

# **ANALYSIS OF ERROR IN THE SURFACE TEMPERATURE MEASUREMENT OF SEMI-INFINITE BODY**

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## **Abstract**

The measurement of surface temperature by a thermocouple is subject to electrical, metallurgical and conduction errors. In this work the parameters which affect the conduction error were theoretically analysed considering a steady-state two-dimensional conduction through the semi-infinite body and the thermocouple wires. Firstly the equation and the boundary conditions of the mathematical model are numerically solved employing the finite element method, and a comparison was made with the analytical results provided by Jacob (1957). At the end of this paper it is analysed some way to minimise the conduction error in the surface temperature measurement.

**Key words:** Conduction error, Surface temperature measurement, Thermocouple.

## **1. INTRODUCTION**

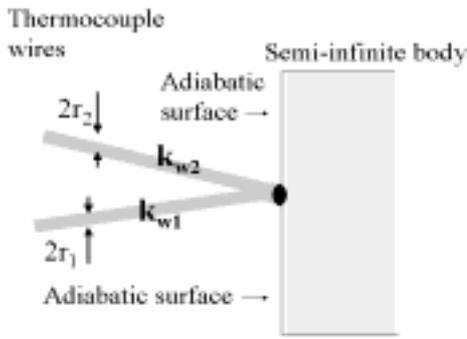
In many applications the surface temperature measurement must be very accurate as for instance in determining local heat-transfer rates like in an exhaust-gas air heater or along an airfoil surface and in experimental heat transfer. The error analysis due to the conduction is extremely important not only to estimate the error but also to find ways to reduce it. Among the causes of conduction error the most significant are: the low thermal conductivity of the body, the thermocouple diameter, the heat loss from the thermocouple to ambient, the way of attachment, the imperfect contact between the thermocouple and the surface and thermal inertia of the thermocouple. A number of methods and simplifying assumptions have been utilized either for transient or steady-state situation and it is essential to mention the study developed by Jacob (1957) whose the analytical solution is utilized in this paper.

Hennecke and Sparrow (1969) investigated the thermal processes associated with the presence of a local heat sink (or source) on the convectively cooled surface of a solid. The sink is due to the presence of a surface-mounted thermocouple, a pin fin or other surface-mounted conductors. The heat transfer results and temperature distributions for the solid are determined without reference to specific applications. The results are then applied to the case of the surface-mounted thermocouple, and the error in the measured temperature owing to the presence of the thermocouple is evaluated. Beck and Keltner (1983) developed mathematical

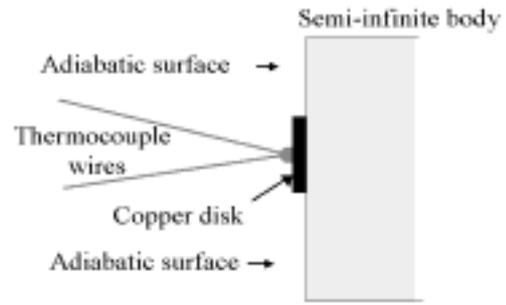
models for the response of the surface mounted thermocouples on a thick wall. These models account for the significant causes of errors in both the transient and steady-state response to changes in the wall temperature. In many cases, closed form analytical expressions are not obtainable.

In this work the error introduced by the thermocouple presence was investigated numerically considering the thermocouple as a single semi-infinite cylinder of homogeneous material and a perfect thermal contact between the thermocouple and the body is assumed, that is, without any thermal resistance. Moreover the body surface is perfectly insulated except at one small circle where the thermocouple is placed. The conduction error was evaluated in two arrangements, in the Figure 1(a) the thermocouple is attached directly to the semi-infinite body whereas in the Figure 1(b) a copper disk is placed between the surface of the body and the junction in order to reduce the conduction error.

The results presented in this work will cover either free and forced convection for the configuration presented in Figure 1(a).



**Figure 1 a.** Direct attachment of the thermocouple.



**Figure 1 b.** Thermocouple attached to the copper disk.

## 2. MATHEMATICAL STATEMENT OF THE PROBLEM

Before going into the equations or even the mathematical models it is shown the steps and assumptions taken to consider the thermocouple wire as a single long-infinite cylinder of homogeneous material with equivalent thermal properties.

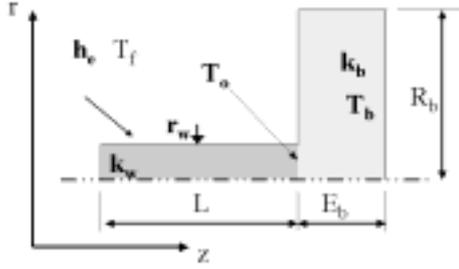
### 2.1 Thermocouple equivalent wire

Taking a careful look at the Figure 1 (a) it is possible to note that two thermocouple wires are attached on the surface of the semi-infinite body and in order to state the two-dimensional mathematical model it is necessary to make some assumptions .

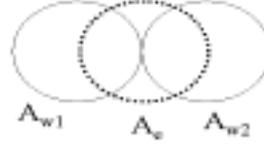
In Figure 2 the two thermocouple wires of radii  $r_1$  and  $r_2$  shown in Figure 1 (a) are replaced by one cylinder with equivalent thermal conductivity  $k_w$  and equivalent radius  $r_w$ . The temperature indicated by the thermocouple is the temperature of the junction  $T_o$  and  $T_f$  is the fluid temperature which is assumed to be uniform over the thermocouple length ( $L$ ). In addition, the body which is also shown in Figure 2 has the thickness  $E_b$  and radius  $R_b$ .

The dotted lines in Figure 3 represent the equivalent cylinder area ( $A_e$ ) whose frontal area is obtained adding the areas of the wires according to expression (1) assuming the same radius  $r_1=r_2$ , for both wires as it follows:

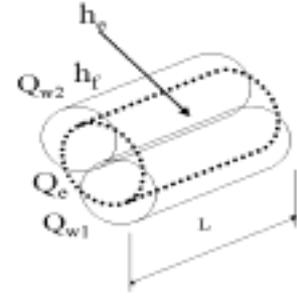
$$\pi r_1^2 + \pi r_2^2 = \pi r_w^2 \rightarrow 2r_1^2 = r_w^2 \rightarrow r_w = r_1 \sqrt{2} \quad (1)$$



**Figure 2.** Body with equivalent cylinder.



**Figure 3.** Equivalent area.



**Figure 4.** Equivalent heat loss.

Once the equivalent radius ( $r_w$ ) is obtained from equation (1), the equivalent convective heat transfer coefficient ( $h_e$ ) is obtained from the heat loss by the equivalent wire ( $\delta Q_e$ ) of length ( $\delta L$ ) which is the sum of the heat loss by each individual wire ( $\delta Q_{w1}$  and  $\delta Q_{w2}$ ), according to Figure 4 and it follows that:

$$2\pi r_w \delta L h_e (T_w - T_f) = 2\pi r_1 \delta L h_f (T_w - T_f) + 2\pi r_2 \delta L h_f (T_w - T_f) \quad (2)$$

where  $h_e$  is the equivalent convective heat transfer coefficient and  $h_f$  is the convective heat transfer coefficient over each wire and  $T_w$  represents the surface temperature of the thermocouple resulting on the following relation for  $h_e$ .

$$h_e = h_f \sqrt{2} \quad (3)$$

To determine the equivalent thermal conductivity ( $k_w$ ) it is necessary to get the long fin equation given by Ozisik, 1985 for the heat flow rate through the fin as:

$$Q = \theta_o \sqrt{PhkA} \quad (4)$$

Summing up the two wires heat losses ( $Q_{w1} + Q_{w2}$ ) and equalizing to the equivalent heat loss ( $Q_e$ ) gives:

$$\theta_o \sqrt{P_e h_e k_w A_e} = \theta_o \sqrt{P_1 h_f k_{w1} A_{w1}} + \theta_o \sqrt{P_2 h_f k_{w2} A_{w2}} \quad (5)$$

where:  $P_e$ ,  $P_1$ ,  $P_2$  are the perimeters of the equivalent wire and of each thermocouple wire;  $k_w$ ,  $k_{w1}$ ,  $k_{w2}$  are the equivalent thermal conductivity and thermal conductivity of each wire and  $\theta_o = (T_o - T_f)$

Then the equivalent thermal conductivity becomes:

$$\sqrt{k_w} = (\sqrt{k_{w1}} + \sqrt{k_{w2}}) / 2 \quad (6)$$

## 2.2 Energy equation

The semi-infinite body is approximated by a finite body with large dimensions and from now it is possible according to the Figure 2 express the governing equations in a cylindrical coordinate system for the two-dimensional steady-state heat conduction with constant thermal conductivity without heat generation, as presented by Ozisik (1985):

$$\frac{1}{r} \frac{\partial}{\partial r} \left( k \frac{r \partial T}{\partial r} \right) + k \frac{\partial^2 T}{\partial z^2} = 0 \quad (7)$$

where  $k$  is the thermal conductivity ( $k=k_w$  in the equivalent thermocouple wire and  $k=k_b$  in the solid region) and  $T$  the temperature in any region.

## 2.3 Boundary conditions

The boundary conditions associated to the heat conduction equation and the heat flux for the domain represented in Figure 2 are given by:

For $r=0 \rightarrow \partial T / \partial n = 0$ (8)	For $r=0 \rightarrow \partial T / \partial n = 0$ (11)
For $r=r_w \rightarrow -k_w \partial T / \partial n = h_e (T - T_f)$ (9)	For $z=L \rightarrow \partial T / \partial n = 0$ (12)
For $r=r_b \rightarrow \partial T / \partial n = 0$ (10)	For $z=L+E_b \rightarrow T=T_b$ (13)

where  $n$  represents the normal unit vector outward to the domain surface.

## 2.4 Surface temperature measurement error

The error ( $E$ ) in surface temperature measurement is defined as the difference between the temperature read by the thermocouple or even the temperature of the junction ( $T_o$ ) and the temperature of the body ( $T_b$ ) which is given by equation (14) whereas ( $T_o$ ) is numerically determined by equation (15), as it follows:

$$E = T_b - T_o \quad (14)$$

$$T_o = \frac{2}{r_w} \int_0^{r_w} T_{(r)} \cdot r dr \quad (15)$$

## 3. NUMERICAL SOLUTION

This problem was numerically solved employing a program based upon the Galerkin finite element method. This program uses a quadratic interpolation polynomial to convert continuous partial differential equations into discrete nodal equations. The program works with a triangular non structured adaptive mesh with six nodes per element. The mesh refinement is automatically processed and presented more intense refinement in regions which have large curvature, geometrically small and subjected to high temperature gradient. The algebraic equations system has been solved through the iterative conjugate-gradient method, using the incomplete Cholesky decomposition as described in Macsyma Inc. (1996).

#### 4. ANALYTICAL MODEL

The temperature measurement error in the arrangement shown in Figure 1 (a) was analysed by Jakob (1957) using one-dimensional approach in which the thermocouple was treated as a pin fin and consequently the heat flow ( $Q_e$ ) from a surface at temperature  $T_o$  into an infinitely long cylinder or wire is:

$$Q_e = \theta_o \sqrt{k_w h_e A_e P_e} = \pi \sqrt{2k_w h_e r_w^3} (T_o - T_f) \quad (16)$$

This amount of heat will be carried away from the semi-infinite body according to the equation given by Groeber (1921):

$$q_0 = 4r_w k_b (T_b - T_o) \quad (17)$$

Comparing equations (16) e (17) gives the final equation for the error determination :

$$E = T_b - T_o = \frac{\pi \sqrt{\frac{k_w h_e r_w}{8k_b^2}} (T_b - T_f)}{\left[ 1 + \pi \sqrt{\frac{k_w h_e r_w}{8k_b^2}} \right]} \quad (18)$$

##### 4.1 Free convection

The Nusselt number depends on Grashof and Prandtl numbers then according to Jakob (1957), when the product of Grashof number ( $Gr$ ) and Prandtl number ( $Pr$ ) is less than  $10^{-5}$  the Nusselt number approaches to a constant value equals to 0.4 as it follows :

$$h_e (2r_w) / k_g \approx 0.4 \quad (19)$$

where  $k_g$  is the fluid thermal conductivity :

Substituting equation (19) in equation (18) results in :

$$E = \pi \sqrt{\frac{k_w k_g}{k_b^2 40}} (T_b - T_f) / \left[ 1 + \pi \sqrt{\frac{k_w k_g}{k_b^2 40}} \right] \quad (20)$$

It is essential to pay closer attention for the natural convection in this specific range of ( $Gr Pr$ ) because the error estimated does not depend on the wire diameter but in the dimensionless group  $k_w k_g / k_b^2$ , so that the error can not be reduced by employing thinner thermocouple wires.

##### 4.2 Forced convection

For forced convection it will be extremely important to know the Reynolds number and the range in which the flow will be studied. It is observed that the term inside the square root

$k_w h_e r_w / k_b^2$  of equation (18) can be correlated with dimensionless parameters of convective heat transfer as Reynolds, Nusselt and Prandtl. According to Ozisik, (1985) it is possible to correlate the Nusselt with the Reynolds number, as it follows:

$$Nu = h_e 2r_w / k_g = (0.4 Re^{0.5} + 0.06 Re^{2/3}) Pr^{0.4} \quad (21)$$

This correlation agrees with the experimental data within  $\pm 25\%$  in the range of variables  $40 < Re < 10^5$  and  $67 < Pr < 300$ , where:  $Re = \text{Reynolds number} = u_\infty 2r_0 / \nu$ ;  $u_\infty =$  fluid free stream velocity;  $\nu =$  cinematic viscosity.

Substituting equation (21) in the term  $k_w h_e r_w / k_b^2$  of equation (18) it results that the surface temperature measurement error depends on the  $k_w k_g / k_b^2$  parameter, the Reynolds number and the Prandtl number. Analysing the equations it is possible to note that unlike the free convection, in forced convection the diameter of the wire does interfere in the estimated error value of the surface temperature measurement.

## 5. RESULTS

In this work, the following numerical values are employed:  $T_f = 273.15$  K,  $T_b = 373.15$  K,  $k_w = 381$  W/m.K,  $r_w = 5.08 \cdot 10^{-4}$  m,  $k_g = 0.0277$  W/m.K (air),  $P_r = 0.7$ . Concerning the body properties two kinds of materials were analysed such as concrete and diatomaceous earth with the thermal conductivities  $k_b = 0.81$  W/m.K and  $k_b = 0.086$  W/m.K respectively.

### 5.1 Free and forced convection

The numerical results are based on geometry and boundary conditions of finite body. When the body dimensions are not much larger than the thermocouple radius ( $r_w$ ), there is a difference between the results from this two-dimensional numerical analysis of this work with the semi-infinite body one-dimensional analytical data of Jakob (1957).

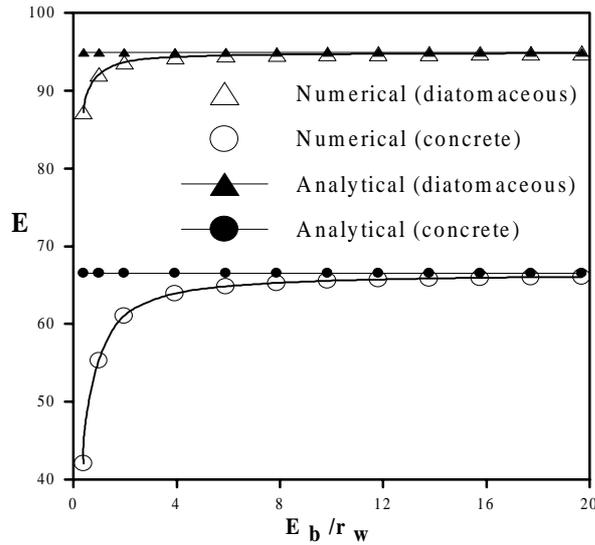
Figure 5 shows the influence of the body thickness in the error on surface temperature measurement and it is observed that for body thickness less than  $E_b/r_w = 8$  the disturbance caused by the thermocouple is more pronounced affecting the temperature distribution on the other side of the body as shown in Figure 6 (a) for  $E_b/r_w = 2$ . Figure 6 (b) shows that the thermocouple attachment do not disturb the temperature near the other surface in the case  $E_b/r_w = 50$ . When  $E_b/r_w > 8$  (diatomaceous earth) and  $E_b/r_w > 16$  (concrete) the finite body behaves as a semi-infinite body according to Figure 5.

Figure 7 also shows that when the body radius  $R_b$  is small, there is an increase in error due to a disturbance in the isothermal lines as a result of the finite body dimensions.

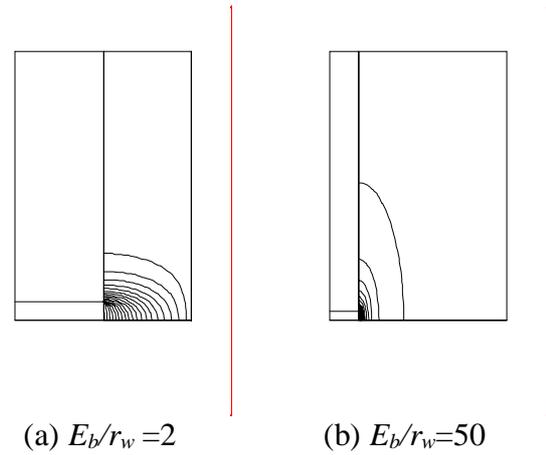
Figure 8 indicates that when the thermocouple length is really small the two-dimensional results are very different from the analytical ones because of the two-dimensional heat conduction near the thermocouple root .

For forced convection, keeping the term  $k_w k_g / k_b^2$  constant and varying Reynolds and consequently the Nusselt number it is verified in Figure 9 that the higher the Reynolds number the higher is the convective heat transfer ( $h_e$ ) then raising the error on the surface temperature measurement .

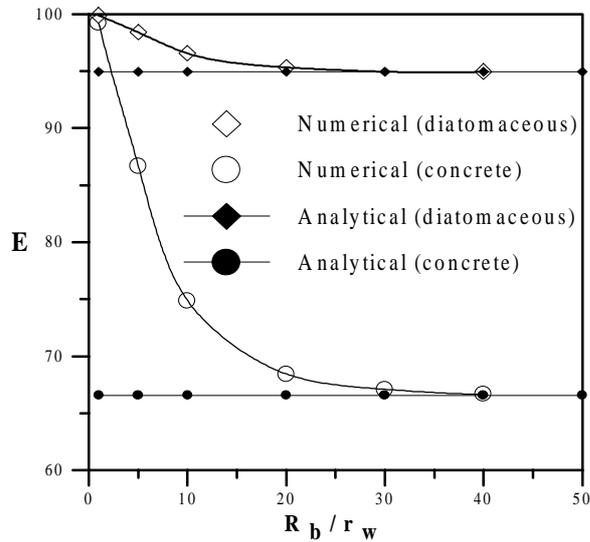
Moreover, the numerical results for ( $E_b/r_w = 50$ ,  $R_b = 200 r_w$ ,  $L = 750 r_w$ ) behaves in the same way as the analytical ones indicating a small difference for high Reynolds numbers.



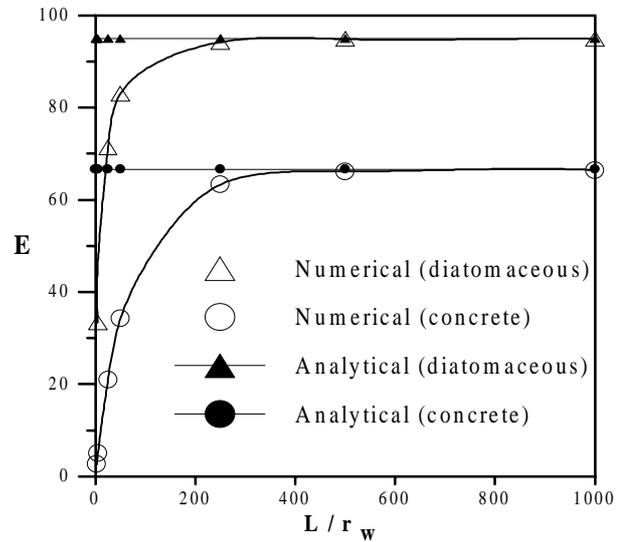
**Figure 5.** Body thickness analysis ( $L=750 r_w$  and  $R_b=200 r_w$ ).



**Figure 6.** Isothermal lines for short (a) and large (b) body thickness ( $L=750 r_w$  and  $R_b=200 r_w$ ).



**Figure 7.** Body radius analysis ( $L=750 r_w$  and  $E_b/r_w=50$ ).

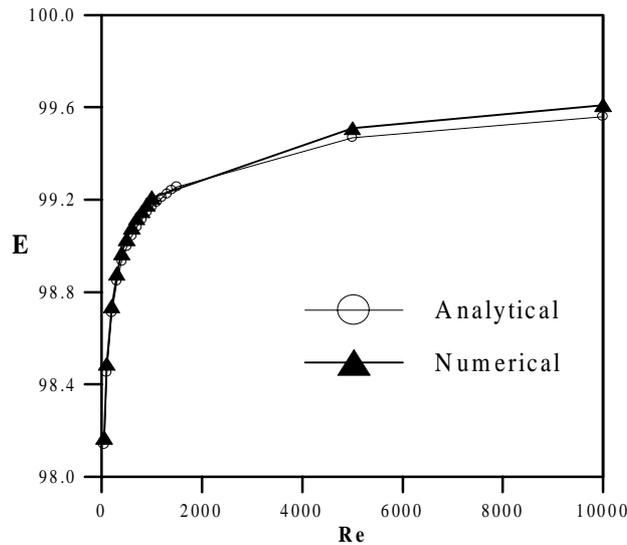


**Figure 8.** Thermocouple length ( $E_b/r_w=50$  and  $R_b=200 r_w$ ).

## 6. REDUCING THE ERROR IN SURFACE TEMPERATURE MEASUREMENT

Among many ways to reduce the error in surface temperature measurement it is important to mention the result obtained when the arrangement of Figure 1 (b) is used, that is, a copper disk is placed between the thermocouple tip and the body surface. Table 1 shows a comparison of the numerical results employing the arrangement in Figure 1 (a) and (b). These results were obtained using the following numerical values:  $k_w=51.92$  W/mK,  $k_g=0.0276$  W/mK,  $r_w=5.08 \cdot 10^{-4}$  m,  $E_b/r_w=50$ ,  $L=750 r_w$ ,  $R_b=200 r_w$ ,  $T_b=373.15$  K,  $T_f=273.15$  K. This analysis were taken for three dissimilar bodies materials such as copper, steel and diatomaceous earth with their respective thermal conductivity equal to: 346.1; 51.92 and 0.086. Furthermore the copper disk placed between the thermocouple and the body has the following properties and dimensions: thermal conductivity  $k=346.1$ , radius  $r=50 r_w$  and thickness  $8 \cdot 10^{-4}$  m. According to Table 1 it is noticed that the lower the thermal conductivity of the body material whose temperature is to be measured the higher is the error in the

measurement. When the body conductivity is high the effect of the copper disk is small, but in the case of diatomaceous earth there is a sharp reduction in the temperature measurement.



**Figure 9.** Reynolds number variation

**Table 1.** Error from the numerical analysis ( $T_b - T_o$ ).

Material	Thermal conductivity (W/mK)	Figure 1 (a)	Figure 1 (b)
Copper	346	0.170	0.170
Steel	51.92	1.107	0.254
Diatomaceous earth	0.086	87.350	8.249

## 7. CONCLUSIONS

The results presented show that the analytical approach is applied for limiting conditions as for instance for body with large dimensions. On the other hand the numerical approach indicates which dimensions both approaches are compatible and shows that for bodies with really small dimensions the errors are larger than the values from the one-dimensional analytical approach.

## 8. ACKNOWLEDGEMENTS

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## 9. REFERENCES

- Beck, J.V. and Keltner, N. R., 1983, "Surface Temperature Measurement Errors", Transactions of the ASME, vol. 105, pp. 312-318.
- Groeber, H., 1921, "Die Grundgesetze der Wärmeleitung und des Wärmeüberganges", Julius Springer, Berlin.
- Hennecke, D.K. and Sparrow, E.M., 1969, "Local Heat Sink on a Convectively Cooled Surface-Application to Temperature Measurement Error", Journal of Heat and Mass Transfer, vol. 13, pp. 287-304.
- Jakob, M., 1957, "Heat Transfer", vol. II, John Wiley & Sons.
- Macsyma Inc., 1996, "PDEase2D™ 3.0 Reference Manual", 3<sup>rd</sup> Ed..
- Ozisik, M.N., 1985, "Heat Transfer A Basic Approach", McGraw-Hill.