MODELLING THE CYCLIC SOFTENING BEHAVIOUR FOR THE ASTM A471 STEEL TROUGH A ELASTO-PLASTIC CONTINUUM THEORY

Fulvio E. G. Chimisso
Depto. de Materiais e Construção
FURG – Fundação Universidade Federal do Rio Grande – RS, Brasil
Gianni Caligiana
Dipartimento di Ingegneria delle Costruzioni Meccaniche, Aeronautiche e di Metallurgia
DIEM - Facoltà di Ingegneria Università di Bologna, Bologna, Italia
Heraldo Costa Mattos
Depto. de Engenharia Mecânica
UFF - Universidade Federal Fluminense, Niteroi, RJ, Brasil

Abstract*. The present work is concerned with the modelling of the cyclic softening behaviour for the ASTM A471 steel (24CrNiMoV14-6, vacuum-treated alloy steel forgings) trough a elasto-plastic continuum theory. This alloy is used in turbine generator shafts, turbine rotor disks and wheels. The experimental work has been performed by strain controlled, completely reversed push-pull low cycle fatigue tests, at room temperature, to determine the stable hysteresis loops and the cyclic curve. The tests have been performed using a servo-hydraulic INSTRON testing machine on ASTM standard specimens. The cyclic inelastic behaviour is characterised by a non-linear coupling between plasticity, isotropic softening and kinematic hardening. The aim of this paper is to present an adequate phenomenological description for such kind of coupling. The constitutive parameters that appear in the model are identified experimentally and the theoretical results are compared with experimental data, showing a very good agreement.*

Key words*:, elasto-plasticity, kinematic hardening and isotropic softening.*

1. INTRODUCTION

The use of continuum mechanics models into design and structural integrity assessment of inelastic mechanical components, is often restricted by the difficulties to find references about material parameters obtained experimentally. The determination of the stable hysteresis loop in an elasto-plastic behaviour is an important step concerning low cycle fatigue of metallic structures. The present work is concerned with the determination of material parameters that appear in a internal variables elasto-plastic theory proposed to model the inelastic behaviour of an ASTM A 471 (vacuum-treated alloy steel forgings), very often utilised for turbine generator shafts, turbine rotor disks and wheels. Low cycle fatigue is significant in the study of turboalternator shafts behaviour because they suffer very heavy local strain during transients and accidental emergencies (short circuits, out of phase synchronisation, etc.).

The experimental data are obtained by strain controlled low cycle fatigue push-pull tests to determine the stable hysteresis loops and the associated cyclic curve. The analysis of the experimental stress-strain hysteresis loops show us a cyclic isotropic softening due to the material behaviour.

A continuum elasto-plastic model with internal variables is used to describe the cyclic softening and the theoretical results are compared with experimental data to check the model accuracy. The plastic deformation, the isotropic softening and the kinematic hardening are strongly coupled for this kind of material. A first step towards a complete modelling of the material behaviour was made in Chimisso & Caligiana (1999). The present paper is mainly concerned with an adequate description of the cyclic softening observed experimentally. A method to identify the material parameters that appear in the evolution laws proposed for the isotropic softening and kinematic hardening is presented and discussed.

2. MATERIAL

The experimental data considered in this paper, according Chimisso and Caligiana (1999), have been derived from a broad study undertaken with the aim of extending knowledge about the fatigue behaviour of turboalternator shafts. The chemical composition and the typical heat treatments for A 471 are reported in Table 1. Mechanical properties are listed in Table 2.

Table 1 – Chemical composition and typical heat treatments of A 471 rotor steel

Chemical Composition (%)										
C	Si	Mn	P	S	Cr	Mo	Ni	V	Al	Cu
0.28	0.07	0.23	0.008	0.004	1.63	0.42	3.59	0.09	0.009	0.06
Heat Treatment										
Tempered			940°C x 28 hrs.;		870°C x 28 hrs.;			630°C x 28 hrs.		
Quality Tempered			845°C x 19 hrs.;		630°C x 25 hrs.;					
Stress-relieving			570°C x 21 hrs.		furnace cooled			(17.2 °C/hrs.)		
			200°C		air cooled					

Table 2 – Mechanical properties of A471 rotor steel

Mechanical Properties					
0.02 Yield Strength (MPa)	Tensile Strength (MPa)	Elongation (%)	Reduction of Area (%)	KV (J)	FATT (°C)
520-655	585-725	20	70	136-193	-12/-14

3. EXPERIMENTAL PROCEDURES

Experimental data utilised in the paper are obtained by push-pull strain controlled low cycle fatigue tests to determine the stable hysteresis loops and the cyclic curve. The tests have been performed, Caligiana (1987), with an INSTRON servo-hydraulic testing machine, series 8000, on ASTM standard 12 mm diameter specimen. A triangular strain waveform at a constant total strain rate (1%/s) was applied and a clip-gage was used for measuring the total strain at room temperature.

The interpolation of the experimental values of isotropic parameters versus accumulated plastic strain has been performed by utilising non linear regression tools of the MATHEMATICA package. The mean values of Young modulus and the elastic limit have been determined, according to normal distribution, by using the same package.

A set of 6 specimens (E1 to E6) has been cycled with standard strain-controlled low cycle fatigue test, with strain amplitudes ranging between 0.46% and 1.27%. Here, due to the limited space, only the development of hysteresis behaviour for specimen E5 is shown in Figure 1, where the continuous softening phenomenon can be observed.

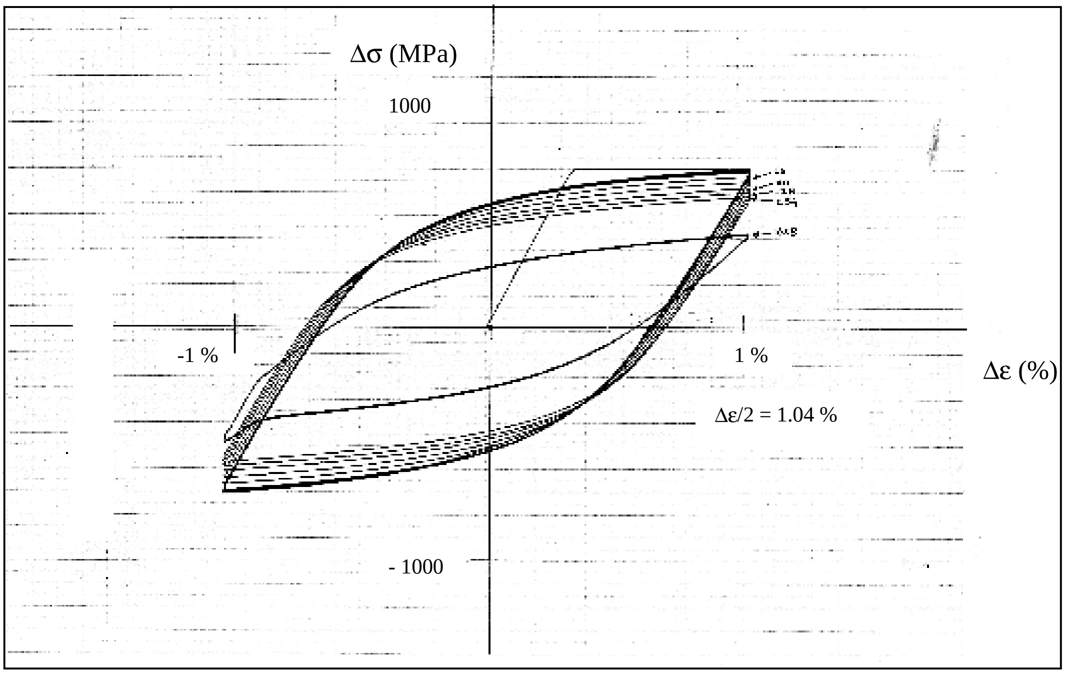

Fig.1 Strain-controlled low cycle fatigue test, specimen E5: Δε/2 = 1.04%

The companion specimen test (CST) is used to determine the experimental cyclic curve. Usually the cyclic curve is obtained by the tips connection of the hysteresis loops at saturation (when it is found) or at the half-life of test specimens.

Considering the little sample, a Student distribution are used to certify that a normal distribution of the Young modulus and of the elastic limit obtained in the experimental tests are a good representation of those parameters. Consequently, mean values E = 186.5 GPa , and elastic limit (at $\varepsilon = 0.02\%$) $S_y = 545$ MPa, are used.

4. THE ELASTO-PLASTIC CONTINUUM MODEL

The set of elasto-plastic constitutive equations with internal variables used in this work describes the mechanical behaviour of metallic materials submitted to non-monotonic loading. We use the concept of the free energy in the constitutive theory with a strong thermodynamic basis, Lemaitre (1990). The free energy is defined as a differentiable function of the state variables ε, ε^p, β: $\psi = \psi(\varepsilon - \varepsilon^p, \beta)$. ε^p is the plastic strain and β is a generic representation of internal variables related to other dissipative mechanisms. The elastic dominion defined by the existence of a plastic potential F such that

$$F(\sigma, B^\beta, \varepsilon^p, \beta) < 0 \quad \Rightarrow \quad \dot{\varepsilon}^p = 0 \;\; \text{e} \;\; \dot{\beta} = 0 \;; \quad B^\beta = \frac{\partial \psi}{\partial \beta}$$

A complete set of constitutive equations is then given by:

$$\sigma = \frac{\partial \psi}{\partial \varepsilon} \qquad (1)$$

$$x = \frac{\partial \psi}{\partial c} \qquad (2)$$

$$y = \frac{\partial \psi}{\partial p} \qquad (3)$$

$$\dot{\varepsilon}^p = \lambda \frac{\partial F}{\partial \sigma} \qquad (4)$$

$$\dot{p} = -\lambda \frac{\partial F}{\partial y} \qquad (5) \text{ and}$$

$$\dot{c} = -\lambda \frac{\partial F}{\partial x} \qquad (6)$$

where $\lambda \geq 0$, $F \leq 0$, $\lambda F = 0$, are the complementary conditions and λ is the Lagrange multiplier related with the restriction (plasticity criterion) $F \leq 0$.

Equations (1) – (3) are called state laws and equations (4) – (6) the evolution laws. For an adequate modelling of the physical behaviour of the ASTM A471 steel, two additional variables p and c related, respectively, with the isotropic softening and kinematic hardening mechanisms are introduced and the following potentials are chosen:

$$\psi = \frac{1}{2} E \left(\varepsilon - \varepsilon^p \right)^2 + v_1 \left(p + \frac{e^{-v_2 p}}{v_2} \right) + p S_y + \frac{1}{2} a c^2 \qquad (7)$$

$$F = J(\sigma - x) - y + \frac{b}{2a} x^2 - \frac{ab}{2} c^2 \qquad (8)$$

where $J(\sigma - x)$ is the von Mises equivalent stress, S_y is the elastic limit, v_1, v_2 are material constants and a, b are non-linear functions of the variable p:

$$a = A_i \, \varepsilon^i \quad \text{and} \quad b = B \, \varepsilon^{-\gamma} \qquad (9)$$

Hence, using (1) – (6),

$$\sigma = \frac{\partial \psi}{\partial \varepsilon} = E(\varepsilon - \varepsilon^p) \qquad (10)$$

$$x = \frac{\partial \psi}{\partial c} = ac \qquad (11)$$

$$y = \frac{\partial \psi}{\partial p} = S_y - v_1 (1 - e^{-v_2 p}) \qquad (12)$$

$$\dot{\varepsilon}^p = \lambda \frac{\partial F}{\partial \sigma} = \lambda \frac{(\sigma - x)}{|\sigma - x|} = \lambda \mathrm{Sg} \qquad (13)$$

$$\dot{p} = -\lambda \frac{\partial F}{\partial y} = \lambda = |\dot{\varepsilon}^p| \qquad (14) \text{ and}$$

$$\dot{c} = -\lambda \frac{\partial F}{\partial x} = \dot{\varepsilon}^p - \frac{b}{a}(x\lambda) \qquad (15)$$

y is called the isotropic hardening variable, x is the kinematic hardening variable, p and c are internal variables associated to the isotropic and kinematic hardening respectively. The main difference between this model and the one proposed by Marquis (1978), which is discussed in Lemaitre and Chaboche (1990), is the expression for the variable y. Generally, the cyclic softening or hardening is described through the kinematic hardening, considering the variable b in equation (15) as a nonlinear function of p. For the ASTM A471 steel, such modeling is not adequate since the softening is basically isotropic.

5. MODELLING THE MATERIAL BEHAVIOUR

From the experimental curves, figure 1, a cyclic softening behaviour of the material has been observed. The material never stabilises and softens continuously until complete failure. Once a crack of significant magnitude was formed, the hysteresis loops were observed to become asymmetric and, in the most of cases, the tests were stopped. In those situations of continuously softening, it is a usual way consider the standard half-life values for a hypothetical stable hysteresis loop.

In the experimental tests, as well, a change is observed in the plastic strain amplitude with the increasing number of cycles. Therefore cyclic softening implicates a change in the anelastic dominion with a consequential change in the accumulated plastic strain, p, cycle by cycle.

Considering the continuum elasto-plastic model (where the isotropic hardening, y, related to the accumulated plastic strain, plays an important role during the life of the specimen), the life history is important since the beginning to the end-failure to model the behaviour of themselves. The strong indication of damage effect presence in the specimens observed by a local change of Young modulus (identified by the slope change in the stress-strain loop), is another important consideration. Consequently, in this case, it isn't appropriate estimate a standard stable hysteresis loop at one-half specimen's fatigue life.

Observing the development of the hysteresis loops, the difference between the accumulated plastic strain in a cycle to the consecutive is small: the plastic strain increases slowly. So, the isotropic hardening or softening is very important in the progressive behaviour cycle by cycle but for a single cycle it may be considered constant. Thereby, during a stable loop, the isotropic hardening will be taken constant and only the non-linear kinematic hardening is significant.

Taking into account the above considerations, the choice of the first hysteresis loop are be justified as an hypothetical stable cycle to build the kinematic hardening behaviour. In the elasto-plastic model represented by the set of equations above, the isotropic hardening, equation (12) was constant (v1 = v2 = 0) and equal to the elastic limit obtained in the first hysteresis loop:

$$y = S_y \quad \Rightarrow \quad \dot{y} = 0 \qquad (12.a)$$

Figure 2 shows the first experimental complete hysteresis loop for the E1 to E6 specimens and figure 3 shows the monotonic, cyclic at middle life and cyclic for the first cycle curves where cyclic softening can be observed.

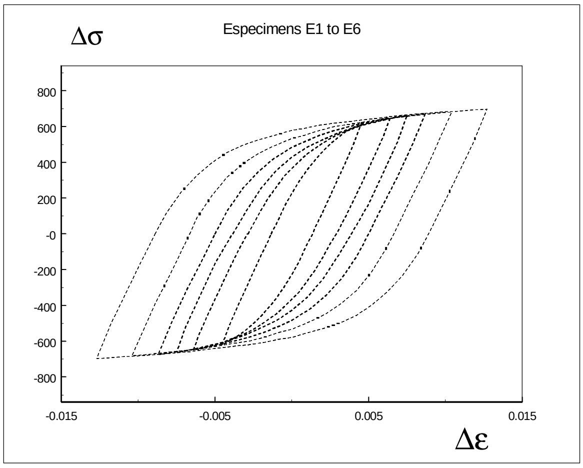

Fig.2 First cycles for the E1 to E6 tests

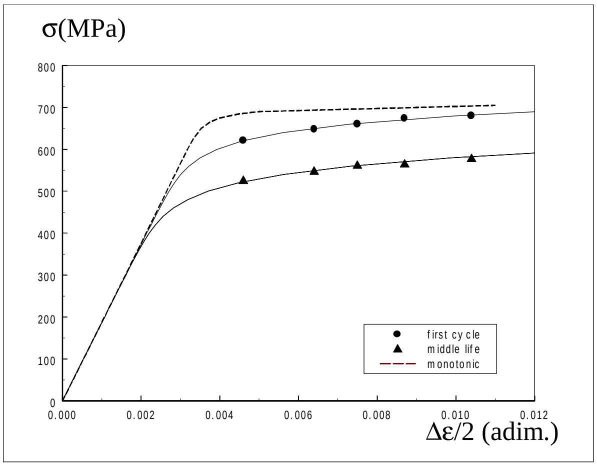

Fig.3 Monotonic vs. cyclic curves

6. PARAMETERS DETERMINATION

6.1. Kinematic Hardening

The kinematic hardening parameters, a and b, are obtained at the first complete hysteresis loop, for the tests E1 to E6, using a least-square method with sufficiently accurate convergence and determination coefficient. For a detailed discussion, see Chimisso & Caligiana (1999). So, the best representation of themselves can be obtained:

$$a = 174 - 34950\varepsilon + 2{,}884{,}840\,\varepsilon^2 - 84{,}160{,}460\,\varepsilon^3 \qquad (16)$$

$$b = 4.4729\,\varepsilon^{-0.7955} \qquad (17)$$

It is interesting to observe that when the total strain amplitude tends to values greater than 1.5 % the parameters a and b tend to asymptotic values. It signifies the saturation of the kinematic hardening curve for large strain. Those behaviour are observed in the experimental tests too. Consequently, when the cyclic strain applied exceed 1.5 % ($\Delta\varepsilon/2 > \pm 1.5\%$), the asymptotic values tend to: a = 15 GPa and b = 126.

Figures 4 and 5 show, for the E4 and E5 specimens, the model prevision and experimental results for the non-linear kinematic hardening behaviour, related to plastic strain.

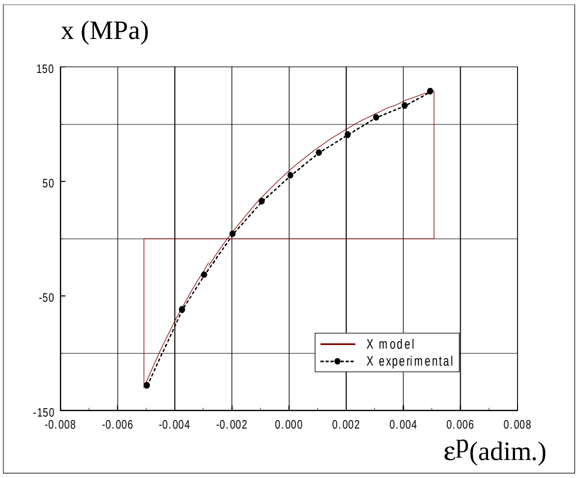

Fig.4 E4: model x test kinematic hardening

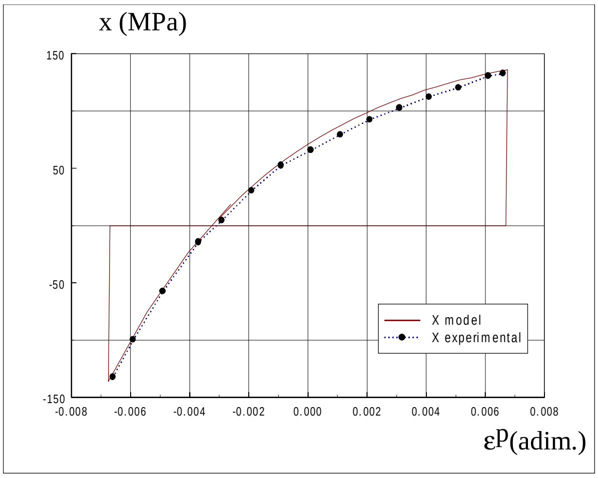

Fig.5 E5: model x test kinematic hardening

6.2. Isotropic Softening

To determine the parameters v_1 e v_2, was considered the equation (12):

$$y = S_y - v_1(1 - e^{-v_2 p})$$

When $p \to 0$, $y \to S_y$, and when $p \to \infty$, $y \to S_y - v_1$
And its derivative related to the accumulated plastic strain, p

$$\frac{dy}{dp} = -v_1 v_2 e^{-v_2 p}$$

When $p \to 0$, $dy/dp \to -v_1 v_2$, and when $p \to \infty$, $dy/dp \to 0$ (y = constant)

The limit values of the variables and its derivatives above can be showed graphically.

In the experiments, the softening denote in a inelastic domain change with a modification in the accumulated plastic strain, cycle by cycle. So, the parameters v_1 e v_2, can be determinate through the cyclic plastic strain variation, related with the accumulated plastic strain, p.

From the figure 1(E5 specimen test) where the isotropic softening is show since the first cycle, the plastic strain can be measured in the following cycles. Supposing a little linear growth of the cyclic plastic strain, the accumulated plastic strain growth, p, is estimated in the following form:

$$p_i = p_{i-1} + 2\Delta\varepsilon_i^p \qquad (18)$$

Where p_i is is the accumulated plastic strain at the i cycle.

Thus, for the E5 test was obtained:

- At the first cycle, y = 545 MPa,
- p_{16} = 42,9 % and y_{16} = 525 MPa , at N = 16 cycles,
- $p_{64} = p_\infty$ = 180 % and y_{64} = 475 MPa, at N = 64cycles.

In this way, the isotropic softening parameters are obteined:

v_1 = 70 MPa and v_2 = 0,784

So, equation (12) give us:

$$y = 545 - 70(1 - e^{-0,784p}) \qquad (19)$$

7. RESULTS

The model and the experimental first hysteresis loop were compared. For example the following figures 6 and 7 show a good agreement of the model results, obtained from numerical simulation, to the experimental results.

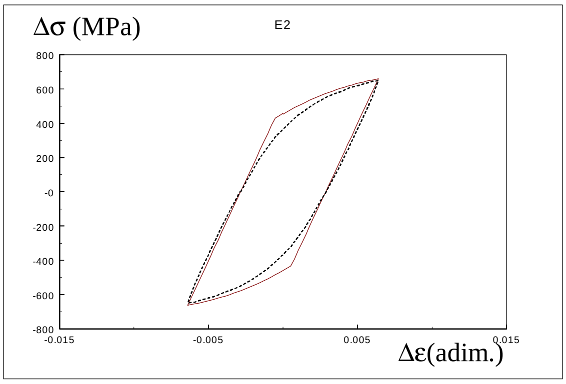

Fig.6 E2: Δσ x Δε Δε/2 = 0.64%

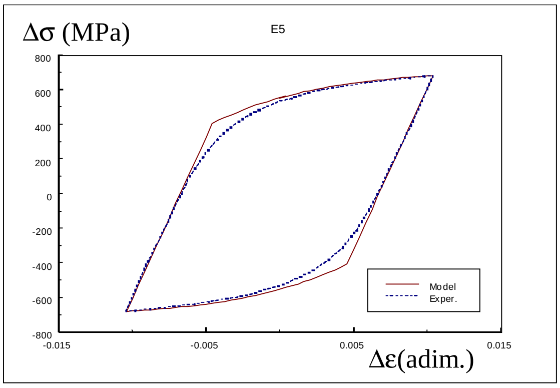

Fig.7 E5: Δσ x Δε Δε/2 = 1.04%

As it can be seen in Chimisso & Caligiana (1999) the results between the model cyclic curve and the experimental cyclic curve taken at the first hysteresis loop are so closed.

Figure (8) show the model results, for the E5 test, obtained for the cyclic softening from the first to de 64th cycle. Figure (9) compare the E2 experimental result with the model result for the 32th cycle.

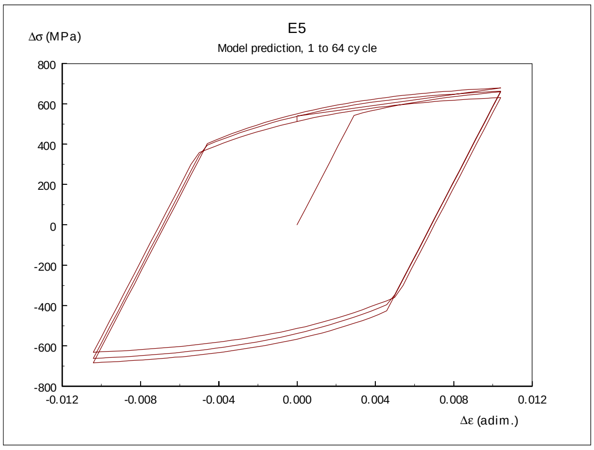

Fig. 8 E5 test: Δε/2 = 1.04%. 1 th to 64th cycle

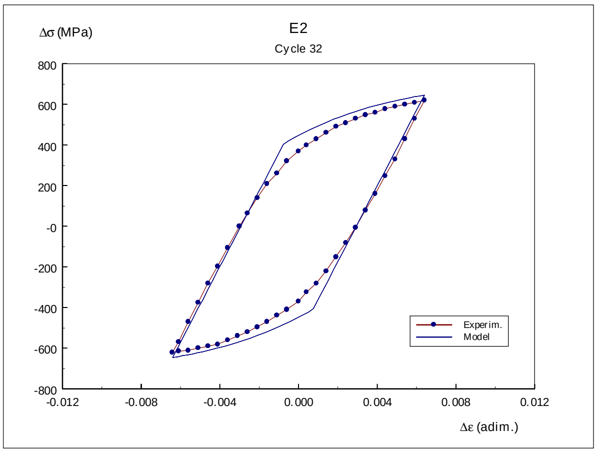

Fig. 9 E2 test: Δε/2 = 0.64%. 32 th cycle

Figures 10 and 11 compare the E5 experimental result with the model result for the 16th and 64th respectively.

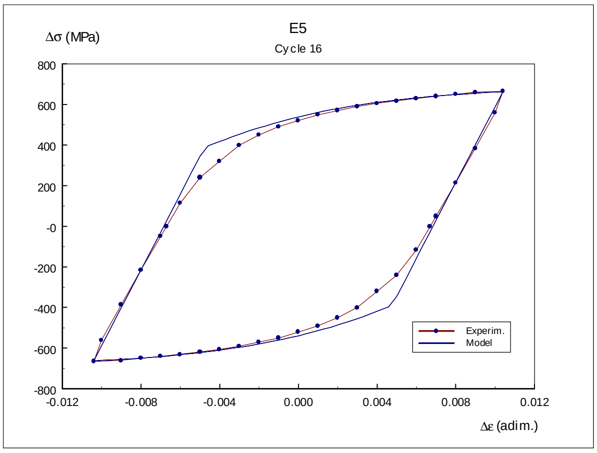

Fig. 10 E5 test: Δε/2 = 1.04%. 16 th cycle

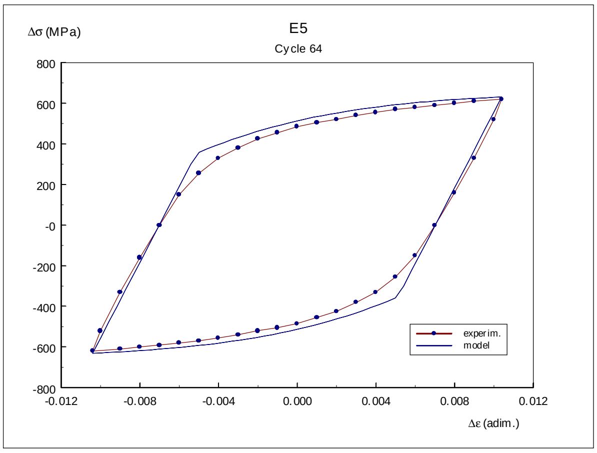

Fig. 11 E5 test: Δε/2 = 1.04%. 64 th cycle

8. CONCLUDING REMARKS

The ASTM A 471 study becomes interesting due to its continuous softening until fracture occurs. In this work the softening behaviour of this steel by a continuum elasto-plastic damage approach is considered. Assuming that the first cycle as a stable cycle, the isotropic hardening/softening influence must be neglected and a non-linear kinematic hardening law is sufficient to find the model hysteresis loop.

The isotropic behaviour is considered with an appropriate softening evolution law. The model allows a good results for the elasto-plastic hysteresis loop behaviour and for the continuously softening phenomenon and this is fundamental to extend the results into the continuum damage model.

Acknowledgement

Data and tests are obtained in the DIEM- Università degli Studi di Bologna, Italia. Our special gratitude to Professor Sergio Curioni the Dean of the Department. The first author is grateful to the **CNPq** (Brazilian Council for Scientific and Technological Development) support.

REFERENCES

- ASTM Standard Practice for Strain-Controlled Fatigue Testing, E 606-92.
- Bhambri, S.K., Prasad, C.R., Vasudevan, R., Low Cycle Fatigue Behaviour of a 1Cr-Mo-V Steelwith Bainite and Ferrite Microstructure, Int. J. Fatigue, 9 No 3, pp. 239-246, Butterworth & Co.,October 1987.
- Caligiana, G., Curioni, S. Freddi, A., Fatigue life evaluation in the motor vehicle industries, Int. J. of Vehicle Design, ed. M.A. Dorgham, Inderscience Ltd., U.K., 1987.
- Chimisso, F., Caligiana, G., Parameter Determination for Modelling the ASTM A471 Steel Behaviour in an Elasto-plastic Continuum Approach, Anais XV COBEM – Congresso Brasileiro de Engenharia Mecanica, Aguas de Lindoia, SP, Brasil, novembro 1999.
- Lemaitre, J. and Chaboche, J.L., Mechanics of Solid Materials, Cambridge University Press, Great Britain, 1990.
- Marquis, D., Modelisation et Identification de l' Ecrouissage Anisotrope des Mètaux, Thése de Troisieme Cycle, Universitè de Paris 6, France, 1978.
- Moosbrugger, J. C., and McDowell, D.L.,On a Class of Kinematic Hardening Rules for Nonproportional Cyclic Plasticity, J. of Eng. Materials and Technology, Trans. of the ASME,Vol. 111, pp. 87-98, January 1989.