

## ELASTIC MODULI PREDICTION OF TEXTILE POLYMERIC MATRIX COMPOSITES BY FINITE ELEMENT METHOD

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### Abstract

In this paper we would like to address the computational issues on the numerical implementation of a micro-mechanical model based on the unit cell approach. As most of the engineering applications are related with the plain weave configuration, we decide to focus our attention in this type of fiber arrangement. Once the unit cell is selected and all geometric characteristics an algorithm was developed to create a three dimensional representation of the unit cell. The numerical implementation of such algorithm was fulfilled by using the AutoLisp language which allowed us to model the unit cell into a Computer Aided Design tool, namely Autodesk Mechanical Desktop software. Then, the unit cell geometry is transferred to ANSYS. The numerical simulations are performed considering the waviness ratio variation from 0.165 to 0.5. The data are compared against Whitcomb and Naik's results with good agreement. Finally, some computational issues of the CAD and FEM compatibility are address and discussed.

Key words: Textile composites, Computer modeling, Micro-mechanics analysis.

### 1. INTRODUCTION

Nowadays, unidirectional fibers composites are used in the aerospace industry for secondary structures, such as engine casings, flaps and rudders, where their high strength-weight ratio reduces the overall weight and hence the operating costs of the aircraft. Textile and 3-D woven composites have great potential applications in the aerospace industry, through cost savings due ease of handling, reduced scrap rate, and decreasing of problems of cracks, such as damage and delamination.

In order to increase the use of 3-D woven composite materials in the aerospace industry, it is needed to settle its physical properties. It is also important to know the composite behavior under different applied loads of the wide range of structures available, and tailors the composite to the structure requirements. Variables such as fiber type, dimensions and their arrangement within the composite fabric need to be optimized for each different application. The costs of preparing and analyzing the various possibilities of composite structures are prohibitive. By using the new methodology proposed the number of tests need are dramatically reduced. The designer can change the fiber components and/or matrix to search

for the best architecture of the composite for a specific application. Such task can be done with a computational method without any cost and saving manpower and time. This new computational method will be helpful for light airplanes industry, where the designer wants to get smallest weight and reduced cost of material and fabrication.

This paper discusses a new methodology to calculate the mechanical proprieties of a plain weave woven textile polymeric composite. It is applied the concept of tridimensional unit cell (Whitcomb, 1991; Naik & Stembekar, 1992; Dasgupta *et al.*, 1996) which represents the smallest portion of the entire woven textile. The unit cell model is analyzed by applying the finite element method to compute the effective elastic moduli. This approach differs from classical ones (Ishikawa & Chou, 1982; Ishikawa & Chou, 1983; Cox & Flanagan, 1997) due to the application of a numerical technique, which allows us to describe the stress and strain distribution over the fibers and the matrix. Moreover, the effects of ondulation on the elastic moduli and Poisson's ratio can also be studied by applying the finite element formulation in conjunction with the unit cell concept.

The computational aspects of finite element implementation and its pros and cons are also addressed in this paper. As an engineering tool the new methodology can bring the followings advantages:

- Flexibility to test various combinations of materials (fibers and matrix) to make a specific composite material.
- Pre-determine the stress concentration factors on fibers, as a function of ondulation.

## 2. WEAVE CONFIGURATIONS

In any weave, the warp yarns run along the length of the textile and the weft yarns run across the width. The three main types of single layer weave geometry are plain, satin and twill weave and in each case the warp and weft yarns are oriented at 0 and 90<sup>0</sup>, respectively.

For plain weave, one warp alternatively crosses over and under consecutive weft yarn, while the next warp yarn crosses under and over the wefts. This pattern is classified as 1/1 and is illustrated in Fig.1-a.

For a satin weave textile, each warp yarn crosses at least tree weft yarns and interlaces with fourth weft yarn, with a progression of interlacings of two to the left or right – a 3/1 4/1 or 7/1 weave. These geometries are also known as Crows Foot Satin, Five Harness and Eighth Harness Satin and are shown in Fig.1-b,c,d.

The basket weave have two warp yarns interlaced with two weft yarns, it is classified as 2/2 and is illustrated in Fig.1-f. Twill weave is characterized by two or more warp yarns crossed by a weft yarn, with a progression of interlacings of one weft yarn to the right or left to form a distinctive diagonal line. Twill weave can be even-sided, i.e. the same amount of warp and weft on each side of textile, or it may be warp- or weft-faced, with a predominance of warp or weft yarns on the upper face. An even weave, with each warp crossing two wefts and vice versa, is classified as a 2/2 twill and is show in Fig.1-e.

## 3. UNIT CELL GEOMETRY

In the Figure 1, the poligonal lines define the boundaries of unit cells. As we are interested into plain weave composite materials we select the unit cell proposed by Naik & Stembekar (1992), see Figure 2.

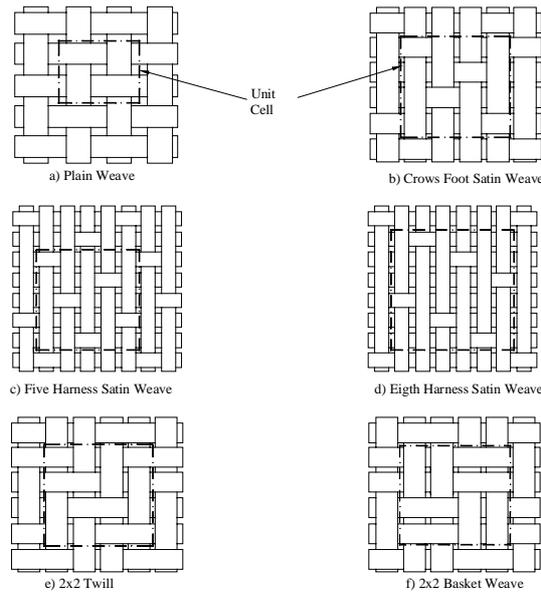


Figure 1 – Unit Cells for principals types of 2D textiles

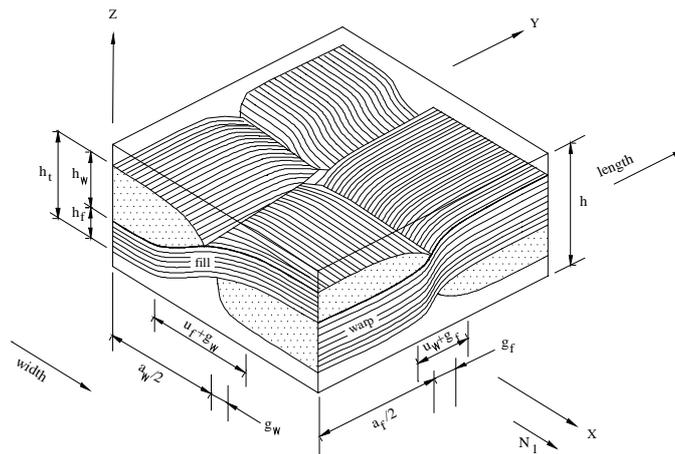


Figure 2 – The unit cell

In Figure 2 the following dimensions are defined:  $a_w$  is the width of warp yarn,  $a_f$  is the width of weft yarn, while  $g_w$  is the distance between two warp yarns. The distance between two weft yarns is represented by  $g_f$ ,  $h_w$  and  $h_f$  are the thicknesses of warp yarn and weft yarn, respectively. Finally, the ondulation length of warp yarn, the ondulation length of weft yarn and the lamina thickness are represented by  $u_w$ ,  $u_f$  and  $h$ .

By considering in Figure 3 the section DC as the mirror of section AB it is possible to assume that section BC is also the mirror of section AD. In each section of unit cell it can be identified three different regions. The first region is the one where the fiber is straight, the second is where the fiber is curved, and the third one is where the matrix is located. To be able to compute the unit cell dimensions Naik & Stembekar (1992) defined five sections  $a_1$ ,  $a_2$ ,  $a_3$ ,  $a_4$  and  $a_5$  parallel to DC and five other sections  $b_1$ ,  $b_2$ ,  $b_3$ ,  $b_4$  and  $b_5$  parallel to AD. The following equations are used to calculate  $a_1$  to  $a_5$  and  $b_1$  to  $b_5$ :

$$a_1 = \frac{a_w - u_f}{2} \quad a_2 = \frac{a_w}{2} \quad a_3 = \frac{a_w + g_w}{2} \quad a_4 = \frac{a_w}{2} + g_w \quad a_5 = \frac{a_w + u_f}{2} + g_w \quad (1)$$

$$b_1 = \frac{a_f - u_w}{2} \quad b_2 = \frac{a_f}{2} \quad b_3 = \frac{a_f + g_f}{2} \quad b_4 = \frac{a_f}{2} + g_f \quad b_5 = \frac{a_f + u_w}{2} + g_f \quad (2)$$

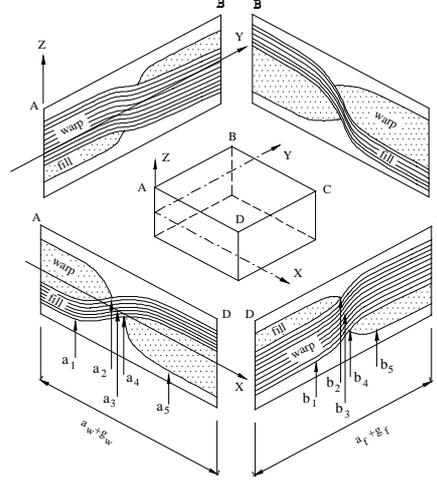


Figure 3: Plain weave unit cell representation

The fibers' configuration in section DC is calculated with the  $h_{y1}(y)$  and  $h_{y2}(y)$  form functions. The fibers' configuration on section AD is calculated with the  $h_{x1}(x,y)$ ,  $h_{x2}(x,y)$  and  $h_{x3}(x,y)$  functions. It is out of the scope of this work to reproduce these form functions, it is shown the resultants figures 4a and 4b. For more details see Oliveira (1999).

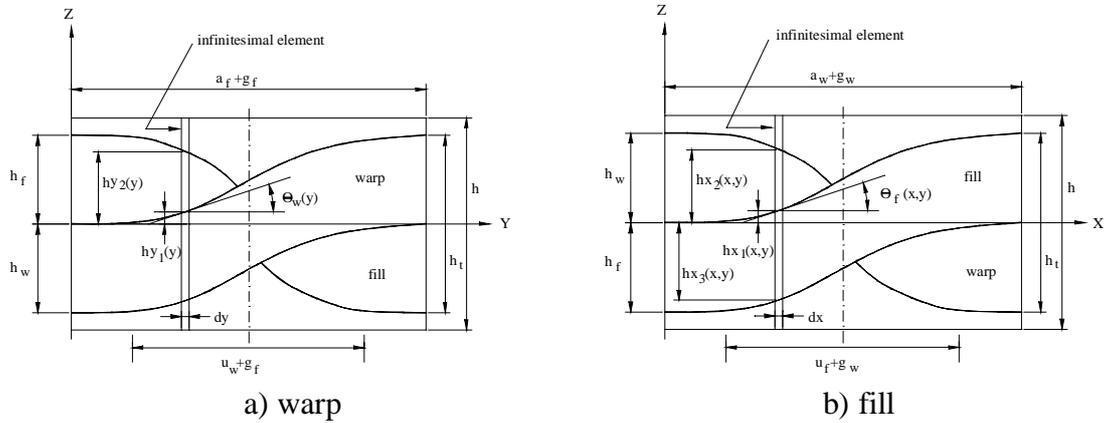


Figure 4: a) sections parallel to DC, b) sections parallel to AD fiber

#### 4. PROPOSED ALGORITHM

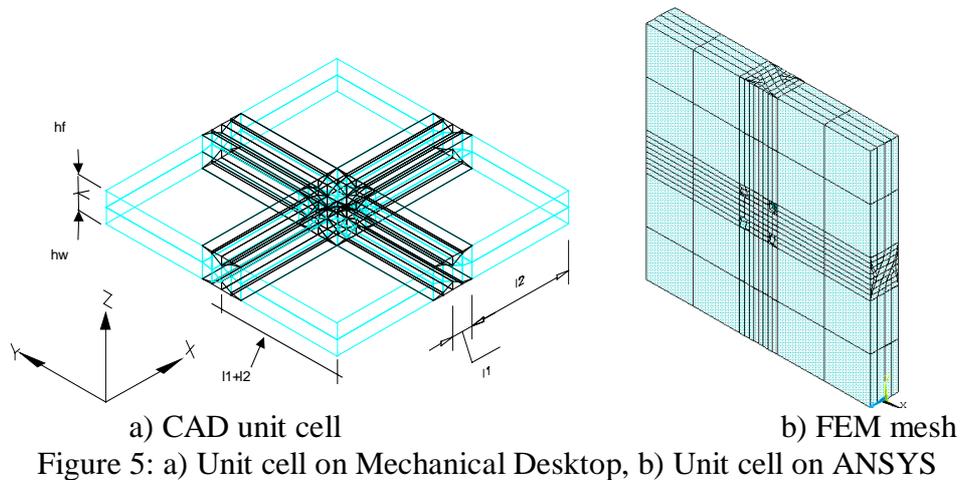
An algorithm was developed based on such form functions to construct the unit cell. Before proceed it is needed to establish some geometric definitions. The yarn width on longitudinal and transverse directions are represented by  $a_f$  and  $a_w$ , while the free distance between two yarns on longitudinal and transverse directions are given by  $g_f$  and  $g_w$ . The weave yarn on longitudinal and transverse direction are represented by  $u_f$  and  $u_w$ . The increment on the y direction is given by  $dy$  and the yarn height on the longitudinal and transverse directions are represented by  $h_f$  and  $h_w$  respectively.

The following steps are performed by the proposed algorithm:

Get  $a_f, a_w, g_f, g_w, u_f, u_w, h_f, h_w, h_t, dy$   
 Compute  $a_1$  through  $a_5$  and  $b_1$  through  $b_5$   
 Draw section D-C  
 Fix the origin (0,0)  $y_i=0; h_{y1}(y_i)=0; h_{y2}(y_i)=h_f$   
 Compute  $k=a_f+g_f$   
 $x1=[1+\sin\{(b_2-b_3)*(\Pi/(u_w+g_f))\}]$   
 $h_{y1}(b_2)=x1*h_f/2+h_t/2-h_f$   
 Increment  $y_{i+1}=y_i+dy$   
 If  $y_{i+1}<b1$  then do  
 $h_{y1}(y_i)=0$   $h_{y2}(y_i)=h_t/2$   
 End if  
 Draw lines  
 $(y_i, h_{y1}(y_i))$   $(y_{i+1}, h_{y1}(y_{i+1}))$   
 $(y_i, h_{y2}(y_i))$   $(y_{i+1}, h_{y2}(y_{i+1}))$   
 If  $b_1<y_{i+1}<b_2$  then do  
 $x1=[1+\sin\{(y-b_3)*(\Pi/(u_w+g_f))\}]$   
 $h_{y1}=x1*h_f/2+h_t/2-h_f$   
 $x2=[h_t/2-h_{y1}(b_2)]*\cos\{(y-b_1)*\Pi/u_w\}$   
 $h_{y2}(y_i)=x2+h_{y1}(b_2)$   
 End if  
 Draw lines  
 $(y_i, h_{y1}(y_i))$   $(y_{i+1}, h_{y1}(y_{i+1}))$   
 $(y_i, h_{y2}(y_i))$   $(y_{i+1}, h_{y2}(y_{i+1}))$   
 Do  $y_{i+1}=b_2$   
 Compute  $h_{y1}(y_i)=h_{y2}(y_i)=h_{y1}(b_2)$   
 Draw lines  
 $(y_{i+1}, h_{y1}(y_{i+1}))$   $(b_2, h_{y1}(b_2))$

$(y_{i+1}, h_{y2}(y_{i+1}))$   $(b_2, h_{y2}(b_2))$   
 If  $b_2<y_{i+1}<b_4$  then do  
 $x1=[1+\sin\{(y-b_3)*(\Pi/(u_w+g_f))\}]$   
 $h_{y1}(y_i)=x1*h_f/2+h_t/2-h_f$   
 End if  
 Draw line  
 $(y_i, h_{y1}(y_i))$   $(y_{i+1}, h_{y1}(y_{i+1}))$   
 Do  $y_{i+1}=b_4$   
 Compute  $h_{y2}(y_i)=-h_{y1}(b_2)$   
 If  $b_4<y_{i+1}<b_5$  then do  
 $x1=[1+\sin\{(y-b_3)*(\Pi/(u_w+g_f))\}]$   
 $h_{y1}(y_i)=x1*h_f/2+h_t/2-h_f$   
 $x1=\cos\{(y-b_5)*\Pi/u_w\}-h_{y1}(b_2)$   
 $h_{y2}(y_i)=-[h_t/2-h_{y1}(b_2)]*x1$   
 End if  
 Draw lines  
 $(y_i, h_{y1}(y_i))$   $(y_{i+1}, h_{y1}(y_{i+1}))$   
 $(b_4, h_{y1}(b_2))$   $(y_{i+1}, h_{y2}(y_{i+1}))$   
 If  $b_5<y_{i+1}<k$  then do  
 $h_{y1}(y_i)=h_t/2$   
 $h_{y2}(y_i)=-h_t/2$   
 End if  
 Draw line  
 $(y_i, h_{y1}(y_i))$   $(y_{i+1}, h_{y1}(y_{i+1}))$   
 $(y_i, h_{y2}(y_i))$   $(y_{i+1}, h_{y2}(y_{i+1}))$

The algorithm listed before was applied to create two codes in *Au oLisp*. Such codes were named *T x2 sp* and *ma rix sp* and they are used to generate the unit cell. Once such unit cell was generated by the *Au od sk M c anica D sk op @ RI I* the file is transferred to a finite element code. Figures 5a and 5b show the unit cell before and after the transferring procedure.



To be able to validate the new computational methodology a set of unit cell configurations are studied. In this study, we are able to calculate the elastic moduli for various

geometric parameters, distinct materials, and different volume fractions of fiber and matrix. The geometric parameters are mainly concerned to the fiber undulation and thickness, and the overall laminate thickness.

**5 . SOME NUMERICAL SIMULATIONS**

The results from the numerical simulation are compared against the data available in the literature. Figures 6 through 8 show the elastic moduli as a function of the fiber undulation. In general, the results are in good agreement with the data available in the literature. It is important to mention that these effective properties are calculated following the methodology proposed by Oliveira (1999).

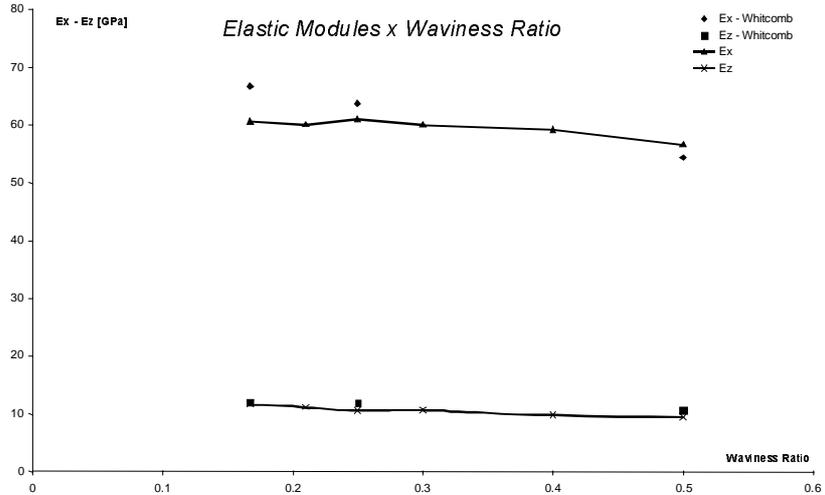


Figure 6 – Effective elastic moduli  $E_x, E_y, E_z$  versus waviness ratio

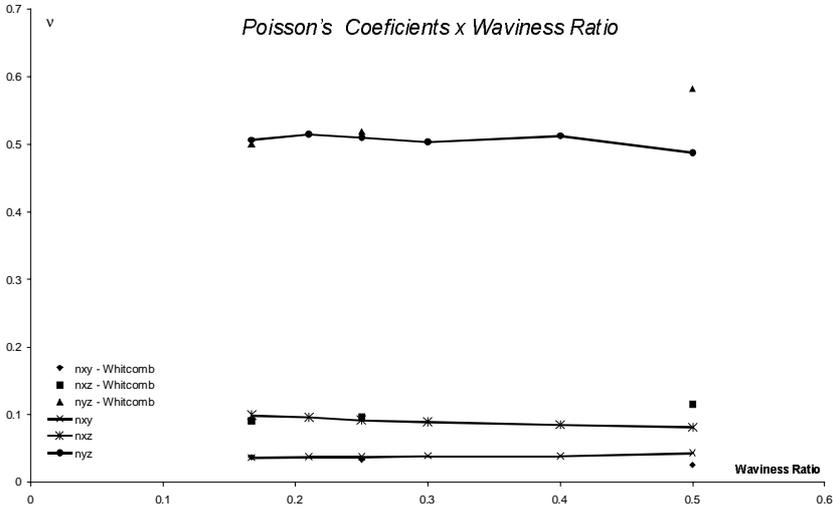


Figure 7 – Poisson's ratios  $\nu_{xy}, \nu_{yz}, \nu_{xz}$  versus waviness ratio

Another advantage of the new methodology is the potentiality of studying the stress concentration into the fibers by showing the effective stress distribution, represented by the Von Mises stress, into the entire unit cell. By analyzing figures 9 and 10 it is possible to observe the influence of undulation on the stress concentration. Plain weave configurations with waviness ratio bigger seems to be less influenced by stress concentrations, as the Von Mises stresses are considerable lower in such type of composite.

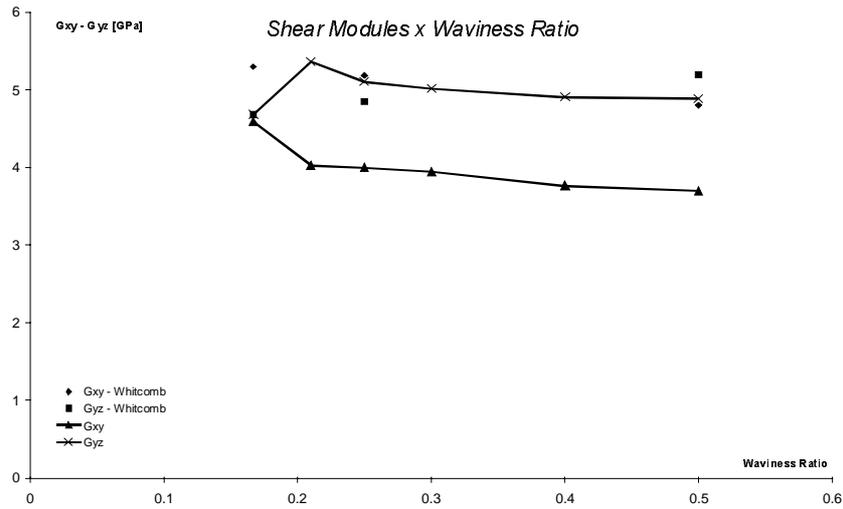
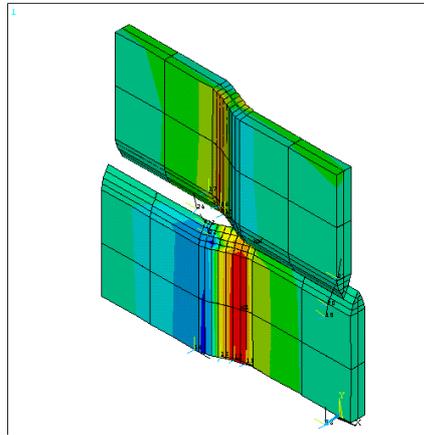


Figure 8 – Shear moduli  $G_{xy}$ ,  $G_{yz}$ ,  $G_{xz}$  versus waviness ratio

```

ANSYS 5.2
FEB 22 1999
18:53:40
MODAL SOLUTION
STEP=1
SUB =1
TIME=1
SEQV (AVG)
DMX =.001007
SMN =77.446
SMX =176.327
77.446
88.433
99.419
110.406
121.393
132.38
143.367
154.354
165.34
176.327

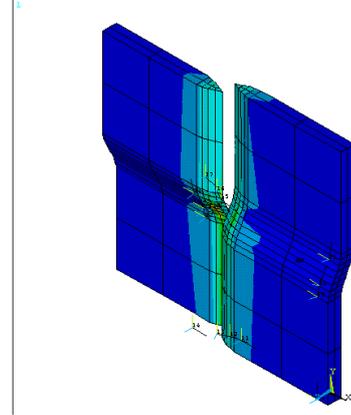
```



```

ANSYS 5.2
FEB 22 1999
20:32:26
MODAL SOLUTION
STEP=1
SUB =1
TIME=1
SEQV (AVG)
DMX =.001
SMN =5.358
SMX =42.792
5.358
9.517
13.676
17.836
21.995
26.154
30.314
34.473
38.632
42.792

```



a) warp

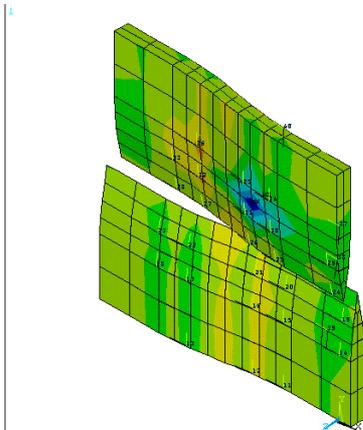
b) fill

Figure 9: Stress distribution on fiber for waviness ratio = 0.167

```

ANSYS 5.2
FEB 22 1999
19:48:54
MODAL SOLUTION
STEP=1
SUB =1
TIME=1
SEQV (AVG)
DMX =.001
SMN =54.883
SMX =180.702
54.883
69.862
84.842
99.822
114.802
129.782
144.762
159.742
174.722
189.702

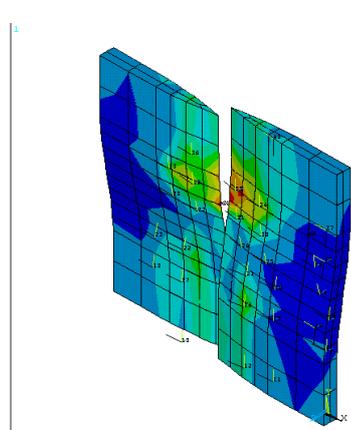
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```

ANSYS 5.2
FEB 22 1999
20:14:46
MODAL SOLUTION
STEP=1
SUB =1
TIME=1
SEQV (AVG)
DMX =.001
SMN =5.985
SMX =16.07
5.985
7.194
8.404
9.613
10.823
12.032
13.242
14.451
15.66
16.07

```



a) warp

b) fill

Figure 10: Stress distribution on fiber for waviness ratio = 0.5

## 6. CONCLUDING COMMENTS

The proposed algorithm seems to be a helpful tool to create numerical representations of textile composites. By applying such type of algorithm is possible to create and analyze different types of textile composites configurations. By using such algorithm the user is allowed to identify possible locations of stress concentrations and correct such problems before hand.

## 7. ACKNOWLEDGEMENTS

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