermal design.

Figure 4 presents the temperature pattern for different values of $S$. Interesting to note is the small effect on $T$, even though $S$ changes by such a large factor of $10^{3}$. Considering the assumed axisymmetry of the flow, a strong rotation will carry fluid tangentially, essentially through zones of equal temperature. On the other hand, an increase in $R e$ or $R a$, shown in Figures 2 and 3, substantially distort the temperature by increasing the ascending cross-flow currents. One should mention that in a real fully three-dimensional flows in industrial equipment ascending currents are quite strong, playing certainly a definite role in establishing the temperature pattern inside such domains. For the simplified flow and geometry here analyzed, however, no such effect was expected.

Residue History. Normalized residues were defined as the norm of the cell residue for mass, energy and tangential velocity equations as,

$$
\begin{equation*}
R_{a b s}=\left\{\sum_{i j}\left(R_{i j}\right)^{2} /(N \cdot M)\right\}^{\frac{1}{2}} \tag{11}
\end{equation*}
$$

where $\phi$ in (11) refers to the general transport variable as defined in Table $1, N$ and $M$ are the number of cells in the $r$ - and $z$-directions, respectively. For continuity equation, $R_{i j}$ can be seen as the difference, for every cell, between the cell outgoing mass flux, $F_{\text {out }}$, and the incoming mass flux, $F_{\text {in }}$. A relative mass residue can then be defined as,

$$
\begin{equation*}
R_{\text {rel }}=\left\{\sum_{i j}\left(\frac{F_{\text {out }}-F_{\text {in }}}{F_{\text {out }}+F_{\text {in }}}\right)^{2} /(N \cdot M)\right\}^{\frac{1}{2}} \tag{12}
\end{equation*}
$$

A discussion on the advantages in simultaneously monitoring $R_{r e l}$ in addition to $R_{a b s}$ is presented in de Lemos 1990, 1992 [11,12] and it is based on the small range of the former $(0,+1)$.

The three parameters here investigated, namely $R e, R a$ and $S$, were analyzed in terms of their influence on the overall convergence rates. Results are shown in Figures 5, 6 and 7, respectively. The iteration counter refers to the total number of sweeps over the domain, that is, the product of the outer counter times the number of inner sweeps. Here, a quick word on the numbers of iterations to convergence seems timely. Other schemes presented in the literature may indicate residue history as a function of outer iteration counters only. Some, use the so-called pseudo-transient approach and plot time steps instead. Each outer iteration, in turn, may consider a great number of internal sweeps, usually controlled by a specified residue reduction rate. Here, in this work, a fixed number of internal sweeps was considered. The relatively large number of necessary iterations seen in the figures below could be associated with the use of a single grid, the tightness of the relaxation parameters and the strong coupling among all variables involved.

Interesting to note is the better convergence performance for the higher Reynolds number (see Figure 5), possibly reflecting the fact that, as $R e$ increases, the flow becomes more forced-convection dominated decreasing the $U W-T$ coupling in relation to the $U W-P$ connection. This, in turn, facilitates the solution of the energy equation once the velocity field is calculated. This idea is supported when Figure 6 is inspected, showing worse convergence rates for a higher Ra. There, the higher degree of coupling between temperature and cross-flow fields makes computation more demanding, reflecting the increase in physical coupling among the flow variables and temperature.

On the other hand, when $S$ is varied, Figure 7 shows a weak dependency of $R_{T}$ on the swirling strength. According to Figure 4, no substantial change on temperature patterns was detected when inlet rotation increases (except for $S=10^{3}$ ). For such small inlet mass flow rates ( $R e=2$ ), viscous shear driven by the incoming swirling motion enhances the cross-flow field which, in turn, distorts isothermal lines. Such an indirect or second-order relationship between $V$ and $\boldsymbol{T}$ is apparently also reflected on the residue histories shown in Figure 7.

Finally, the Figures seems to indicate also that the cross-flow field can quickly adjust itself to changes in the scalar profiles, and that, in the segregated case, those changes are too slowly transferred back to $\Theta-V$. The coupled solution, however, quickly transmits back to $\Theta$ and $V$-equations changes in the cross-flow pattern, more realistic simulating the strong interaction among the variables involved.

## 4. CONCLUDING REMARKS

This paper detailed a fully-coupled technique for numerical prediction of ascending heated swirling flows in a cylindrical chamber. An extension of the numerical method in Vanka, 1986 [9,10] towards a fully implicit solution of the energy, tangential and cross-flow equations was reported. Outlining of the numerical method showed the necessary steps for setting up the residuals and the methodology used to calculate the corrections for all dependent variables. Comparison of partiallysegregated and fully-coupled treatments for the energy and tangential velocity shows a lower computer effort when the latter method was used. The approach herein is promising regarding
numerical stability of the entire equation set since inherent coupling among the variables is implicitly handled. Further, it is also expected that more advanced computer architectures can benefit from the point wise error smoothing operator here described.

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Table 1-Terms in the general transport

|  | $\phi$ | $\Gamma_{\phi}$ | $S_{\phi}$ |
| :---: | :---: | :---: | :---: |
| Continuity | 1 | 0 | 0 |
| Axial <br> Momentum | $W$ | $\mu$ | $-\frac{\partial \mathrm{P}}{\partial \mathrm{z}}+\rho_{0} \mathrm{~g}_{\mathrm{z}} \beta\left(\mathrm{T}-\mathrm{T}_{0}\right)$ |
| Radial <br> Momentum | $U$ | $\mu$ | $-\left(\frac{\partial \mathrm{P}}{\partial \mathrm{r}}+\frac{\mu \mathrm{U}}{\mathrm{r}^{2}}-\frac{\rho \mu \mathrm{V}^{2}}{\mathrm{r}}\right)$ |
| Azimuthal <br> Momentum | $V$ | $\mu$ | $-\left(\frac{\mu \mathrm{V}}{\mathrm{r}^{2}}+\frac{\rho \mathrm{UV}}{\mathrm{r}}\right)$ |
| Energy | $T$ | $\mu / P r$ | 0 |



Figure 1 - Vertical cylindrical chamber


Figure 2 - Effect of $R e$ on temperature field, $S=1, R a=10^{2}$.


Figure 3 - Effect of $R a$ on temperature field,

$$
R e=2, S=1 \text {. }
$$



Figure 4 - Effect of swirling strength on temperature, $R a=10^{4}, \operatorname{Re}=2$.


Figure 5 - Influence of $R e$ on convergence rate of $T$-equation.


Figure 6 - Effect of $R a$ on $R_{T}$


Figure 7 - Effect of swirling strength $S$ on $R_{\mathrm{T}}$.

