WAVE PROPAGATION IN DISCONTINUOUS DAMAGED FIELDS

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Abstract

This paper presents a theoretical investigation, based on numerical simulations, of the wave propagation phenomenon in damageable elasto-viscoplastic one-dimensional medium. The degradation of the medium induced by inelastic deformations is described by a local continuum damage model in a context of internal variable theory. To solve the hyperbolic governing equations, the Glimm's scheme along with a splitting technique were used. Numerical simulations with discontinuous damage initial conditions are presented so that the influence of the damage on the wave propagation phenomenon is highlighted.

Keywords: Wave Propagation, Continuum Damage Mechanics, Elasto-viscoplasticity, Glimm's Scheme.

1. INTRODUCTION

Metallic materials, specially those used in high temperature environments, are known to be susceptible to inelastic deformations when exposed to moderately amplitude impact loading and/or transient complex loading histories. Structural components used in nuclear and thermohydraulics power plants are typical examples of particular interest.

Because of its importance in engineering applications, the analysis of dynamical problems involving the wave propagation in inelastic solids has been the subject of several researches in the past years (Belytshko, et al. 1987, Loret and Prevost, 1990 and Sluys et al., 1993). Even though several important features have been definitively addressed and significant progress has been achieved, a few works (Freitas Rachid et al., 1996 and Freitas Rachid et al., 1997) have been dedicated to the study of the degradation phenomenon (damage induced by inelastic strains) influence on the dynamical response of structures.

This paper reports a theoretical and numerical investigation of longitudinal wave propagation phenomenon in damageable elasto-viscoplastic bars under isothermal and small deformations. The material inelastic response is described by an internal constitutive theory with strong thermodynamics basis and the degradation phenomenon by means of local continuum damage theory.

The constitutive model used in this work exhibits strain-softening phenomenon yet gives rise to a unique solution to the wave propagation initial-boundary-value problem. The primary effect of the damage on the wave propagation analysis is that it affects the tensile wave speed. As a result, dispersive effects appear in the solution whenever the magnitude of inelastic strains is capable to cause damage evolution. To solve the non-linear hyperbolic equations describing the wave propagation phenomenon, a suitable numerical technique based on Glimm's method is proposed.

With this model is possible to analyse cases in which there exists discontinuous damage initial conditions. This is a case of interest when a structure, composed by several members with different loading histories, is exposed to an impact loading. Another relevant case occurs in practice when damaged stretches of a structure are replaced by pieces of virgin material.

2. CONSTITUTIVE EQUATIONS

The constitutive equations used in this work are derived in a context of an internal variable theory (Lemaitre & Chaboche, 1990). Due to the limited space, only its main features will be focused on. For further details, refer to the work of Freitas Rachid & Costa Mattos (1995).

For the isothermal evolution of an elasto-viscoplastic damageable solid, the local state is supposed to be characterized by the total strain tensor $\underline{\underline{\varepsilon}}$, the anelastic strain $\underline{\underline{\varepsilon}}^p$, by an internal variable D related with the damage and by two other internal variables $\underline{\underline{c}}$ and p related to the kinematical and isotropic hardening, respectively. The variable $\underline{\underline{c}}$ is a second-order tensor whereas p is of scalar nature. The variable $D \in [0,1]$ is a macroscopic quantity which can be interpreted as a local measure of the degradation of the material induced by deformation. If $D = 0$, the material is virgin and if $D = 1$ the material loses locally its mechanical strength.

The Helmholtz free energy ψ is assumed to be a differentiable scalar function of the state variables with the following form:

$$\rho\psi\left(\underline{\underline{\varepsilon}},\underline{\underline{\varepsilon}}^p,D,\underline{\underline{c}},p\right)=(1-D)\left\{\psi_e\left(\underline{\underline{\varepsilon}}-\underline{\underline{\varepsilon}}^p\right)+\psi_p(p)+\psi_c\left(\underline{\underline{c}}\right)\right\} \tag{1}$$

$$\psi_e\left(\underline{\underline{\varepsilon}}-\underline{\underline{\varepsilon}}^p\right)=\frac{1}{2}\left(\underline{\underline{\underline{C}}}\left(\underline{\underline{\varepsilon}}-\underline{\underline{\varepsilon}}^p\right)\right)\cdot\left(\underline{\underline{\varepsilon}}-\underline{\underline{\varepsilon}}^p\right),\ \psi_p(p)=b\left(p+\frac{1}{d}\exp(-dp)\right)+\sigma^y p,\ \psi_c\left(\underline{\underline{c}}\right)=\frac{1}{2}a\underline{\underline{c}}\cdot\underline{\underline{c}} \tag{2.1-2-3}$$

The term ψ_e is the elastic strain energy density and the terms ψ_p and ψ_c are the inelastic strain energy densities related to the internal variables $\underline{\underline{c}}$ and p. $\underline{\underline{\underline{C}}}$ is the classical symmetric fourth-order positive definite tensor of elasticity. In the above relations, a, b and d are material parameters. The so-called thermodynamical forces $\left(\underline{\underline{\sigma}},B^D,\underline{\underline{B}}^c,B^p\right)$ related to the internal variables $\left(\underline{\underline{\varepsilon}},\underline{\underline{\varepsilon}}^p,D,\underline{\underline{c}},p\right)$ are defined from the free energy potential by taking its partial derivatives. The relations between the state variables and the thermodynamical forces are the so-called state laws:

$$\underline{\underline{\sigma}}=-\rho\frac{\partial\psi}{\partial\underline{\underline{\varepsilon}}^p}=(1-D)\underline{\underline{\underline{C}}}\left(\underline{\underline{\varepsilon}}-\underline{\underline{\varepsilon}}^p\right) \tag{3.1}$$

$$B^D=-\rho\frac{\partial\psi}{\partial D}=\psi_e+\psi_p+\psi_c \tag{3.2}$$

$$\underline{\underline{B}}^c=-\rho\frac{\partial\psi}{\partial\underline{\underline{c}}}=-(1-D)a\underline{\underline{c}} \tag{3.3}$$

$$B^p = -\rho\frac{\partial\psi}{\partial p} = -(1-D)\Big(+b\big(1-\exp(-dp)\big)+\sigma^y\Big) \tag{3.4}$$

To complete the constitutive equations, evolution laws are required for the internal variables. These are obtained by introducing a differentiable scalar function $\Phi = \Phi\left(\underline{\underline{\sigma}}, B^D, B^p, \underline{\underline{B}}^c, \underline{\underline{\varepsilon}}, \underline{\underline{\varepsilon}}^p, D, p, \underline{\underline{c}}\right)$ of the thermodynamical forces and the internal variables, named dissipation potential. For this particular material Φ has the following form:

$$\Phi = \frac{k}{n+1}\left\langle\frac{F}{k}\right\rangle^{n+1} \tag{4}$$

where $\langle x\rangle = \max(0,x)$ and F is the yield function $F\left(\underline{\underline{\sigma}}, B^D, B^p, \underline{\underline{B}}^c, \underline{\underline{\varepsilon}}, \underline{\underline{\varepsilon}}^p, D, p, \underline{\underline{c}}\right)$:

$$F = f + \frac{\varphi}{2a}\left(\underline{\underline{B}}^c\cdot\underline{\underline{B}}^c - (1-D)^2 a^2 \underline{\underline{c}}\cdot\underline{\underline{c}}\right) + \frac{1}{2S_0}\left(\left(B^D\right)^2 - \left(\psi_e+\psi_p+\psi_c\right)^2\right)$$

$$f = f\left(\underline{\underline{\sigma}}, B^p, \underline{\underline{B}}^c\right) = J\left(\underline{\underline{\sigma}}+\underline{\underline{B}}^c\right) + B^p \text{ and } J\left(\underline{\underline{\sigma}}+\underline{\underline{B}}^c\right) = \left(\tfrac{3}{2}\left(\underline{\underline{\sigma}}+\underline{\underline{B}}^c\right)_{dev}\cdot\left(\underline{\underline{\sigma}}+\underline{\underline{B}}^c\right)_{dev}\right)^{1/2}$$

In the above relations, φ and S_0 are material parameters. $\left(\underline{\underline{\sigma}}+\underline{\underline{B}}^c\right)_{dev}$ is the deviatoric part of $\left(\underline{\underline{\sigma}}+\underline{\underline{B}}^c\right)$. When Φ is differentiated with respect to the arguments $\underline{\underline{\sigma}}$, B^D, B^p and $\underline{\underline{B}}^c$, the evolution laws are obtained:

$$\dot{\underline{\underline{\varepsilon}}}^p = \frac{\partial\Phi}{\partial\sigma} = \frac{3}{2}\left\langle\frac{F}{k}\right\rangle^n \frac{\left(\underline{\underline{\sigma}}+\underline{\underline{B}}^c\right)_{dev}}{J} \tag{5.1}$$

$$\dot{p} = \frac{\partial\Phi}{\partial B^p} = \left(\tfrac{2}{3}\dot{\underline{\underline{\varepsilon}}}^p\cdot\dot{\underline{\underline{\varepsilon}}}^p\right)^{1/2} = \left\langle\frac{F}{k}\right\rangle^n \tag{5.2}$$

$$\dot{\underline{\underline{c}}} = \frac{\partial\Phi}{\partial\underline{\underline{B}}^c} = \dot{\underline{\underline{\varepsilon}}}^p + \frac{\varphi}{a}\underline{\underline{B}}^c\dot{p} \tag{5.3}$$

$$\dot{D} = \frac{\partial\Phi}{\partial B^D} = -\frac{B^D}{S_0}\dot{p} \tag{5.4}$$

Using equations (3), it is easy to verify that $F\left(\underline{\underline{\sigma}}, B^D, B^p, \underline{\underline{B}}^c, \underline{\underline{\varepsilon}}, \underline{\underline{\varepsilon}}^p, D, p, \underline{\underline{c}}\right) < 0$ if and only if $f\left(\underline{\underline{\sigma}}, B^p, \underline{\underline{B}}^c\right) < 0$. If $f < 0$ then $\dot{\underline{\underline{\varepsilon}}}^p = 0$, $\dot{p} = 0$, $\dot{\underline{\underline{c}}} = 0$, $\dot{D} = 0$, and consequently, the material will behave elastically. When $B^p = -\sigma^y$ and $\underline{\underline{B}}^c = 0$, the condition $f \le 0$ is nothing else than the classical Von-Mises criterion. If $B^p = -\sigma^y$ and $\underline{\underline{B}}^c = 0$ at the time t=0, the evolution of the elastic domain (the set of the stress σ such that $f\left(\underline{\underline{\sigma}}, B^p, \underline{\underline{B}}^c\right) < 0$) will be characterized by an homothetical expansion or contraction (due to $B^p(t)$) and by a translation (due to $\underline{\underline{B}}^c(t)$) of the initial elastic domain (defined by the Von-Mises criterion).Equation (5.1) implies that $tr\left(\underline{\underline{\varepsilon}}^p\right) = 0$. This means that the inelastic deformation preserve the volume of the body. If the internal variable $\underline{\underline{c}}$ is zero at time t=0 then, from (3.3), (5.1) and (5.3), it is also easy to verify that $\mathrm{tr}(\underline{\underline{c}}) = \mathrm{tr}(\underline{\underline{B}}^c) = 0$. The variable $\dot{\underline{\underline{\varepsilon}}}^p$ is usually called the plastic strain and the internal variable p is called the cumulated plastic strain.

Equations (3) and (5) describe adequately the mechanical phenomena of damage, elasticity, plasticity (cyclic plasticity, strain-hardening and strain-softening), creep and relaxation observed in many metallic materials at high temperature. Experimental procedures to identify the material parameters a, b, d, σ^y, k, n, φ and S_0 can be found in (Lemaitre & Chaboche, 1990).

3. BALANCE EQUATIONS

The balance of linear momentum, along with the strain-displacement relationship, that governs the motion of a continuum solid body under the assumption of small displacements and deformations are (Salençon, 1990):

$$\rho \frac{\partial \underline{v}}{\partial t} = \mathrm{div}\underline{\underline{\sigma}} + \rho \underline{g}\,, \quad \underline{\underline{\varepsilon}} = 1/2\left(\mathrm{grad}\underline{u} + \mathrm{grad}\underline{u}^T\right) \tag{6.1-2}$$

In the above expressions $\underline{v}$, $\underline{u}$, $\underline{\underline{\sigma}}$ and $\underline{\underline{\varepsilon}}$ represent, respectively, the velocity vector field, the displacement vector field, the stress tensor field and the strain tensor field, which are all functions of the space coordinate x in the reference configuration and of the time t. The external body force per unit of mass $\underline{g}$ is assumed to be zero and the body supposed to be homogeneous, so that the mass density ρ is constant. Once the body geometry and the initial/boundary conditions have been specified, the balance (6) and constitutive (3) and (5) equations describe the problem of wave propagation in damageable elasto-viscoplastic medium.

4. PROBLEM FORMULATION

As a first step towards the analysis of the wave propagation phenomenon, we restrict the complexity of the problem by dealing with the one-dimensional situation. Let us consider a bar of length L of an isotropic damageable elasto-viscoplastic material whose axis coincides with the coordinate x. The origin of the coordinate system is the left end of the bar so that $x \in [0, L]$ Fig.1.

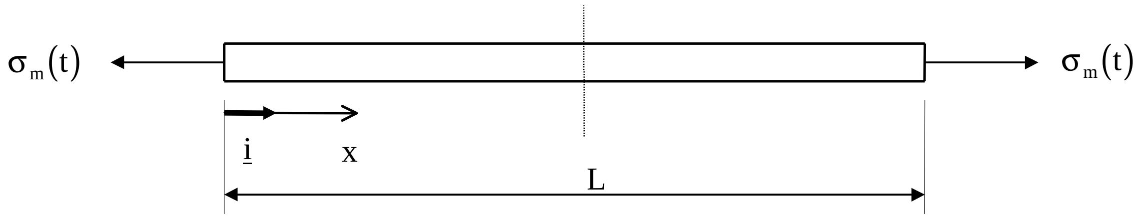


Fig.1 - One-dimensional bar of length L with stress prescribed at both ends

It is further assumed that the velocity and the stress tensor have only one non-vanishing component in the x direction, $\underline{v} = v\underline{i}$ and $\underline{\underline{\sigma}} = \sigma \underline{i} \otimes \underline{i}$, in which $\underline{i}$ is the unit length vector of the positive x direction and $\otimes$ stands for the usual tensorial product. As a consequence, there will exist only one independent component of the tensors $\underline{\underline{\varepsilon}}^p$ and $\underline{\underline{c}}$, $\underline{\underline{\varepsilon}}^p = \varepsilon^p \underline{i} \otimes \underline{i} - \frac{\varepsilon^p}{2} \underline{j} \otimes \underline{j} - \frac{\varepsilon^p}{2} \underline{k} \otimes \underline{k}$ and $\underline{\underline{c}} = c\underline{i} \otimes \underline{i} - \frac{c}{2} \underline{j} \otimes \underline{j} - \frac{c}{2} \underline{k} \otimes \underline{k}$.

Based on the considerations presented so far, equations (3), (5) and (6) can be reduced to the following system of partial differential equations:

$$\frac{\partial v}{\partial t} - \frac{1}{\rho}\frac{\partial \sigma}{\partial x} = 0, \quad \frac{\partial \sigma}{\partial t} - \rho\lambda^2 \frac{\partial v}{\partial x} + \frac{\sigma}{(1-D)}\frac{\partial D}{\partial t} + (1-D)E\frac{\partial \varepsilon^p}{\partial t} = 0 \tag{7.1-2}$$

$$\frac{\partial \varepsilon^p}{\partial t} = f(\sigma, c, p, D), \quad \frac{\partial c}{\partial t} = g(\sigma, c, p, D), \quad \frac{\partial p}{\partial t} = h(\sigma, c, p, D), \quad \frac{\partial D}{\partial t} = l(\sigma, c, p, D) \tag{7.3-4-5-6}$$

where the functions f, g, h and l of the arguments (σ, c, p, D) are obtained from (5) and $\lambda = \lambda(D)$, the wave speed with which disturbances propagate in the bar, is given by $\lambda = \sqrt{(1-D)E/\rho}$.

It is initially assumed that the bar is at rest and free of stress. As boundary conditions, we consider the bar is submitted to a prescribed stress at its edges. These conditions can be stated as follows:

$$v(x, t=0) = \sigma(x, t=0) = \varepsilon^p(x, t=0) = p(x, t=0) = c(x, t=0) = 0 \tag{8}$$

$$\sigma(x=0, t) = \sigma(x=L, t) = \sigma_m(t) \tag{9}$$

$$D(x \le L/2, t=0) = D_0^L; \; D(x > L/2, t=0) = D_0^R \tag{10}$$

It can be shown that the system of equations (7) is merely hyperbolic in time for $D \in [0,1)$ since its eigenvalues are all real (although not distinct) and the associated eigenvectors form a complete set of linear independent vectors (Jeffrey, 1976). The hyperbolicty of (7) ensures the well-posedness of the initial-boundary-value problem (7)+(8)+(9).

5. NUMERICAL METHOD

In order to construct the numerical approximation for the solution $(\sigma, v, \varepsilon^p, D, c, p)$ of (7) from time t^n to time $t^{n+1} = t^n + \Delta t$ the following splitting algorithm is adopted:

i. First, an initial approximation $(\overline{\sigma}, \overline{v})$ is obtained by advancing Δt in time through:

$$\frac{\partial v}{\partial t} - \frac{1}{\rho}\frac{\partial \sigma}{\partial x} = 0, \quad \frac{\partial \sigma}{\partial t} - \rho\lambda^2 \frac{\partial v}{\partial x} = 0 \tag{11}$$

via Glimm's method. As Cauchy data for the Glimm's scheme, the values of (σ, v, D) at time t^n are used.

ii. Once $(\overline{\sigma}, \overline{v})$ has been evaluated, the numerical approximation for the solution at time t^{n+1} is finally obtained by advancing in time with the same time-step Δt through:

$$\frac{\partial\sigma}{\partial t} = -\frac{\sigma}{(1-D)} l - (1-D)Ef\,, \quad \frac{\partial\varepsilon^p}{\partial t} = f\,, \quad \frac{\partial c}{\partial t} = g\,, \quad \frac{\partial p}{\partial t} = h\,, \quad \frac{\partial D}{\partial t} = l \qquad (12)$$

To do this, the Euler's scheme is employed by taking as initial conditions (ε^p, D, c, p) at time t^n and $(\bar{\sigma}, \bar{v})$.

This procedure may be repeated throughout until a desired time of simulation has been reached. Before using Glimm's method for solving equations (11), one must know the solution of the associated Riemann problem.

5.1 Riemann Problem

The Riemann problem associated to (11) is an initial-value problem of the form (Smoller, 1983),

$$\left(\sigma(x, t=0)\;;\;\; v(x, t=0)\right) = \begin{cases} (\sigma_l, v_l)\;; & \text{for } \; x < 0 \\ (\sigma_r, v_r)\;; & \text{for } \; x > 0 \end{cases}$$

with discontinuous coefficient λ such that:

$$\lambda = \begin{cases} \lambda_l = \lambda(D_l) = \text{constant}\;; & \text{if } \; x < 0 \\ \lambda_r = \lambda(D_r) = \text{constant}\;; & \text{if } \; x > 0 \end{cases}$$

where σ_l , v_l , D_l and σ_r , v_r , D_r are constants.

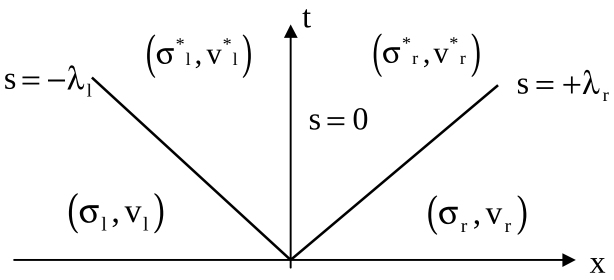
The generalized solution of this particular problem depends only on the ratio $\xi = x/t$ and is constructed by connecting the left state (σ_l, v_l) and the right state (σ_r, v_r) to the intermediate states $(\sigma^*{}_l, v^*{}_l)$ and $(\sigma^*{}_r, v^*{}_r)$, which must be determined, by shock waves (discontinuous solutions). The shock speeds *s* must satisfy the Rankine-Hugoniot jump condition. In this case this relationship is:

$$\left.\begin{aligned} s\rho[v] &= [\sigma] \\ s[\sigma] &= \rho\lambda_l^2[v] \end{aligned}\right\} \text{ if, } \; x < 0 \quad \text{ or } \quad \left.\begin{aligned} s\rho[v] &= [\sigma] \\ s[\sigma] &= \rho\lambda_r^2[v] \end{aligned}\right\} \text{ if, } \; x > 0 \qquad (13)$$

where $[\varsigma]$ denotes the jump of ς across adjacent states.

From equations (13) it is easy to see that shocks propagate with speeds $s = -\lambda_l$ if $x < 0$ or $s = \lambda_r$ if $x > 0$. Thus, the solution is constructed by connecting the left state (σ_l, v_l) to the intermediate state $(\sigma^*{}_l, v^*{}_l)$ with shock speed $s = -\lambda_l$. Similarly, the right state (σ_r, v_r) is connected to the intermediate state $(\sigma^*{}_r, v^*{}_r)$ by a shock with speed $s = \lambda_r$. To complete the solution, however, it remains to connect the state $(\sigma^*{}_l, v^*{}_l)$ and $(\sigma^*{}_r, v^*{}_r)$. This is done by imposing a stationary shock (a shock with speed $s = 0$) at $x = 0$ as illustrated in Fig. 2. It should be mentioned that such kinds of shock appearing in the solution of this problem are actually the so-called contact discontinuities (Smoller, 1983).

The generalized solution of this problem is unique and is summarized in Fig. 2 which displays the regions in the x-t plane where the solutions is defined.



$$(\sigma, v) = \begin{cases} (\sigma_l, v_l), & \text{if } -\infty < \xi < -\lambda_l \\ (\sigma_l^*, v_l^*), & \text{if } -\lambda_l < \xi < 0 \\ (\sigma_r^*, v_r^*), & \text{if } 0 < \xi < \lambda_r \\ (\sigma_r, v_r), & \text{if } \lambda_r < \xi < +\infty \end{cases} \qquad (14)$$

Fig. 2 - Solution of Reimann Problem in the x-t plane.

In the above relations, σ_l^*, v_l^* and σ_r^*, v_r^* are given by the following expressions:

$$\sigma_r^* = \sigma_l^* = \frac{\left(\lambda_r \sigma_r + \lambda_l \sigma_l + \rho\left(\lambda_l^2 v_l - \lambda_r^2 v_r\right)\right)}{\left(\lambda_r + \lambda_l\right)}$$

$$v_l^* = \frac{\lambda_r}{\lambda_l\left(\lambda_r + \lambda_l\right)}\left(\lambda_l v_l + \lambda_r v_r + \frac{\left(\sigma_l - \sigma_r\right)}{\rho}\right), \qquad v_r^* = \frac{\lambda_l}{\lambda_r\left(\lambda_r + \lambda_l\right)}\left(\lambda_l v_l + \lambda_r v_r + \frac{\left(\sigma_l - \sigma_r\right)}{\rho}\right)$$

5.2 Glimm's Method

Glimm's method is a numerical scheme which employs the solution of the associated Riemann problem to generate approximate solutions of the hyperbolic equations, when they are subjected to arbitrary initial data. The main idea behind the method is to appropriately gather the solution of as many Riemann problems as desired to successively march from time t^n to time $t^{n+1} = t^n + \Delta t$. To do this it is first necessary to approximate the data at the time t^n by piecewise constant functions.

Consider a uniform partition $0 = x_1 < \cdots < x_i < \cdots < x_{i+1} < \cdots < x_{N+1} = L$ of the interval [0,L] such that $\Delta x = x_{i+1} - x_i$. Let us also assume that at time t^n there already exists a piecewise constant approximation of (σ, v, D) such as :

$$\sigma(x, t^n) = \sigma_i = \sigma(x_i, t^n), \quad v(x, t^n) = v_i = v(x_i, t^n), \quad D(x, t^n) = D_i = D(x_i, t^n)$$

for $x \in (x_i - \Delta x/2, x_i + \Delta x/2)$ and $1 \le i \le N+1$.

For each two consecutive states (σ_i, v_i, D_i) and $(\sigma_{i+1}, v_{i+1}, D_{i+1})$, $1 \le i \le N$, there is defined and solved a Riemann problem centered at $\overline{x} = x_i - \Delta x/2$, according to the preceding section. Now, with solution of these N Riemann problems and by introducing a random sequence of numbers $\{\theta_n\}$, $\theta_n \in (0,1)$, it is finally obtained a piecewise constant approximate solution of (11) at time t^{n+1} as follows:

$$(\overline{\sigma}, \overline{v})(x, t^{n+1}) = \left(\overline{\sigma}_j(\xi_i), \overline{v}_j(\xi_i)\right), \text{ where } \quad \xi_i = \frac{\left(x_i + \theta_n \Delta x - \overline{x}\right)}{\Delta t} \qquad (15)$$

for $x \in (x_i - \Delta x/2, x_i + \Delta x/2)$ and $1 \le i \le N+1$ with j defined as $j = i$, if $\theta_n \le 1/2$ or $j = i+1$, if $\theta_n > 1/2$.

In these expressions $(\overline{\sigma}_j, \overline{v}_j)$ stands for the j-th Riemann problem with initial data (σ_i, v_i, D_i) and $(\sigma_{i+1}, v_{i+1}, D_{i+1})$ whose solution is given by equation (14).

In order that nearby shocks of adjacent Riemann problem do not interact with each other, the time-step Δt is chosen to satisfy the Courant-Friedrichs-Lewy condition:

$$\Delta t = t^{n+1} - t^n \leq \frac{\Delta x}{2|\lambda|_{max}}$$

where $|\lambda|_{max}$ is the maximum absolute value of the shock speeds, taking into account the N Riemann problems at time t^n.

With reference to the algorithm described at the beginning of this section, the aforementioned approximation $(\overline{\sigma}, \overline{v})$ is the one provided by equation (15).

6. NUMERICAL EXAMPLES

Aiming to better understand the phenomenon of wave propagation in a damageable elasto-viscoplastic medium, the physical problem, described in section 4 is numerically simulated here. To characterize severe impact loading, we consider as boundary conditions triangular-shape stress pulses, having a rise time of 0.5 ms and duration of 1 ms, so that:

$$\sigma(x=0,t) = \sigma(x=L,t) = \begin{cases} \sigma_m t\,, & \text{if } 0 \leq t \leq 0.5\text{ms} \\ -\sigma_m t + 2\sigma_m\,, & \text{if } 0.5\text{ms} \leq t \leq 1\text{ms} \\ 0\,, & \text{if } t \geq 1\text{ms} \end{cases}$$

where σ_m represents the stress pulse amplitude

The bar, which is 16m long, is made an AISI 316L stainless steel and is submitted to a temperature of 600ºC. At this temperature the material parameters are (Lemaitre and Chaboche, 1990):E=130 GPa, ν=0,27, σ^y =6 MPa, k=150 MPas , b=80 MPa, d=10, n=12, a=17 GPa, φ =300 and S_0 =2 kPa.

After time t=0, the impulsive stress pulses generated at both extremities will propagate towards the midpoint of the bar. Afterward these pulses will interact with each other and will be reflected at the bar's ends until the steady state has been reached or rupture has occurred. The wave speed in the virgin material is $\lambda(D=0) = 4082\ m/s$.

Evidences of the influence of the degradation on the bar response can be found at (Freitas Rachid et al., 1997). Here we are concerned about the response of the bar when we have discontinuous damage initial conditions. We consider as damage initial condition $D_0^L = 0$ and $D_0^R = 0.9$, as shown in equation (10).

In what follows, the dynamical response of the bar will be analyzed for an input stress pulse σ_m = 190 MPa. Spatial dimensionless stress distributions along the bar are displayed in fig. 3 for different instants. The evolution of the wave propagation is depicted in an horizontal sequence.

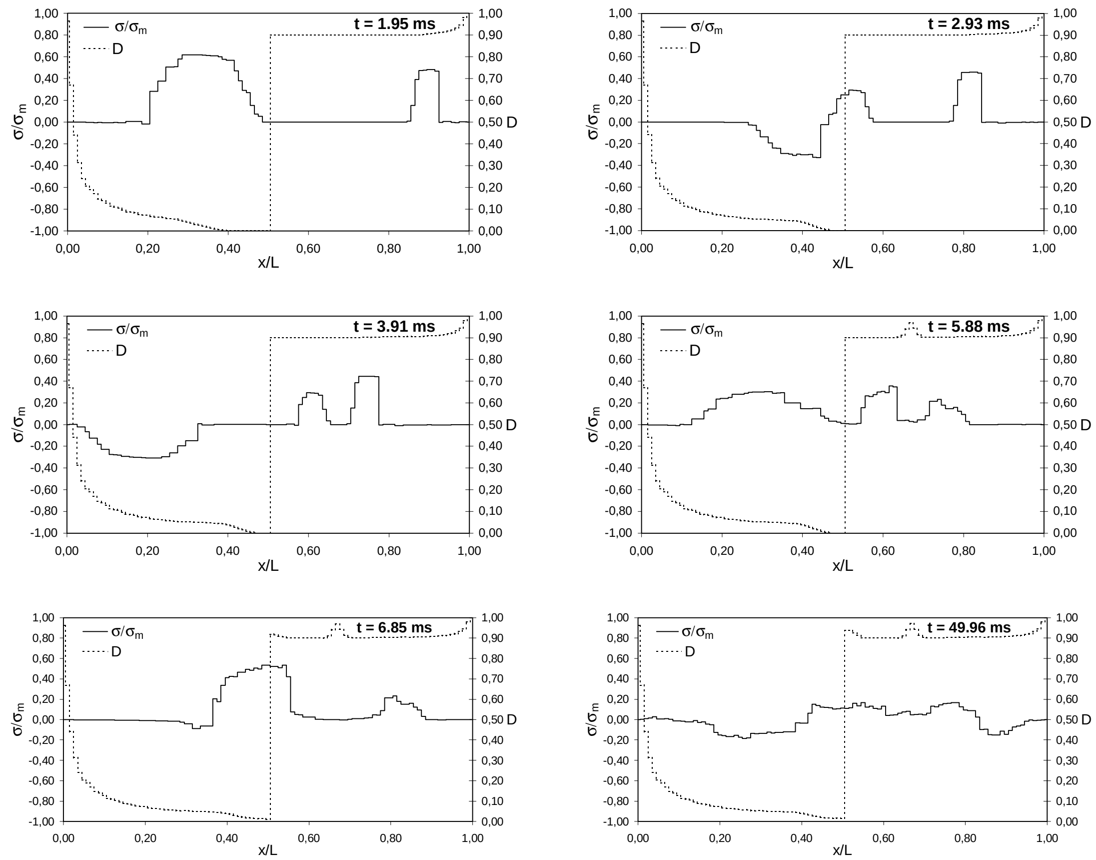


Fig. 3 - Dimensionless stress and damage fields at several time instants for σ_m = 190 MPa

At time t = 1.95 ms both stress pulses are propagating towards the midpoint of the bar. The right pulse is smaller than the left one because damage is an additional dissipative mechanism. In addition, at time t = 2.93 ms, the right stress pulse reaches the midpoint before the other. The pulses propagate with different speeds due to the reduction of the local wave speed induced by damage. So, in the left side of the bar the wave speed is greater than in the right one. At the same instant, t = 2.93 ms, when the left pulse reaches the midpoint, we can see the reflection of the part of the pulse. Notice that it occurs before the first superposition of the stress pulses. So, at t = 3.91 ms, we can see clearly three stress pulses propagating in the rod. t = 5.88 ms is the instant after the first superposition and we can see the damage evolution around the point x/L = 0.66. When the right pulse reaches the midpoint, at t = 6.85 ms, it occurs another superposition. The reflected pulse reaches the midpoint at the same time.

The last picture, at t = 49.96 ms, shows the final stress and damage fields. The most critical regions susceptible to damage evolution are not only those close to the extremities and to the center of the bar. There exist another region susceptible to damage evolution whose localization and dimension depends on damage initial conditions.

7. FINAL REMARKS

The influence of the degradation on the wave propagation phenomenon in an elasto-viscoplastic medium has been characterized by means of numerical simulations in a bar submitted to impact stress pulses at both ends. Besides of attenuating the stress waves and introducing significant delay in the signals, the discontinuous damage initial condition may cause the reflection of the part of the stress pulse where the damage discontinuity is sufficiently great.

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