# METHOD OF RECOVERY SYSTEM VALUES OPTIMIZATION AND ITS APPLICATION 

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#### Abstract

The present work deals with the preliminary design features of a high performance ground recovery system for small orbital payloads based on parachutes with an air bag for impact attenuation. The paper shows and discusses the method for determination of system optimum area of the drag parachute, necessity of parachute reefing and optimum sea-level rate of descent which provide the minimum system mass. After this method the parachute construction is characterized by a certain parameter, which takes into account aerodynamic characteristics and main aspects of rational design, such as relative sizes, sections load, strength loss and expenses. The sequences of the system optimum values calculations for the SARA orbital platform are given. Use of this method provides the increase of the system safety and system total mass and volume decrease in comparison with the basic system.


Keywords: parachute design, system mass, optimum values.

## 1. INTRODUCTION

The orbital platform SARA will be used to perform microgravity experiments in space (Moraes, 1998), and its recovery at ground must be safe and soft, in order to protect the payload inside it from high ground impact. Systems based on parachutes with impact attenuator are the most adequate choice, due to its proven reliability and low cost. The development of such a system includes the consideration of determination of parachutes area and mass, drag and stress analysis of the parachutes, flight and impact (crash) simulation, materials selection and testing. A good concept and design leads to maximization of performance and, consequently, to weight minimization of the complete system, that is of great importance for space systems (Deweese, Schultz \& Nutt, 1978).

The design of recovery systems makes varied demands on the state-of-the art over a broad range of operational conditions and complexity of performance requirements characteristics of modern aerospace vehicles and research instruments. The recovery system technology has benefited from rapid improvement in the rigor of analytical methods made possible and practical by development of complex and flexible computer programs. While empirical data and full-scale testing are still of major importance to the design process, the facility with which system and component designs can be executed and analyzed, their performance predicted, and test data reduced and evaluated has both speeded the design process and improved depth and quality of results. A number of different parachute design programs are now developed in great detail (Peterson, 1990). When the input of new basic system parameters, comprehensive series of similar designs can be compared with respect to performance, weight, and drag efficiency for a single application.

Development of a design procedure starts with essential design criteria, sizing, staging, performance, opening loads analysis, strength of materials and parachute mass, packing volume, and choice of methods for parameters optimization. Taking system mass as the design criteria, the optimum values of the system parameters are determined by means of equalization to zero of partial derivative of system mass function with respect to the parameter which is optimized.

## 2. RECOVERY SYSTEM MASS DETERMINATION

On the whole the recovery system mass can be presented as the sum of its main parts.

### 2.1 Parachute framework mass

The parachute framework mass can be calculated as a sum of its elements in all sections:

$$
\begin{align*}
& \mathrm{m}_{\mathrm{f}}=\sum \mathrm{l}_{\mathrm{i}} \mathrm{w}_{\mathrm{i}} \mathrm{n}_{\mathrm{i}}  \tag{1}\\
& \mathrm{l}_{\mathrm{i}}=\mathrm{l}_{\mathrm{i}}^{*} \mathrm{~S}^{1 / 2}  \tag{2}\\
& \mathrm{w}_{\mathrm{i}}=\mathrm{p}_{\mathrm{i}} /\left(\mathrm{g} \mathrm{l}_{\mathrm{p}} \mathrm{i}\right): \tag{3}
\end{align*}
$$

where $l_{i} \quad$ length of parachute elements, $m$
$\mathrm{n}_{\mathrm{i}} \quad$ number of section elements
$\mathrm{w}_{\mathrm{i}} \quad$ material specific mass, $\mathrm{kg} / \mathrm{m}$
$1_{i}^{*} \quad$ element specific length, dimensionless
S parachute canopy area, $\mathrm{m}^{2}$
$\mathrm{p}_{\mathrm{i}} \quad$ section element strength, N
$\mathrm{g} \quad$ gravity acceleration, $\mathrm{m} / \mathrm{s}^{2}$
$1_{p i}$ material break length, $m$.
For Nylon cords and ribbons $\mathrm{l}_{\mathrm{pi}}=\mathrm{p}_{\mathrm{i}} /\left(\mathrm{g} \mathrm{w}_{\mathrm{i}}\right) \approx 30 \mathrm{~km}$, for Nylon cloth $\mathrm{l}_{\mathrm{pi}} \approx 20 \mathrm{~km}$ and for Kevlar materials $1_{\mathrm{pi}} \approx 58 \mathrm{~km}$ with consideration that for Kevlar elements the dynamic load is approximately in 1.5 times more than for Nylon elements (Knacke, 1992).

The number of elements $n_{i}$ can be determined from parachute section strength condition:

$$
\begin{align*}
& \alpha_{\mathrm{i}} \mathrm{p}_{\mathrm{i}} \mathrm{n}_{\mathrm{i}}=\eta_{\mathrm{i}} \mathrm{r}_{\mathrm{i}} \mathrm{f}_{\mathrm{s}} \mathrm{~F}_{\mathrm{x}}  \tag{4}\\
& \mathrm{~F}_{\mathrm{x}}=\mathrm{C}_{\mathrm{k}} \mathrm{C}_{\mathrm{D}} S \mathrm{R}_{\mathrm{R}} \rho \mathrm{~V}_{\mathrm{o}}^{2} / 2  \tag{5}\\
& \mathrm{R}_{\mathrm{R}}=\left(\mathrm{C}_{\mathrm{D}} S\right)_{\mathrm{R}} /\left(\mathrm{C}_{\mathrm{D}} \mathrm{~S}\right) \tag{6}
\end{align*}
$$

where $\alpha_{i}$ loss factor due to joint, abrasion, fatigue, water, oil, temperature ( $\alpha_{i}=0.6 \div 0.9$ )
$\mathrm{f}_{\mathrm{s}} \quad$ safety factor ( $\mathrm{f}_{\mathrm{s}}=1.3 \div 2.5$ )
$\mathrm{F}_{\mathrm{x}} \quad$ maximum parachute force, N
$r_{i} \quad$ section load factor (section load ratio to parachute force), dimensionless
$\eta_{\mathrm{i}} \quad$ section strength margin, dimensionless
$\mathrm{C}_{\mathrm{k}} \quad$ parachute opening force factor, dimensionless
$C_{D} \quad$ parachute nominal drag coefficient, dimensionless
$\rho \quad$ air density, $\mathrm{kg} / \mathrm{m}^{3}$
$\mathrm{V}_{\mathrm{o}} \quad$ initial parachute velocity, $\mathrm{m} / \mathrm{s}$
$\mathrm{R}_{\mathrm{R}} \quad$ parachute reefing ratio, dimensionless.

Taking (2), (3), (4) and (5) for (1) we have, (Koldaev, 1989):

$$
\begin{align*}
& \mathrm{m}_{\mathrm{f}}^{*}=\mathrm{m}_{\mathrm{f}} / \mathrm{m}_{\mathrm{c}}=\mathrm{R}^{2}(\mathrm{~V}) \mathrm{K}(\mathrm{~S}) \mathrm{K}_{\mathrm{m}} \mathrm{R}_{\mathrm{R}}  \tag{7}\\
& \mathrm{R}(\mathrm{~V})=\mathrm{V}_{\mathrm{o}}\left(2 \mathrm{~g} \mathrm{l}_{\mathrm{p}}\right)^{-1 / 2}  \tag{8}\\
& \mathrm{~K}(\mathrm{~S})=\mathrm{C}_{\mathrm{k}} \mathrm{R}_{\mathrm{m}}  \tag{9}\\
& \mathrm{~K}_{\mathrm{m}}=\mathrm{f}_{\mathrm{S}} \mathrm{C}_{\mathrm{D}}^{-1 / 2} \Sigma \mathrm{l}_{\mathrm{i}}^{*} \mathrm{r}_{\mathrm{i}} \eta_{\mathrm{i}} /\left(\alpha_{\mathrm{i}} \mathrm{l}_{\mathrm{pi}}^{*}\right)  \tag{10}\\
& \mathrm{R}_{\mathrm{m}}=\rho\left(\mathrm{C}_{\mathrm{D}} \mathrm{~S}\right)^{3 / 2} / \mathrm{m}_{\mathrm{c}}  \tag{11}\\
& \mathrm{l}_{\mathrm{pi}}^{*}=\mathrm{l}_{\mathrm{pi}} / \mathrm{l}_{\mathrm{p}} \tag{12}
\end{align*}
$$

where $R(V)$ parachute velocity ratio, dimensionless
K(S) parachute structure parameter, dimensionless
$\mathrm{K}_{\mathrm{m}} \quad$ parachute framework mass parameter, dimensionless
$\mathrm{R}_{\mathrm{m}} \quad$ parachute mass ratio, dimensionless
$\mathrm{l}_{\mathrm{pi}}{ }^{*} \quad$ relative material break length, dimensionless
$1_{p} \quad$ basic break length $\left(l_{p}=30000 \mathrm{~m}\right)$
$\mathrm{m}_{\mathrm{c}} \quad$ capsule (platform) mass, kg .
The value $K(S)$ as approximation functions of $R_{m}$ ratio and parachute opening dynamic factor $\mathrm{C}_{\mathrm{x}}$ is shown in Table 1, (Koldaev \& Moraes, 1997). Dynamic factor $\mathrm{C}_{\mathrm{x}}$ depends on type and porosity of parachute and is determined as ratio of $\mathrm{F}_{\max }$ to medium drag force in infinite mass condition by wind tunnel test. For several parachute types $C_{x}=1.1 \div 1.5$, (Lobanov, 1975).

Table 1. Approximation of parachute structure parameter K(S)

| $\mathrm{R}_{\mathrm{m}}$ value | $\mathrm{R}_{\mathrm{m}}<10^{-2}$ | $10^{-2}<\mathrm{R}_{\mathrm{m}}<1$ | $\mathrm{R}_{\mathrm{m}}>1$ |
| :--- | :---: | :--- | :---: |
| Parachute type | Drag parachute | Intermediate parachute | Main parachute |
| Reefed | $\mathrm{C}_{\mathrm{x}} \mathrm{R}_{\mathrm{m}}$ | $\mathrm{R}_{\mathrm{m}}\left(0.12+\left(0.06-\mathrm{C}_{\mathrm{x}} / 2\right) \lg \mathrm{R}_{\mathrm{m}}\right)$ | 0.12 |
| Non - reefed | $\mathrm{C}_{\mathrm{x}} \mathrm{R}_{\mathrm{m}}$ | $\mathrm{R}_{\mathrm{m}}\left(0.25+\left(0.125-\mathrm{C}_{\mathrm{x}} / 2\right) \lg \mathrm{R}_{\mathrm{m}}\right)$ | 0.25 |
| Des reefed | $\mathrm{C}_{\mathrm{x}} \mathrm{R}_{\mathrm{m}}$ | $\mathrm{R}_{\mathrm{m}}\left(0.6+\left(0.3-\mathrm{C}_{\mathrm{x}} / 2\right) \lg \mathrm{R}_{\mathrm{m}}\right)$ | 0.60 |

The $K_{m}$ parameter from (10) means dimensionless universal parachute quality parameter that takes into account as parachute aerodynamic characteristics as main aspects of rational design and can be used to compare parachutes of different types, areas and purposes of their use. $\mathrm{K}_{\mathrm{m}}$ value for same systems is shown in Table 2. It was determined from (5) and (7) as:

$$
\begin{equation*}
\mathrm{K}_{\mathrm{m}}=\mathrm{m}_{\mathrm{f}} \mathrm{l}_{\mathrm{p}} \mathrm{~g}\left(\mathrm{C}_{\mathrm{D}} \mathrm{~S}\right)^{-1 / 2} / \mathrm{F}_{\mathrm{x}} \tag{13}
\end{equation*}
$$

Table 2. Parachutes data (Knacke, 1992) and parameters $K_{m}$ and $K_{b}$

| $\mathrm{m}_{\mathrm{c}}(\mathrm{kg})$ | $\mathrm{m}_{\mathrm{p}}(\mathrm{kg})$ | $\mathrm{m}_{\mathrm{f}}(\mathrm{kg})$ | $\mathrm{C}_{\mathrm{D}} \mathrm{S}\left(\mathrm{m}^{2}\right)$ | $\mathrm{V}_{\mathrm{o}}(\mathrm{m} / \mathrm{s})$ | $\mathrm{V}_{\mathrm{e}}(\mathrm{m} / \mathrm{s})$ | $\mathrm{C}_{\mathrm{k}}$ | $\mathrm{K}_{\mathrm{m}}$ | $\mathrm{K}_{\mathrm{b}}$ |
| :---: | :---: | :---: | :---: | :--- | :--- | :--- | :--- | :--- |
| 50 | 2.5 | 1.4 | 18 | 120 | 6.7 | 0.13 | 9.2 | 0.66 |
| 150 | 6.8 | 5.0 | 28 | 120 | 9.1 | 0.21 | 10 | 0.40 |
| 300 | 13.2 | 9.7 | 56 | 120 | 9.15 | 0.15 | 9.6 | 0.46 |
| 500 | 21.0 | 14.9 | 95 | 120 | 9.15 | 0.11 | 9.4 | 0.50 |

### 2.2 Air bag mass

The air bag casing mass we can calculate after the following formulas (Koldaev \& Moraes, 1998):

$$
\begin{align*}
& m_{a}=w_{a} S_{a}=w_{a} H_{b} 2 \pi R_{c}=w_{a} H_{b}\left(S_{c} \pi\right)^{1 / 2}  \tag{14}\\
& m_{c} V_{e}^{2} / 2+m_{c} g H_{b}=\eta_{a} G m_{c} g H_{b}  \tag{15}\\
& T_{a}=f_{s a} R_{c} P_{\max }=f_{s a}\left(S_{c} / \pi\right)^{1 / 2} G m_{c} g / S_{c}=w_{a} G l_{a} \tag{16}
\end{align*}
$$

where $\mathrm{w}_{\mathrm{a}}$ specific cloth mass, $\mathrm{kg} / \mathrm{m}^{2}$
$\mathrm{S}_{\mathrm{a}} \quad$ area of air bag casing, $\mathrm{m}^{2}$
$\mathrm{S}_{\mathrm{c}} \quad$ capsule done area, $\mathrm{m}^{2}$
$\mathrm{H}_{\mathrm{b}} \quad$ required height of air bag, $m$
$\mathrm{V}_{\mathrm{e}} \quad$ system rate of descent at impact initial moment, $\mathrm{m} / \mathrm{s}$
$\eta_{\mathrm{a}} \quad$ efficiency of air bag, dimensionless $\left(\eta_{\mathrm{a}} \approx 0.65\right.$, Knacke, 1992)
$\mathrm{T}_{\mathrm{a}} \quad$ stress tension for cylindrical air bag, $\mathrm{N} / \mathrm{m}$
G capsule maximum acceleration, $g$
$\mathrm{P}_{\text {max }} \quad$ air bag maximum pressure, Pa
$1_{a} \quad$ break length of air bag casing cloth, $m$.
Taking $\mathrm{H}_{\mathrm{b}}$ from (15) and $\mathrm{w}_{\mathrm{a}}$ from (16) for (14) we have:

$$
\begin{equation*}
\mathrm{m}_{\mathrm{a}}^{*}=\mathrm{m}_{\mathrm{a}} / \mathrm{m}_{\mathrm{c}}=\mathrm{V}_{\mathrm{e}}^{2} \mathrm{f}_{\mathrm{sa}} /\left(\mathrm{g} \mathrm{l}_{\mathrm{a}}\left(\eta_{\mathrm{a}}-1 / \mathrm{G}\right)\right) \tag{17}
\end{equation*}
$$

### 2.3 Total mass of recovery system

$$
\begin{equation*}
\mathrm{m}_{\mathrm{s}}^{*}=\left(\Sigma\left(\mathrm{R}_{\mathrm{j}}^{2}(\mathrm{~V}) \mathrm{K}_{\mathrm{j}}(\mathrm{~S}) \mathrm{K}_{\mathrm{mj}} \mathrm{R}_{\mathrm{Rj}}+\mathrm{S}_{\mathrm{j}} \mathrm{w}_{\mathrm{cj}} / \mathrm{m}_{\mathrm{c}}\right)+\mathrm{N}_{\mathrm{R}} \mathrm{~m}_{\mathrm{R}}+\mathrm{V}_{\mathrm{e}}^{2} \mathrm{f}_{\mathrm{sa}} /\left(\mathrm{g}_{\mathrm{a}}\left(\eta_{\mathrm{a}}-1 / \mathrm{G}\right)\right)\right)\left(1+\mathrm{K}_{\mathrm{b}}\right) \tag{18}
\end{equation*}
$$

where $\mathrm{m}_{\mathrm{s}}{ }^{*}$ recovery system mass ratio to capsule mass, dimensionless
j parachute number (stage)
$\mathrm{w}_{\mathrm{cj}} \quad$ specific mass of parachute canopy cloth, $\mathrm{kg} / \mathrm{m}^{2}$
$\mathrm{N}_{\mathrm{R}} \quad$ number of reefed parachutes
$m_{R} \quad$ mass of each complete set of reefing equipment, kg
$\mathrm{K}_{\mathrm{b}} \quad$ relative mass of binding and packing equipment, dimensionless.
Mass of each complete set of reefing equipment lies in the interval $m_{R}=0.1 \div 0.4 \mathrm{~kg}$ and depends on the type of pyrotechnic knife and parachute opening force.

Binding equipment includes pieces for putting the parachutes and air bag into action, risers for joining system components and other furniture. Packing equipment includes parachutes bags, swivels, system container. According to statistic data, relative total mass of binding and packing equipment to system mass lies in the value $\mathrm{K}_{\mathrm{b}}=$ $0.4 \div 1.0$ (depends on the type of system composition). Value of $\mathrm{K}_{\mathrm{b}}$ for some recovery systems is shown in the Table 2. Calculated system mass versus rate $\mathrm{V}_{\mathrm{e}}$ is shown in the Figure 1.


Figure 1. Components mass of SARA recovery system

## 3. OPTIMIZATION OF SYSTEM VALUES

Settled (critical) rate of system descent depends on parachute and capsule drag area sum, is equal to initial velocity of the next stage and may be calculated from force equilibrium as:

$$
\begin{align*}
& \mathrm{m}_{\mathrm{c}} \mathrm{~g}=\left(\left(\mathrm{C}_{\mathrm{D}} S\right)_{\mathrm{j}}+\mathrm{C}_{\mathrm{c}} \mathrm{~S}_{\mathrm{c}}\right) \rho_{\mathrm{j}+1} \mathrm{~V}_{\mathrm{crj}}^{2} / 2  \tag{19}\\
& \mathrm{~V}^{2}{ }_{\mathrm{crj}}=\mathrm{V}_{\mathrm{oj}+1}^{2}=2 \mathrm{~m}_{\mathrm{c}} \mathrm{~g} /\left(\rho_{\mathrm{j}+1}\left(\left(\mathrm{C}_{\mathrm{D}} \mathrm{~S}\right)_{\mathrm{j}}+\mathrm{C}_{\mathrm{c}} \mathrm{~S}_{\mathrm{c}}\right)\right) \tag{20}
\end{align*}
$$

where $j$ index of the first (drag) parachute
$j+1$ index of the next (main) parachute
$\mathrm{V}_{\mathrm{crj}}$ settled (critical) rate of descent, $\mathrm{m} / \mathrm{s}$.
Hence, taking (20) for (8) we can write:

$$
\begin{equation*}
\mathrm{R}^{2}{ }_{\mathrm{j}+1}(\mathrm{~V})=\mathrm{m}_{\mathrm{c}} /\left(\rho_{\mathrm{j}+1} \mathrm{l}_{\mathrm{pj}+1}\left(\left(\mathrm{C}_{\mathrm{D}} \mathrm{~S}\right)_{\mathrm{j}}+\mathrm{C}_{\mathrm{c}} \mathrm{~S}_{\mathrm{c}}\right)\right) \tag{21}
\end{equation*}
$$

### 3.1 Optimum area of drag parachute

Having taken the system total mass $\mathrm{m}_{\mathrm{s}}$ from (18) as the optimization criteria we can determine optimum $\mathrm{X}_{\mathrm{opt}}$ value of the system which guarantees $\mathrm{m}_{\text {smin }}$. Then $\mathrm{X}_{\mathrm{opt}}$ is the root of the equation:

$$
\begin{equation*}
\mathrm{dm}_{\mathrm{s}}^{*} / \mathrm{dX}=0 \tag{22}
\end{equation*}
$$

When the area of the main parachute is given we can write :

$$
\begin{equation*}
\mathrm{m}_{\mathrm{s}}^{*}=\Sigma \mathrm{R}_{\mathrm{j}}^{2}(\mathrm{~V}) \mathrm{K}_{\mathrm{j}}(\mathrm{~S}) \mathrm{K}_{\mathrm{mj}} \mathrm{R}_{\mathrm{Rj}}+\mathrm{const} \tag{23}
\end{equation*}
$$

Taking (11), (21) and (23) for non-reefed drag and main parachutes from (18) we have:

$$
\begin{equation*}
\mathrm{m}_{\mathrm{s}}^{*}=\mathrm{R}_{\mathrm{j}}^{2}(\mathrm{~V}) \mathrm{K}_{\mathrm{mj}} \mathrm{C}_{\mathrm{kj}} \rho_{\mathrm{j}}\left(\mathrm{C}_{\mathrm{D}} S\right)_{\mathrm{j}}^{3 / 2} / \mathrm{m}_{\mathrm{c}}+\mathrm{K}_{\mathrm{j}+1}(\mathrm{~S}) \mathrm{K}_{\mathrm{mj}+1} \mathrm{~m}_{\mathrm{c}} /\left(\rho_{\mathrm{j}+1} \mathrm{l}_{\mathrm{pj}+1}\left(\left(\mathrm{C}_{\mathrm{D}} S\right)_{\mathrm{j}}+\mathrm{C}_{\mathrm{c}} \mathrm{~S}_{\mathrm{c}}\right)\right)+\text { const } \tag{24}
\end{equation*}
$$

Let us take for the first step of drag area $\left(\mathrm{C}_{\mathrm{D}} \mathrm{S}\right)_{\mathrm{j}}$ optimization the value of $\mathrm{C}_{\mathrm{kj}}=\mathrm{C}_{\mathrm{xj}}$ as for drag parachute and $\left(\mathrm{C}_{\mathrm{D}} \mathrm{S}\right)_{\mathrm{j}} \gg \mathrm{C}_{\mathrm{c}} \mathrm{S}_{\mathrm{c}}$ as for a relative small capsule. Then from (22) and (24):

$$
\begin{equation*}
\mathrm{dm}_{\mathrm{s}}^{*} / \mathrm{d}\left(\mathrm{C}_{\mathrm{D}} S\right)_{\mathrm{j}}=3 \mathrm{C}_{\mathrm{xj}} K_{\mathrm{mj}} \mathrm{R}_{\mathrm{j}}^{2}(\mathrm{~V}) \rho_{\mathrm{j}+1}\left(\mathrm{C}_{\mathrm{D}} S\right)_{\mathrm{j}}^{1 / 2} / 2 \mathrm{~m}_{\mathrm{c}}-K_{\mathrm{mj+1}} K_{\mathrm{j}+1}(\mathrm{~S}) \mathrm{m}_{\mathrm{c}} /\left(\rho_{\mathrm{j}+1} l_{\mathrm{pj}+1}\left(\mathrm{C}_{\mathrm{D}} S\right)_{\mathrm{j}}\right) \tag{25}
\end{equation*}
$$

$\left(C_{D} S\right)_{j \text { jopt1 }}=\left(\left(2\left(K_{m j+1} / K_{m j}\right) m_{c} K_{j+1}(S)\right) /\left(3 C_{x j} \rho_{j} R_{j}^{2}(V) \rho_{j+1} l_{\mathrm{p} j+1}\right)\right)^{2 / 5}$
If $\mathrm{K}_{\mathrm{mj}}=\mathrm{K}_{\mathrm{mj}+1}, \mathrm{l}_{\mathrm{pj}}=\mathrm{l}_{\mathrm{pj}+1}$ and $\mathrm{K}_{\mathrm{j}+1}(\mathrm{~S})=0.25$ (for main parachute, see Table 1) we have:
$\left(\mathrm{C}_{\mathrm{D}} \mathrm{S}\right)_{\text {jopt } 1}=\left(\mathrm{m}_{\mathrm{c}}{ }^{2} \mathrm{~g} /\left(3 \mathrm{C}_{\mathrm{xj}} \rho_{\mathrm{j}} \mathrm{V}_{\mathrm{o}}{ }^{2} \rho_{\mathrm{j}+1}\right)\right)^{2 / 5}$
For the second step of drag area optimization we can determine $C_{k j+1}=f\left(R_{m}, C_{x}\right)$. Then:
$\left(\mathrm{C}_{\mathrm{D}} \mathrm{S}\right)_{\text {jopt2 }}=\left(\mathrm{C}_{\mathrm{D}} \mathrm{S}\right)_{\text {jopt1 }}\left(\mathrm{C}_{\mathrm{kj+1}}\left(1+\mathrm{C}_{\mathrm{c}} \mathrm{S}_{\mathrm{c}} /\left(\mathrm{C}_{\mathrm{D}} \mathrm{S}\right)_{\mathrm{jopt1}}\right)\right)^{-2 / 5}$

### 3.2 Optimum sea-level rate of system descent

When the drag area of the drag parachute $\left(\mathrm{C}_{\mathrm{D}} \mathrm{S}\right)_{\mathrm{j}}$ is known we can write from (18):
$\mathrm{m}_{\mathrm{s}}{ }^{*}=\left(\mathrm{w}_{\mathrm{c} j+1} \mathrm{~S}_{\mathrm{j}+1} / \mathrm{m}_{\mathrm{c}}+\mathrm{V}_{\mathrm{e}}^{2} \mathrm{f}_{\mathrm{sa}} /\left(\mathrm{g} \mathrm{l}_{\mathrm{a}}\left(\eta_{\mathrm{a}}-1 / \mathrm{G}\right)\right)\right)\left(1+\mathrm{K}_{\mathrm{b}}\right)+$ const ${ }^{\prime}$

Taking $V_{e}^{2}=X$ and area of main parachute $S_{j+1}$ from (19) if $\left(C_{D} S\right)_{j+1} \gg C_{c} S_{c}$ we have:

$$
\begin{align*}
& S_{j+1}=2 \mathrm{~m}_{\mathrm{c}} \mathrm{~g} /\left(\mathrm{C}_{\mathrm{Dj}+1} \rho_{\mathrm{o}} \mathrm{~V}_{\mathrm{e}}^{2}\right)  \tag{30}\\
& \mathrm{m}_{\mathrm{s}}^{*}=\left(\mathrm{w}_{\mathrm{c} j+1} 2 \mathrm{~g} /\left(\mathrm{C}_{\mathrm{Dj}+1} \rho_{\mathrm{o}} \mathrm{X}\right)+\mathrm{Xf}_{\mathrm{sa}} /\left(\mathrm{g} \mathrm{l}_{\mathrm{a}}\left(\eta_{\mathrm{a}-}-1 / \mathrm{G}\right)\right)\right)\left(1+\mathrm{K}_{\mathrm{b}}\right)+\mathrm{const}^{\prime} \tag{31}
\end{align*}
$$

Let us define $\mathrm{V}_{\text {eopt }}{ }^{2}=\mathrm{X}_{\text {opt }}$ as the root of the equation (22). Then from (31) we have:

$$
\begin{align*}
& -2 X^{-2} w_{c j+1} g /\left(C_{D j+1} \rho_{o}\right)+f_{s a} /\left(g l_{\mathrm{a}}\left(\eta_{\mathrm{a}}-1 / \mathrm{G}\right)\right)=0  \tag{32}\\
& X_{\mathrm{opt}}=\left(\mathrm{w}_{\mathrm{cj+1}} \mathrm{~g} /\left(\mathrm{C}_{\mathrm{Dj}+1} \rho_{\mathrm{o}}\right) /\left(2 \mathrm{f}_{\mathrm{sa}} /\left(\mathrm{g} \mathrm{l}_{\mathrm{a}}\left(\eta_{\mathrm{a}}-1 / \mathrm{G}\right)\right)\right)^{1 / 2}\right.  \tag{33}\\
& \mathrm{V}_{\text {eopt }}=\left(\mathrm{w}_{\mathrm{cj+1}} \mathrm{~g}^{2} l_{\mathrm{a}}\left(\eta_{\mathrm{a}}-1 / \mathrm{G}\right) /\left(2 \mathrm{f}_{\mathrm{sa}} \mathrm{C}_{\mathrm{Dj}+1} \rho_{\mathrm{o}}\right)\right)^{1 / 4} \tag{34}
\end{align*}
$$

### 3.3 Necessity of parachute reefing

Let us make the convention to evaluate reefing necessity by comparison of the system mass gain with, inevitable for reefing, complication of construction and possible decrease of the reliability of the system. Within this, each non-reefed parachute of the system being developed should have a certain mass increase barrier, the excess of which would mean the reefing necessity. The gain in system mass due to use of the parachute reefing can be calculated as:

$$
\begin{equation*}
\Delta \mathrm{m}=\mathrm{m}_{\mathrm{f}}-\mathrm{m}_{\mathrm{f}}^{\mathrm{R}}-\mathrm{m}_{\mathrm{R}} \tag{35}
\end{equation*}
$$

where $\Delta \mathrm{m} \quad$ mass gain, kg
$\mathrm{m}_{\mathrm{f}} \quad$ mass of parachutes framework without reefing, kg
$\mathrm{m}_{\mathrm{f}}^{\mathrm{R}} \quad$ mass of parachutes framework with reefing, kg
$m_{R} \quad$ mass of reefing equipment, kg .
From (7) the parachute framework masses with and without reefing are:

$$
\begin{align*}
& m_{f}=m_{c} K_{m} R^{2}(V) K(S)=K_{m} \rho V_{o}^{2}\left(C_{D} S\right)^{3 / 2} C_{k} /\left(2 g l_{p}\right)  \tag{36}\\
& m_{f}^{R}=K_{m} R^{2}(V) K^{R}(S)=K_{m} \rho V_{o}^{2}\left(R_{R} C_{D} S\right)^{3 / 2} C_{k}^{R}  \tag{37}\\
& \Delta m=m_{f}\left(1-R_{R}^{3 / 2} C_{k}^{R} / C_{k}\right)-m_{R} \tag{38}
\end{align*}
$$

## 4. CALCULATIONS FOR SARA RECOVERY SYSTEM

Let us take the following system parameters and requirements for recovered from orbit SARA capsule (Moraes, 1999):

Capsule mass, $\mathrm{m}_{\mathrm{c}}$
Capsule drag area, $\mathrm{C}_{\mathrm{c}} \mathrm{S}_{\mathrm{c}}$
Capsule speed at moment of the system inflation, $\mathrm{V}_{\mathrm{o}}$
Air density at altitude of drag parachute inflation, $\rho_{6}$
Air density at altitude of main parachute inflation, $\rho_{2}$
Load factor (capsule maximum acceleration), G

$$
\begin{aligned}
& 215 \mathrm{~kg} \\
& 0,4 \mathrm{~m}^{2} \\
& 135 \mathrm{~m} / \mathrm{s} \\
& 0.67 \mathrm{~kg} / \mathrm{m}^{3}(\mathrm{H}=6 \mathrm{~km}) \\
& 1.0 \mathrm{~kg} / \mathrm{m}^{3}(\mathrm{H}=2 \mathrm{~km}) \\
& 8 \mathrm{~g} .
\end{aligned}
$$

### 4.1 Calculations for basic system

Let us consider the system, which guarantees the sea-level rate of descent of $6 \mathrm{~m} / \mathrm{s}$, as the basic one. Taking index j for the drag parachute and index $(\mathrm{j}+1)$ for the main parachute, coefficients $\mathrm{C}_{\mathrm{kj}}=1.2, \mathrm{C}_{\mathrm{Dj}}=0.55$ for ribbon drag parachute and $\mathrm{C}_{\mathrm{Dj}+1}=0.8$ for cross main parachute we can determine maximum force and required parachutes areas from (5) and (30):

$$
\begin{align*}
& \mathrm{F}_{\mathrm{xj}}=\mathrm{C}_{\mathrm{kj}} \mathrm{C}_{\mathrm{Dj}} \mathrm{~S}_{\mathrm{j}} \rho_{6} \mathrm{~V}_{\mathrm{o}}^{2} / 2 \leq \mathrm{Gm}_{\mathrm{c}} \mathrm{~g}=8215 \quad 9.81=16900 \mathrm{~N}  \tag{39}\\
& \mathrm{~S}_{\mathrm{j}} \leq \mathrm{Gm}_{\mathrm{c}} \mathrm{~g} /\left(\mathrm{C}_{\mathrm{kj}} \mathrm{C}_{\mathrm{Dj}} \rho_{6} \mathrm{~V}_{\mathrm{o}}^{2} / 2\right)=82159.812 /\left(1.20 .55 \quad 0.67 \quad 135^{2}\right)=4.19 \mathrm{~m}^{2} \tag{40}
\end{align*}
$$

$$
S_{j+1} \geq 2 \mathrm{~m}_{\mathrm{c}} \mathrm{~g} /\left(\mathrm{C}_{\mathrm{Dj}+1} \rho_{\mathrm{o}} \mathrm{~V}_{\mathrm{e}}^{2}\right)=22159.81 /\left(\begin{array}{lll}
0.8 & 1.225 & 6^{2} \tag{41}
\end{array}\right)=120 \mathrm{~m}^{2}
$$

Taking drag area $\left(\mathrm{C}_{\mathrm{D}} \mathrm{S}\right)_{\mathrm{j}}=4.19 \quad 0.55=2.3 \mathrm{~m}^{2}$ the initial velocity $\mathrm{V}_{\mathrm{oj}+1}$ from (20) is:

$$
\begin{equation*}
\mathrm{V}_{\mathrm{oj}+1}=\left(2 \mathrm{~m}_{\mathrm{c}} \mathrm{~g} / \rho_{2} /\left(\left(\mathrm{C}_{\mathrm{D}} \mathrm{~S}\right)_{\mathrm{j}}+\mathrm{C}_{\mathrm{c}} \mathrm{~S}_{\mathrm{c}}\right)\right)^{1 / 2}=(22159.81 / 1.0 /(2.3+0.4))^{1 / 2}=40 \mathrm{~m} / \mathrm{s} \tag{42}
\end{equation*}
$$

It is necessary to use the reefing because within the ratio $\mathrm{V}_{\mathrm{oj}+1} / \mathrm{V}_{\mathrm{e}}=40 / 6=6.7>5$ the apparent air recontact is possible (Koldaev, Guimaraes \& Moraes, 1999). Taking reefing ratio $R_{R j}=0.2$ and $\left(C_{D} S\right)_{R j}=2.3 \mathrm{~m}^{2}$, from (6) the value $\left(\mathrm{C}_{\mathrm{D}} S\right)_{\mathrm{j}}=2.3 / 0.2=11.5 \mathrm{~m}^{2}$. For parameters $\mathrm{K}_{\mathrm{b}}=0.75, \mathrm{l}_{\mathrm{p}}=30 \mathrm{~km}, \mathrm{~K}_{\mathrm{m}}=10($ Table 2$), \mathrm{w}_{\mathrm{c}}=0,065 \mathrm{~kg} / \mathrm{m}^{2}($ MIL-C-7350I $), \mathrm{f}_{\mathrm{sa}}=2.5, \mathrm{l}_{\mathrm{a}}=20$ $\mathrm{km}, m_{R}=0.3 \mathrm{~kg}, \eta_{\mathrm{a}}=0.65$, and $\mathrm{R}_{\mathrm{j}+1}(\mathrm{~V})$ from (21) the basic system mass from (18) is:

$$
\begin{align*}
& \mathrm{m}_{\text {sbas }}=\left(\mathrm{V}_{\mathrm{o}}^{2} \mathrm{C}_{\mathrm{xj}} \rho_{6}\left(\mathrm{C}_{\mathrm{D}} S\right)_{\mathrm{j}}^{3 / 2} \mathrm{R}_{\mathrm{Rj}} \mathrm{~K}_{\mathrm{m}} /\left(2 \mathrm{~g} \mathrm{l}_{\mathrm{p}}\right)+0.25 \mathrm{~K}_{\mathrm{m}} \mathrm{~m}_{\mathrm{c}}^{2} /\left(\rho_{2} \mathrm{l}_{\mathrm{p}}\left(\left(\mathrm{C}_{\mathrm{D}} S\right)_{\mathrm{j}}+\mathrm{C}_{\mathrm{c}} \mathrm{~S}_{\mathrm{c}}\right)\right)+\right.  \tag{43}\\
& \left.+\mathrm{S}_{\mathrm{j}+1} \mathrm{~W}_{\mathrm{c}}+\mathrm{N}_{\mathrm{R}} \mathrm{~m}_{\mathrm{R}}+\mathrm{V}_{\mathrm{e}}^{2} \mathrm{f}_{\mathrm{as}} \mathrm{~m}_{\mathrm{c}} /\left(\mathrm{g}_{\mathrm{a}}\left(\eta_{\mathrm{a}}-1 / \mathrm{G}\right)\right)\right)\left(1+\mathrm{K}_{\mathrm{b}}\right)= \\
& =\left(135^{2} 1.20 .6711 .5^{3 / 2} 100.2 /(29.8130000)+0.2510215^{2} /(130000(11.5+0.4))+\right. \\
& \left.+1200.065+0.3+6^{2} 2.5215 /(9.8120000(0.65-1 / 8))\right)(1+0.75)=18.4 \mathrm{~kg} .
\end{align*}
$$

### 4.1 Calculations of optimum system values

Taking the requirements equal to basic system for the optimum sea-level rate of capsule descent from (34), the drag and main parachute optimum areas from (27) and (30), we have:

$$
\left.\left.\begin{array}{l}
V_{\text {eopt }}=\left(w_{\mathrm{c}} g^{2} l_{\mathrm{a}}\left(\eta_{\mathrm{a}}-1 / \mathrm{G}\right) /\left(2 \mathrm{f}_{\mathrm{sa}} \mathrm{C}_{\mathrm{Dj}+1} \rho_{\mathrm{o}}\right)\right)^{1 / 4}= \\
=\left(\begin{array}{llll}
0.065 & 9.81^{2} & 20000 & (0.65-1 / 8) /(2
\end{array} 2.50 .8\right. \\
1.225
\end{array}\right)\right)^{1 / 4}=10.8 \mathrm{~m} / \mathrm{s} .
$$

Taking $\left(C_{D} S\right)_{j}=2.3 \mathrm{~m}^{2}$ the parachute mass gain due reefing use from (36) and (38) is:

$$
\begin{equation*}
\Delta m_{j}=K_{m} \rho_{\mathrm{j}} \mathrm{~V}_{\mathrm{o}}^{2}\left(\mathrm{C}_{\mathrm{D}} S\right)_{\mathrm{j}}^{3 / 2}\left(\mathrm{C}_{\mathrm{kj}}-\mathrm{R}_{\mathrm{Rj}}^{3 / 2} \mathrm{C}_{\mathrm{kj}}^{\mathrm{R}}\right) /\left(2 \mathrm{~g} \mathrm{l}_{\mathrm{p}}\right)-\mathrm{m}_{\mathrm{R}}=0.56 \mathrm{~kg} \tag{47}
\end{equation*}
$$

It is not necessary to use the parachutes reefing as the mass gain $\Delta \mathrm{m}_{\mathrm{j}}$ is small and the ratio $\mathrm{V}_{\mathrm{oj}+1} / \mathrm{V}_{\mathrm{e}}=40 / 10.8=3.7<5$. Then the mass of optimum recovery system from (43) is:

$$
\begin{aligned}
& \mathrm{m}_{\text {sopt }}=\left(135^{2} 1.20 .672 .3^{3 / 2} 10 /(29.8130000)+0.2510215^{2} /(130000(2.3+0.4))+\right. \\
& \left.+370.065+10.8^{2} 2.5215 /(9.8120000(0.65-1 / 8))\right)(1+0.75)=8.94 \mathrm{~kg}
\end{aligned}
$$

## 5. CONCLUSIONS

Preliminary design features of a high performance ground recovery system for small orbital payloads based on parachutes with an air bag for impact attenuation have been analyzed and discussed. A method for determination of system optimum values for the parachute recovery systems was proposed. With the use of this method to design the recovery system of the orbital platform SARA, the areas of drag and main parachutes have been determined, which guarantee minimum system mass and volume. Compared with the basic system, the so optimized system has approximately two times less mass (see Figure 1), more simple construction and higher reliability of function due to exclusion of the parachute reefing with equal load during parachutes opening and landing impact.

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