LUMPED-DIFFERENTIAL FORMULATIONS IN HYPERBOLIC HEAT CONDUCTION

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ABSTRACT

The present work aims at applying of the ideas on the so-called Coupled Integral Equations Approach (CIEA) to one-dimensional thermal wave propagation problem in a finite solid medium, offering an improved lumped-differential formulations. The non-stationary heat conduction problem is studied by assuming values of the thermal relaxation time for the solid medium, and boundary conditions of the prescribed heat flux and convection heat transfer are used to illustrate the powerful of this approach. The employment of the CIEA in the hyperbolic heat conduction equation results in a system of two or three ordinary differential equations for the average temperature, surface temperature and a combination of the surface temperature with time surface temperature derivative, respectively. The Runge-Kuta methods, from DIVPRK routine of the IMSL (1987), is used to obtain results for the average temperature in the medium as function of the thermal relaxation time and of the boundary conditions adopted.

Keywords: Lumped analysis, Improved Differential Formulations, Hyperbolic heat conduction.

1. - INTRODUCTION

The classical theory of heat conduction is based on Fourier's law: $\mathbf{q} = -k\nabla\mathbf{T}$. This constitutive equation relates the heat flux to the temperature gradient. In accord to this law, heat propagates with an infinite speed within a conducting medium. In despite such an unacceptable physical mechanism notion of energy transport in solids, Fourier's Law is accurate in describing heat conduction in most engineering situations encountered in daily life. To circumvent the known deficiencies of Fourier's law to describe of problems involving a high rate of temperature change, the concept of heat transmission by waves has been introduced (Cattaneo, 1958; Joseph and Prezioasi, 1989, 1990; Özisik and Tzou, 1994; Kronberg et al., 1998). However, there are practical situations in which the effects of the finite speed on heat propagation becomes important. For such situations, a constitutive equation which allows a time lag between the heat flux vector and the temperatures gradients is given by:

$$\mathbf{q} + \tau_r \frac{\partial \mathbf{q}}{\partial t} = -k\nabla\mathbf{T} \qquad (1)$$

where τ is the relaxation time, an intrinsic property of medium. This equation, combined with the energy equation

$$\rho c_p \frac{\partial T}{\partial t} = -\nabla \mathbf{q} \tag{2}$$

gives a hyperbolic heat conduction equation, which assuming constant physical properties as follow:

$$\tau_r \frac{\partial^2 \mathbf{T}}{\partial t^2} + \frac{\partial \mathbf{T}}{\partial t} = \alpha \nabla^2 \mathbf{T} \tag{3}$$

this equation describes the heat propagation in solids with a finite speed $\nu = (\alpha / \tau_r)^{1/2}$, where α is the thermal diffusivity.

The hyperbolic heat conduction equation has also been applied for heat transfer in materials with non-homogeneous inner structures, such as suspensions, pastes and meat products (Brazhnikov et al. 1975; Kaminski's, 1990). The values of the relaxation time τ_r for homogeneous materials have been showed by Sieniutycz (1977) to range from 10^{-8} to 10^{-10} sec for gases, and from 10^{-10} to 10^{-12} sec for liquids and dielectrics solids. For materials with non-homogeneous inner structure, Brazhnikov et al. (1975) report τ_r = 20 to 30 sec for meat products and experimental results of Kaminski's (1990) show τ_r = 20 sec for sand and τ_r = 29 sec for $NaCCO_3$. According to Barletta (1995) in most experimental studies relating the phenomenon of finite-speed propagation of thermal signals, often called *second sound,* have been performed at low temperatures. For instance, this phenomenon has been observed in NaF at $\approx$ 10 K (Jackson and Walker, 1971), while it has been shown that the speed of second sound in Bismute at $\approx$ 3.4 K is 780 m s^{-1} (Narayanamurti and Dynes, 1972). Further experimental validations of the hyperbolic heat conduction equation could be based on the comparison between solutions of the equation and measurements of the temperature field performed by suitable experimental apparatuses. Then, values of the thermal relaxation time could be obtained using parameter estimation method. In this context, Orlande and Ozisik (1994) have developed an inverse analysis for simultaneous estimation of thermal diffusivity and relaxation time associated with a hyperbolic heat conduction equation, by using simulated temperature recordings taken in a semi-infinite medium subjected to a heat flux boundary condition.

In the literature, many others works, experimental and theoretical, have been carried in years. For example, Bartella and Zanchini (1996) developed an analysis of the compatibility of Cattaneo-Vernotte's constitutive equation for the heat flux density vector with the hypothesis of local thermodynamic equilibrium, this compatibility is checked out by determining the entropy production rate per unit volume. Antaki (1997) have obtained a solution for a semi-infinite slab with surface convection for the cases of heating and cooling of a slab. Liao (1997) has applied the general boundary element method to solve 2D unsteady non-linear heat transfer problems of inhomogeneous materials, governed by the so-called hyperbolic heat conduction equation.

The solution of multidimensional heat conduction problems presents difficulties associated with a marked analytical involvement and also requires a considerable computational effort. Considering these facts, it becomes of interest engineering practice, to propose simpler formulations of the original partial differential systems, through a reduction of the number of independent variables in the multidimensional problems, by integration of the original partial differential system in one or more space variables, but retaining some information in the direction, whereas integration was performed, provided by the related boundary conditions (Cotta and Mikhailov, 1997; Correa and Cotta, 1999). Different levels of approximation in such mixed lumped-differential formulations can be used, starting from the plain and classical lumped system analysis, towards improved formulations obtained through Hermite-type approximations for integrals (Hermite, 1878). In this work, the so-called coupled integral equations approach (CIEA) (Cotta and Mikhailov, 1997; Correa and Cotta, 1999) is employed to improve lumped-differential formulations in a problem of the hyperbolic heat conduction by considering a slab subject to the boundary conditions of prescribed heat flux and convection heat transfer in the boundaries.

In the present work, we consider the following three approximations for integrals (Hermite, 1878):

$H_{0,0}$ Approximation (Trapezoidal Rule)

$$\int_0^h f(x)dx \cong \frac{h}{2}\big(f(0)+f(h)\big) \tag{4.a}$$

$H_{1,1}$ Approximation (Corrected Trapezoidal Rule)

$$\int_0^h f(x)dx \cong \frac{h}{2}\big(f(0)+f(h)\big)+\frac{h^2}{12}\big(f'(0)-f'(h)\big) \tag{4.b}$$

$H_{0,1}$ Approximation

$$\int_0^h f(x)dx \cong \frac{h}{3}\big(f(0)+2f(h)\big)-\frac{h^2}{6}f'(h) \tag{4.c}$$

2. - ANALYSIS

For illustration of the proposed approach, we consider a hyperbolic heat conduction in a slab of thickness L, initially at the uniform temperature T_0, subject to a prescribed heat flux at boundary x = 0 and dissipating heat by convection from the boundary surface at x = L into a fluid maintained at a constant temperature, T_∞., and with a heat transfer coefficient h. Assuming constant thermophysical properties k and α, and no internal generation, this transient formulation in dimensionless form is written as:

$$\frac{\partial\theta}{\partial\tau}+\tilde{\tau}_r\frac{\partial^2\theta}{\partial\theta^2}=\frac{\partial^2\theta}{\partial\eta^2}, \qquad 0<\eta<1, \quad \tau>0 \tag{5.a}$$

$$\theta(\eta,\tau=0)=1, \qquad \frac{\partial\theta(\eta,0)}{\partial\tau}=0, \qquad 0\le\eta\le 1 \tag{5.b, c}$$

$$\frac{\partial(\eta=0,\tau)}{\partial\eta}=-F(\tau), \qquad \tau>0 \tag{5.d}$$

$$\frac{\partial\theta}{\partial\eta}+Bi\theta+Bi\tilde{\tau}_r\frac{\partial\theta}{\partial\tau}=0, \qquad \eta=1, \quad \tau>0 \tag{5.e}$$

where the various dimensionless groups are given by:

$$\theta=\frac{T-T_\infty}{T_0-T_\infty}; \quad \eta=\frac{x}{L}; \quad \tau=\frac{\alpha t}{L^2}; \quad Bi=\frac{hL}{k}; \quad \tilde{\tau}_r=\frac{\alpha\tau_r}{L^2}; \quad Q(\tau)=\frac{Lq(t)}{k(T_0-T_\infty)} \tag{6.a-f}$$

and the function F in right side of the eq. (5.d) is

$$F(\tau)=Q(\tau)+\tilde{\tau}_r Q'(\tau) \tag{7}$$

2.1 – The Classical Lumped System Analysis

We seek now for a simplified formulation for the partial differential system (5), through elimination of the spatial dependence, i.e., by integration out the independent variable η over the domain $0 \leq \eta \leq 1$, in eq. (5.a), so that a system of ordinary differential equations is obtained for the average and surface temperatures, $\bar{\theta}(\tau)$ and $\theta(1,\tau) \equiv \theta_1(\tau)$, respectively. From the definition of $\bar{\theta}(\tau)$, given bellow:

$$\bar{\theta}(\tau) = \int_0^1 \theta(\eta,\tau)d\eta \tag{8}$$

eq. (5.a) is operated on with $\int_0^1 d\eta$, to yield, after, invoking the boundary conditions (5.d,e):

$$\tilde{\tau}_r \frac{d^2\bar{\theta}(\tau)}{d\tau^2} + \frac{d\bar{\theta}(\tau)}{d\tau} + \mathrm{Bi}G(\tau) = F(\tau), \qquad \tau > 0 \tag{9.a}$$

$$\bar{\theta}(0) = 1; \qquad \frac{d\bar{\theta}(0)}{d\tau} = 0 \tag{9.b,c}$$

where

$$G(\tau) \equiv \theta_1(\tau) + \bar{\tau}_r \frac{d\theta_1(\tau)}{d\tau} \tag{10}$$

Making the usual assumption in the classical lumped system analysis that the surface temperature at $\eta = 1$ is essential equal to the average value, or

$$\theta_1(\tau) \cong \bar{\theta}(\tau) \tag{11}$$

which provides the approximate classical formulation

$$\tilde{\tau}_r \frac{d^2\bar{\theta}(\tau)}{d\tau^2} + \left(1 + \mathrm{Bi}\tilde{\tau}_r\right)\frac{d\bar{\theta}(\tau)}{d\tau} + \mathrm{Bi}\bar{\theta}(\tau) = F(\tau)\,; \qquad \tau > 0 \tag{12.a}$$

$$\bar{\theta}(0) = 1; \qquad \frac{d\bar{\theta}(0)}{d\tau} = 0 \tag{12.b,c}$$

Eqs. (12. b, c) were obtained by the same process, i.e., eqs. (5. b, c) were also operated with $\int_0^1 d\eta$ operator. The approximation in eq. (11) imposes very strict applicability limits, reflected in terms of the Biot number value. As a rule of thumb the classical lumping approach is in general restricted to problems with Bi < 0.1.

2.2 – Improved Lumped-Differential Formulations

Now, the objective is to retain more information about the physical phenomenon in the direction to be eliminated through the application of lumping procedures. Thus, the basic idea behind the improved approach is finding a relation

$$\theta_1(\tau) = f\left[\bar{\theta}(\tau)\right] \ \text{ or } \ \frac{d\theta_1(\tau)}{d\tau} = f\left[\bar{\theta}(\tau), \theta_1(\tau)\right] \tag{13}$$

developed after applying the Hermite approximations, $H_{\alpha,\beta}$, given by eqs. (4), on the integrals that define the average temperature and heat flux in the spatial coordinate to be eliminated. Depending on the problem formulation, different levels of approximation can be achieved with increasing analytical involvement.

$H_{0,0}$ / $H_{0,0}$ Approximation

Considering the $H_{0,0}$ - approximation, eq. (4.a), one finds the approximate relations bellow for the auxiliary averaged temperature and heat flux, respectively:

$$\bar{\theta}(\tau) = \int_0^1 \theta(\eta,\tau)d\eta \cong \theta(1,\tau) - \theta(0,\tau) \quad ; \quad \int_0^1 \frac{\partial\theta(\eta,\tau)}{\partial\eta}d\eta \cong \frac{1}{2}\left[\frac{\partial\theta(1,\tau)}{\partial\eta} - \frac{\partial\theta(0,\tau)}{\partial\eta}\right] \tag{14.a, b}$$

making use of the boundary conditions (5.d, e) and $\theta(1,\tau) \equiv \theta_1(\tau)$, and after substituting in eq. (9.a), we obtain the following ordinary differential system for the average and surface temperatures, respectively.

$$\tilde{\tau}_r \frac{d^2\bar{\theta}(\tau)}{d\tau^2} + \frac{d\bar{\theta}(\tau)}{d\tau} = F(\tau) - BiG(\tau), \qquad \tau > 0 \tag{15.a}$$

$$Bi\tilde{\tau}_r \frac{d\theta_1(\tau)}{d\tau} + (4+Bi)\theta_1(\tau) = 4\bar{\theta}(\tau) - F(\tau), \qquad \tau > 0 \tag{15.b}$$

$$\bar{\theta}(0) = 1; \quad \theta_1(0) = 1, \quad \frac{d\bar{\theta}(0)}{d\tau} = 0 \tag{15.c, d}$$

where $G(\tau)$ is defined by eq. (10).

$H_{0,0}$ / $H_{1,1}$ Approximation

In this formulation, the heat flux is approximated through the corrected trapezoidal rule, eq. (4.b), taking into account the temperature derivatives at the boundaries

$$\int_0^1 \frac{\partial\theta(\eta,\tau)}{\partial\eta}d\eta \cong \frac{1}{2}\left[\frac{\partial\theta(1,\tau)}{\partial\eta} + \frac{\partial\theta(0,\tau)}{\partial\eta}\right] + \frac{1}{12}\left[\frac{\partial^2\theta(0,\tau)}{\partial\eta^2} - \frac{\partial^2\theta(1,\tau)}{\partial\eta^2}\right] \tag{16}$$

using the PDE (5.a) in the limit when $\eta \to 0$ and $\eta \to 1$, respectively, to obtain $\partial^2\theta/\partial\eta^2$ at the boundaries and by making use of eq. (13.a) together with boundary conditions (5.d,e), we obtain the following equations for $\theta_1(\tau)$ beyond of eq. (9.a) for $\bar{\theta}(\tau)$:

$$\tilde{\tau}_r \frac{d^2\theta_1(\tau)}{d\tau^2} + (1 + 4Bi\tilde{\tau}_r)\frac{d\theta_1(\tau)}{d\tau} + (12+4Bi)\theta_1(\tau) = 12\bar{\theta}(\tau) - 2F(\tau), \quad \tau > 0 \tag{17.a}$$

$$\bar{\theta}(0) = 1; \quad \frac{d\bar{\theta}(0)}{d\tau} = 0; \quad \theta_1(0) = 1; \quad \frac{d\theta_1(0)}{d\tau} = 0 \tag{17.b-e}$$

$H_{1,1}$ / $H_{0,0}$ Approximation

In this case, the average temperature is approximated through the corrected trapezoidal rule, eq. (4.b), to yield:

$$\int_0^1 \theta(\eta,\tau)d\eta \cong \frac{1}{2}[\theta(1,\tau) + \theta(0,\tau)] + \frac{1}{12}\left[\frac{\partial\theta(0,\tau)}{\partial\eta} - \frac{\partial\theta(1,\tau)}{\partial\eta}\right] \tag{18}$$

and by making use of eq. (14.b) together with boundary conditions (5.d,e), we obtain the following additional equation for θ_1:

$$Bi\tilde{\tau}_r \frac{d\theta_1(\tau)}{d\tau} + (3+4Bi)\theta_1(\tau) = 3\bar{\theta}(\tau) - \frac{F(\tau)}{2}, \qquad \tau > 0 \tag{19.a}$$

$$\bar{\theta}(0)=1; \quad \frac{d\bar{\theta}(0)}{d\tau}=0; \quad \theta_1(0)=1, \tag{19.b-d}$$

$H_{1,1}$ / $H_{1,1}$ Approximation

Now, the average temperature and heat flux are approximated through the corrected trapezoidal rule, eq. (4.b), and given by eq. (16) for the heat flux, and eq.(18) for average temperature. Combining these two equations with boundary conditions, we obtain the following additional equations for $\theta_1(\tau)$ e $G(\tau)$:

$$\tilde{\tau}_r \frac{d^2G(\tau)}{d\tau^2}+13\frac{dG(\tau)}{d\tau}=-36\big(F(\tau)+BiG(\tau)\big)-144\big(\theta_1(\tau)-\bar{\theta}(\tau)\big)+ \\ -\left(\frac{dF(\tau)}{d\tau}+\bar{\tau}_r\frac{d^2F(\tau)}{d\tau^2}\right), \quad \tau>0 \tag{20.a}$$

$$\tilde{\tau}_r \frac{d\theta_1(\tau)}{d\tau}+\theta_1(\tau)+=G(\tau), \quad \tau>0 \tag{20.b}$$

$$\bar{\theta}(0)=1; \quad \frac{d\bar{\theta}(0)}{d\tau}=0; \quad \theta_1(0)=1; \quad G(0)=1; \quad \frac{dG(0)}{d\tau}=0 \tag{20.c-g}$$

$H_{0,1}$ / $H_{0,0}$ Approximation

In this case, the average temperature is approximated through eq. (4.c):

$$\int_0^1 \theta(\eta,\tau)d\eta \cong \frac{1}{3}\big[2\theta(1,\tau)+\theta(0,\tau)\big]-\frac{1}{6}\frac{\partial\theta(1,\tau)}{\partial\eta} \tag{21}$$

by making use of eq. (14.b) and of boundary conditions (5.d,e), we obtain the following additional equation for $\theta_1(\tau)$

$$2Bi\tilde{\tau}_r \frac{d\theta_1(\tau)}{d\tau}+(6+2Bi)\theta_1(\tau)=6\bar{\theta}(\tau)-F(\tau), \quad \tau>0 \tag{22.a}$$

$$\bar{\theta}(0)=1; \quad \frac{d\bar{\theta}(0)}{d\tau}=0; \quad \theta_1(0)=1 \tag{22.b-d}$$

$H_{0,1}$ / $H_{1,1}$ Approximation

Applying the approximation given by eq. (4.c) in average temperature (see eq. (21)), and by making use of boundary conditions, the following additional equations for $\theta_1(\tau)$ e $G(\tau)$, are obtained:

$$Bi\tilde{\tau}_r \frac{d^2G(\tau)}{d\tau^2}+\big(6+Bi\big)\frac{dG(\tau)}{d\tau}+30BiG(\tau)=72\big(\bar{\theta}(\tau)-\theta_1(\tau)\big)-6F(\tau), \quad \tau>0 \tag{23.a}$$

$$\tilde{\tau}_r \frac{d\theta_1(\tau)}{d\tau}+\theta_1(\tau)+=G(\tau), \quad \tau>0 \tag{23.b}$$

$$\bar{\theta}(0)=1; \quad \frac{d\bar{\theta}(0)}{d\tau}=0; \quad \theta_1(0)=1; \quad G(0)=1; \quad \frac{dG(0)}{d\tau}=0 \tag{23.c-g}$$

3. - RESULTS AND DISCUSSION

Numerical results for average temperature were computed for different values of Biot number (Bi), dimensionless relaxation time ($\tilde{\tau}_r$) and several wave pulse form for $Q(\tau)$. The ordinary differential equations systems showed above are solved by routine DIVPRK from IMSL Library (IMSL LIBRARY, 1987) with a relative error target of 10^{-4} prescribed by the user, for all potential of the system.

Table 1. - Comparison of the approximate formulations, for $Q(\tau) = 0$ and $\tilde{\tau}_r < 10^{-6}$, against the exact solution for $Q(\tau) = 0$ and $\tilde{\tau}_r = 0$.

Bi = 0.1								
τ	**Classical**	$H_{0,0}/H_{0,0}$	$H_{0,0}/H_{1,1}$	$H_{1,1}/H_{0,0}$	$H_{1,1}/H_{1,1}$	$H_{0,1}/H_{0,0}$	$H_{0,1}/H_{1,1}$	**Exact**
0.01	0.9990	0.9990	0.9990	0.9990	0.9990	0.9990	0.9990	0.9990
0.10	0.9900	0.9903	0.9901	0.9904	0.9902	0.9904	0.9902	0.9902
0.50	0.9512	0.9524	0.9522	0.9528	0.9525	0.9528	0.9525	0.9526
1.00	0.9048	0.9070	0.9069	0.9078	0.9075	0.9078	0.9075	0.9076
2.00	0.8187	0.8227	0.8226	0.8240	0.8238	0.8240	0.8238	0.8239
3.00	0.7408	0.7463	0.7461	0.7480	0.7479	0.7480	0.7478	0.7479
Bi = 1.0								
0.01	0.9900	0.9920	0.9902	0.9925	0.9902	0.9925	0.9902	0.9907
0.10	0.9048	0.9231	0.9138	0.9277	0.9157	0.9277	0.9150	0.9196
0.50	0.6065	0.6703	0.6641	0.6873	0.6779	0.6873	0.6759	0.6811
1.00	0.3679	0.4493	0.4477	0.4724	0.4680	0.4724	0.4660	0.4704
2.00	0.1353	0.2019	0.2034	0.2231	0.2231	0.2231	0.2215	0.2244
3.00	0.0498	0.0907	0.0924	0.1054	0.1063	0.1054	0.1053	0.1070
Bi = 5.0								
0.01	0.9512	0.9780	0.9546	0.9814	0.9541	0.9814	0.9532	0.9640
0.10	0.6065	0.8007	0.7363	0.8290	0.7320	0.8290	0.7056	0.7814
0.50	0.0821	0.3292	0.3285	0.3916	0.3591	0.3916	0.3314	0.3852
1.00	0.0067	0.1084	0.1208	0.1533	0.1506	0.1533	0.1362	0.1625
2.00	4.55E-5	0.0117	0.0163	0.0235	0.0265	0.0235	0.0230	0.0289
3.00	3.23E-7	0.0013	0.0022	0.0036	0.0047	0.0036	0.0039	0.0051

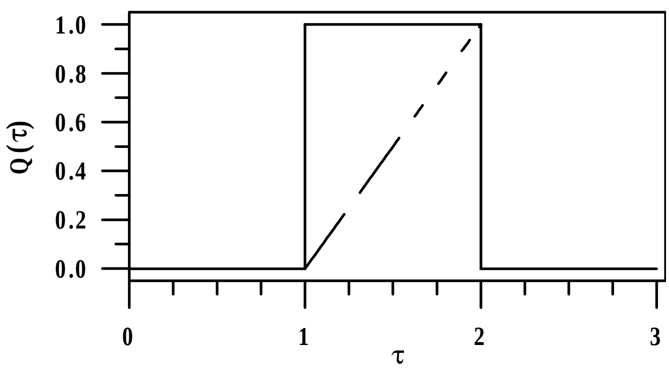

Fig. 1 - Wave pulses for $Q(\tau)$.

In order to the validate the numerical code developed here, it is showed in Table 1 a comparison of results for the average temperature obtained with improved formulations against those computed from an analytical solution for the case $Q(\tau) = 0$ and $\tilde{\tau}_r = 0$. In Table 1 it is observed that for the range of Bi < 0.1, all the approximate formulations show good agreement against exact solution, and can be accepted as sufficiently accurate for most engineering purposes. However, when

Biot number is increased, the classical approximation is already markedly inaccurate, especially for increasing values of the dimensionless time, while the $H_{1,1}/H_{0,0}$, $H_{1,1}/H_{1,1}$, $H_{0,1}/H_{0,0}$ and $H_{0,1}/H_{1,1}$ formulations are still quite reasonable.

In order to verify powerfulness of these improved formulations, two types of wave pulse for $Q(\tau)$ were studied, i. e., square and triangular wave pulses. Figure 1 shows the square and triangular wave pulses adopted here in the range $0 \leq \tau \leq 3$ for the dimensionless time. In Figure 2 and 3 it is analyzed the influence of the relaxation time and Biot number in the average temperature for $Q(\tau)$ given by a square wave pulse, respectively.

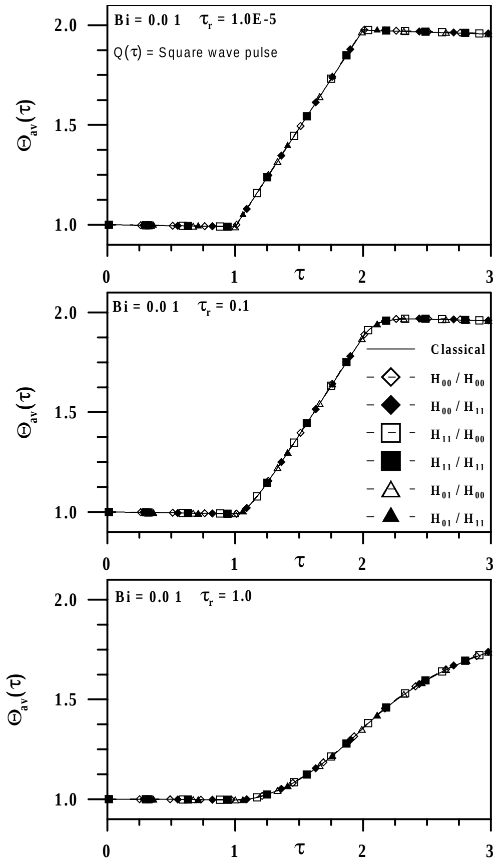

Fig. 2 - Average Temperature for Square Wave pulse and Bi = 0.01.

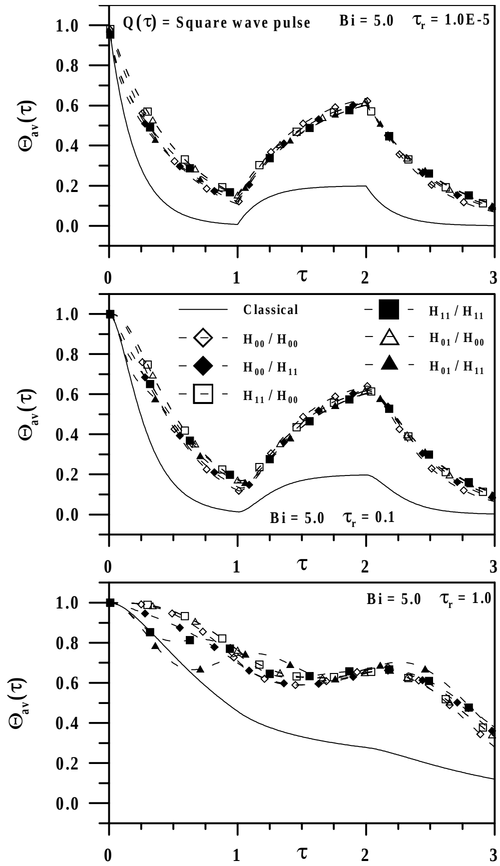

Fig. 3 - Average Temperature for Square Wave pulse and Bi = 5.

More specifically, in the Fig. 2 it is noticed that for Bi = 0.01 the Classical formulation is in good agreement with all other approximate formulations for the three dimensionless relaxation time adopted. It can also be observed that as the dimensionless relaxation time increases there is a decrease in the value of the average temperature for τ = 3. This fact can be explained by the low speed of propagation of thermal signal, and because for square wave pulse the function $F(\tau) = Q(\tau)$ due to $Q'(\tau) = 0$ (see eq. (7)). In Fig. 3 for the case of Bi = 5, a higher discrepancy among the results of the classical formulation and those from other approximations can be observed. The temperature decreases until the beginning of the perturbing wave (τ = 1), then it start to increase, due to an input of energy flux at η = 0, until the dimensionless time τ = 2. The temperature begins to decrease again after the end of perturbing wave. The influence of the perturbing wave on amplitude of the average temperature is less important for great values of the dimensionless relaxation time.

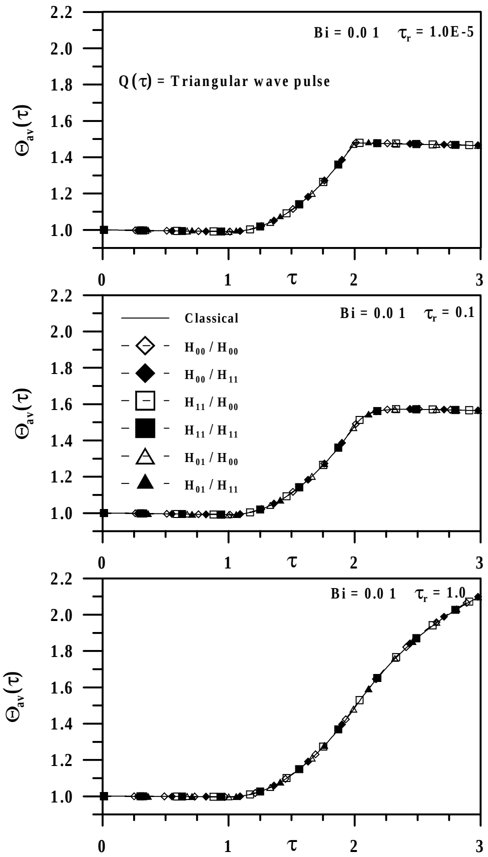

Fig. 4 - Average Temperature for Triangular Wave pulse and Bi = 0.01.

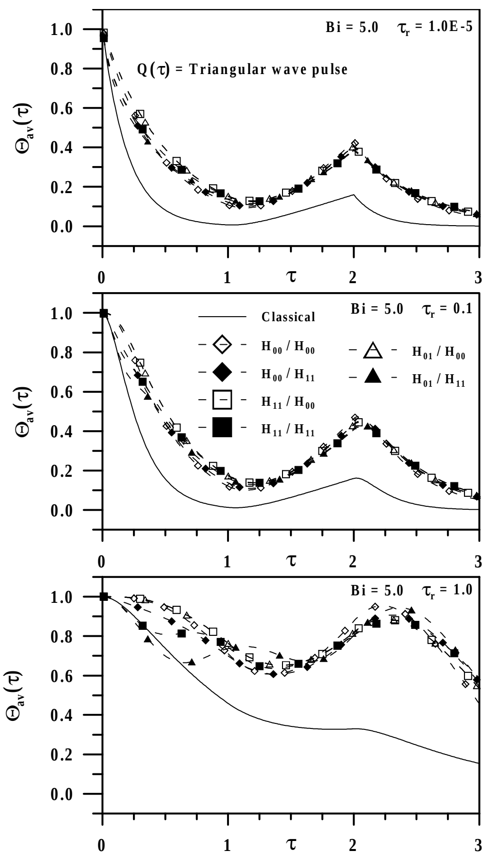

Fig. 5 - Average Temperature for Triangular Wave pulse and Bi = 5.

Similar analysis can be done for Figures 4 and 5. However, in Fig. 4, for the case of Bi =0.01 it is verified that as the dimensionless relaxation time increases the average temperature also increase. Specially at the dimensionless relation time $\bar{\tau}_r = 1$ the average temperature increases according to a rate of approximately 0.7, until to reach the dimensionless time τ = 3, because for the case of a triangular wave pulse there is a linear dependence between the function $F(\tau)$ and $\bar{\tau}_r$, i.e., $F(\tau) = Q(\tau) + \bar{\tau}_r$ due to $Q'(\tau) = 1$. For the case of Bi = 5 and $\bar{\tau}_r \leq 0.1$ (see Fig. 5) the shape of the average temperature curve suffers a direct influence of the triangular wave pulse prescribed at the boundary η = 0. For small relaxation time this effect is caused by the immediate response of the material to the perturbation on its boundary (approximately Fourie's law) and for higher dimensionless relaxation time the influence of the perturbation is dumped as can be seen in figures above.

4. - CONCLUSIONS

The hyperbolic heat conduction in a slab, subject to boundary conditions of the prescribed heat flux and convection heat transfer has been analyzed by employing the ideas in the so-called Coupled Integral Equations Approach (CIEA), offering reliable results for the average temperature in range of Biot numbers and dimensionless relaxation times analyzed. Results were computed for two different wave pulses demonstrating that the average temperature predicted with hyperbolic heat conduction equation can be significantly different form those of the Fourier equation for higher values of the relaxation time.

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