ON THE HEIGHT OF MAXIMUM SPEED-UP IN ATMOSPHERIC BOUNDARY LAYERS OVER LOW HILLS

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Abstract
In this paper, we present a study on the height of maximum speed-up, *l*, for flows over low hills under neutral atmosphere. We consider the four most well known expressions to calculate *l*, due to Jackson and Hunt (JH), Jensen (JEN), Claussen (CL) and Beljaars and Taylor (BT). In the analysis, we present a formal demonstration of the fact that *l* can, in fact, be calculated as the inner layer depth, where inertia and turbulent forces balance. The need for such a demonstration has received little attention by researchers over the years. We also propose a new value for the constant in CL's expression and confirm that JEN's expression gives better results than JH's one. Regarding this fact, we suggest that JH's expression should definitively be substituted by JEN's or CL's with the proposed constant.

Key-words: inner-layer depth, maximum speed-up, flow over hills, atmospheric boundary layers.

1. INTRODUCTION

There has been a remarkable interest over the years on estimating the height above the ground where wind speed-up is a maximum in the atmospheric boundary layer (ABL) over low hills. The idea is strongly appealing for wind power specialists and for those who want to calculate wind loads on various kinds of structures.

Many expressions to calculate this height, often denoted by *l*, have been proposed since the idea appeared. The most well known expressions come from the pioneering work of Jackson and Hunt (1975) and from later works by Jensen et al., (1984) Claussen (1988) and Beljaars and Taylor (1989) (hereafter JH, JEN, CL and BT, respectively). The expressions obtained by these authors, respectively, read

$$(l/L_h)\ln(l/z_0) = 2\kappa^2, \tag{1}$$

$$(l/L_h)\ln^2(l/z_0) = 2\kappa^2, \tag{2}$$

$$(l/L_h)\ln(l/z_0) = \text{const.}, \tag{3}$$

$$(l/L_h)\ln^n(l/z_0) = \text{const.}, \tag{4}$$

where L_h is the half-length of the hill, defined following JH as 'the distance from the hilltop to the upstream point where the elevation is half its maximum'; z_0 is the roughness length and k is the von Karman's constant, adopted as 0.39, as suggested by a recent work by Frenzen and Voguel (1995). A comparative study of the relative merits of the four expressions can be found in Walmsley and Taylor (1996). If we divide the four expressions by z_0 and rewrite the constants in (3) and (4) as $C_1 k^2$ and $C_n k^2$, respectively, we get:

$$l^+ \ln(l^+) = 2\kappa^2 L_h{}^+ , \quad (5)$$

$$l^+ \ln^2(l^+) = 2\kappa^2 L_h{}^+ , \quad (6)$$

$$l^+ \ln(l^+) = C_1 \kappa^2 L_h{}^+ , \quad (7)$$

$$l^+ \ln^n(l^+) = C_n \kappa^2 L_h{}^+ , \quad (8)$$

where $l^+ \equiv l/z_0$ and $L^+ \equiv L/z_0$. Based on only one experimental result, CL suggests that $C_1(x)k^2 = 0.09$, which means that $C_1 = 0.59$ for κ=0.39. After comparison with model results, BT suggests that n=1.4 to 1.6, depending on the turbulence closure assumed, and that $C_n(x)\kappa^2 = 0.26$ to 0.55 (also depending on closure), which yields $C_n = 3.62$ to 1.71.

In his work, JH divides the ABL in two regions. In the first, more external region, the effects of inertia dominate and in the second, more internal, turbulent forces have to be considered too. In the same work, they identify l as the depth of the inner layer. Although this idea was adopted in most of the works that followed, none of the works we had access to demonstrate that l, calculated as the inner layer depth (where inertia and turbulent forces balance), was also the height of maximum speed-up (hereafter called l_{max}, to avoid confusion). All works considered restricted themselves to verify that the field data for l_{max} confirmed the proposed expressions for l. In a review paper, Taylor et al. (1987) state that 'l is probably best considered as a scale height for the inner layer rather than the height at which something specific occurs.' The authors, however, follow JH's hypothesis and compared the results of their expression for l with field data for $l_{max.}$ In a more recent paper, Beljaars and Taylor (1989) say that 'since the inner-layer depth, l, has been introduced by means of order of magnitude considerations, its practical definitions is somewhat arbitrary'. Apart from that, a lot of discussion is found in the literature about the relative merits of expressions (1)-(3). The following points where summarised from Walmsley and Taylor (1996):

- independent of which expression is used, l is always considered to be the height of maximum speed-up;
- values predicted by the JH expression are too high when compared to field results and no reasonable adjustment of z_0 can fix the problem;
- values predicted by the JEN expression agree very well with observed values in the whole range of variation of L_h/z_0;
- CL's expression gives better agreement to observed values than JEN's at the specific value of L_h/z_0 with which it was calibrated;
- Model results suggest a value for n between 1 and 2 in the BT expression;
- More observational data is required to solve definitively the question.

In this work, we present a new deduction for the JEN's expression obtained through slightly modified order of magnitude arguments applied to the hypothesis that l is the height where inertia and turbulence forces balance in the ABL. To our present knowledge, this deduction is both new and simpler than the previous ones. We also present, for the first time, a formal demonstration that *l*, calculated this way is, in fact, the height of maximum speed-up. We compare our results to JH's, CL's and observational data showing that JEN's expression agrees better with observational data than JH's and that the constant in CL's expression can be calibrated to agree well with field data. We also ratify Walmsley and Taylor's conclusion that more observation is needed at some ranges of the parameter L_h/z_0.

2. DEFINITIONS

Consider one isolated 2D hill in the middle of an otherwise flat terrain, of constant roughness and under a neutrally stratified atmosphere. For our purposes, we consider a hill to be a topographical variation with characteristic length about 5 Km and height less than 500m. A hill is called low when it slope never exceeds 20^o. Fig. 1 illustrates the main features of a typical low hill. The vertical co-ordinate z, is defined as the height above the local terrain rather than the vertical height above sea level.

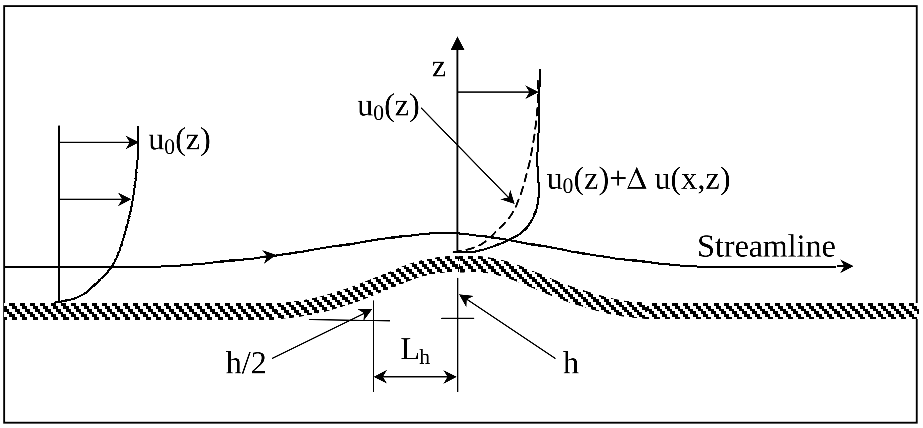


Fig.1. Definitions of h, L_h, Δu, u_0 and z.

In the case under study, we assume that the vertical profile of the horizontal mean wind is essentially logarithmic far from the hill. Hereafter we refer to this profile as $\bar{u}_0(z)$, and the location upwind of the hilltop (HT) where it is found as the reference site (RS). The RS profile suffers the influence of the hill in such a way that it is modified by a speed-up quantity $\Delta\bar{u}(x,z)$ and becomes $\bar{u}(x,z)$ at a given point over the hill. Thus:

$$\bar{u}(x,z) \equiv \bar{u}_0(z) + \Delta\bar{u}(x,z), \tag{9}$$

where Δu is positive at HT, because the flow is accelerated to satisfy the continuity equation. If we divide the speed-up by the RS velocity, we have the relative speed-up, ΔS:

$$\Delta S(x,z) \equiv \frac{\bar{u}(x,z)}{\bar{u}_0(x,z)} - 1. \tag{10}$$

The height where $\Delta\bar{u}$ is maximum, l_{max}, is defined as

$$l_{max} \equiv z(\Delta\bar{u} = \Delta\bar{u}_{max})\,. \tag{11}$$

Conversely, we can write $\Delta\bar{u}_{max} \equiv \Delta\bar{u}(x, l_{max})$.

In the next section, we develop an order of magnitude analysis of the governing equations to obtain an expression for l_{max}. We also establish the co-ordinate system most appropriate for our purposes.

3. ORDER OF MAGNITUDE ANALYSIS

To obtain the expression for the height of maximum speed-up we proceed in two steps: first we obtain the expression for the inner layer depth, l, and second, we show that this depth is really the maximum speed-up height, l_{max}. In order to establish a co-ordinate system suitable for the task, we follow the work of Kaimal and Finnigan (1994), which recommends the use of the streamline co-ordinates for distorted flows of this type. The work of Finnigan (1983) gives the mean mass conservation and the mean x-momentum equations for 2D flows in this system, respectively, as

$$\frac{\partial\bar{u}}{\partial x} + \frac{\partial\bar{w}}{\partial z} = 0\,, \tag{12}$$

$$\bar{u}\frac{\partial\bar{u}}{\partial x} = -\frac{1}{\bar{\rho}}\frac{\partial\bar{p}}{\partial x} - \frac{\partial\overline{u'^2}}{\partial x} - \frac{\partial\overline{u'w'}}{\partial z} + \frac{\overline{u'^2} - \overline{w'^2}}{L_a} + 2\frac{\overline{u'w'}}{R} - g_x\frac{\bar{T}}{\bar{T}_0} + V_x\,. \tag{13}$$

In eqs. (12) and (13), x is the direction parallel to the streamlines and $\bar{u}$ and u’ are the mean and turbulent velocities in this direction, respectively. The direction normal to the streamlines is z, and $\bar{w}$ and w’ are the corresponding velocities. The thermodynamic mean pressure is denoted by $\bar{p}$, the mean density by $\bar{\rho}$, the mean temperature by $\bar{T}$, the reference mean temperature by $\bar{T}_0$, the x-component gravity acceleration by g_x and the x-component mean viscous force by V_x. R and L_a are flow length scales and they are related to the mean variables through $R = \bar{u}/(\Omega + \partial\bar{u}/\partial z)$ and $L_a = \bar{u}/(\partial\bar{u}/\partial z)$, where Ω is the mean vorticity component in the z direction in the original Cartesian co-ordinate system.

Supposing the existence of a region where inertia and turbulence terms balance we write

$$\bar{u}\frac{\partial\bar{u}}{\partial x} \sim \frac{\partial\overline{u'w'}}{\partial z}\,. \tag{14}$$

To evaluate expression (14) we assume that $x \sim L_h$ and that inertia and turbulence terms balance in the region where $z \sim l$. We also assume that $\bar{u} \sim \bar{u}_0$ which means that $\Delta\bar{u} << \bar{u}$. Finally, we assume that $u' \sim w' \sim u_*$, where u_* is the friction velocity. With these assumptions we have

$$\frac{\bar{u}_0^{\,2}(l)}{L_h} \sim \frac{u_*^{\,2}}{l}\,. \tag{15}$$

To transform this order of magnitude relation in an equality, we introduce an unknown function $C_2(x)$ of order one, such that $\bar{u}_0{}^2(l)/L_h = C_2(x)\cdot u_*{}^2/l$ and, thus,

$$\frac{l}{L_h} = C_2 \frac{u_*{}^2}{\bar{u}_0{}^2(l)}\,. \tag{16}$$

This relation is presumably valid for all incident wind profiles. For a logarithmic profile of the form $\bar{u}_0/u_* = (1/\kappa)\ln(l/z_0)$, we can write $l/L_h = C_2\,\kappa^2/\ln^2(l/z_0)$ which, upon dividing and multiplying by z_0, gives

$$l^+ \ln(l^+) = C_2 \kappa^2 L_h{}^+\,. \tag{17}$$

This equation is identical to that of JEN except for the constant C_2, to be determined. In the mainframe of an order of magnitude analysis, this can only be accomplished through a comparison with observational data. We do that in the next section. First, however, we shall show that l, calculated from (17), is indeed the maximum speed-up height.

Consider the mass conservation and x-momentum equations in Cartesian co-ordinates. They are essentially the same as eqs. (12) and (13), except for the curvature terms containing R and L_a in eq. (13). Returning to the hypothesis that the inertia and turbulence terms balance in the inner region, we have

$$\bar{u}\frac{\partial \bar{u}}{\partial x} + w\frac{\partial \bar{u}}{\partial z} \sim \frac{\partial \overline{u'w'}}{\partial z}\,. \tag{18}$$

As we now want to obtain results about the height where $\Delta\bar{u}$ is maximum, we substitute $\bar{u} = \bar{u}_0 + \Delta\bar{u}$ into (18) to obtain

$$\bar{u}_0 \frac{\partial \Delta\bar{u}}{\partial x} + \overline{w}\frac{\partial \bar{u}_0}{\partial z} \sim \frac{\partial \overline{u'w'}}{\partial z}\,, \tag{19}$$

after considering that $\partial\bar{u}_0(z)/\partial x = 0$ and that $\Delta\bar{u} << \bar{u}$. If we are to obtain the height where $\Delta\bar{u}$ is maximum, we must impose that $\partial\Delta\bar{u}/\partial z = 0$ at $z = z_h + l_{max}$ (in Cartesian co-ordinates). Differentiating (19) with respect to z allows us to substitute this condition in the resulting equation and find

$$\frac{\partial \bar{u}_0}{\partial z}\frac{\partial \Delta\bar{u}}{\partial x} + \frac{\partial \overline{w}}{\partial z}\frac{\partial \bar{u}_0}{\partial z} + w\frac{\partial^2 \bar{u}_0}{\partial z^2} \sim \frac{\partial^2 \overline{u'w'}}{\partial z^2}\,, \tag{20}$$

at $z = z_h + l_{max}$. Substituting for the orders of the individual terms yields

$$\frac{\bar{u}_0}{(z_h + l)}\frac{\Delta\bar{u}}{L_h} + \frac{\overline{w}\bar{u}_0}{(z_h + l)^2} \sim \frac{u_*{}^2}{(z_h + l)^2}\,. \tag{21}$$

The magnitude order for *w* can be obtained from the mass conservation equation as $\overline{w} \sim \overline{u}_0 (z_h + l_{max})/L_h$. Substituting this expression on eq. (21), multiplying by $(z_h + l_{max})^2$ and introducing a function $C_3(x)$ to obtain an equality, we get

$$\overline{u}_0 (\Delta\overline{u} + \overline{u}_0) \frac{(z_h + l_{max})}{L_h} = C_3 u_*^{\,2} . \tag{22}$$

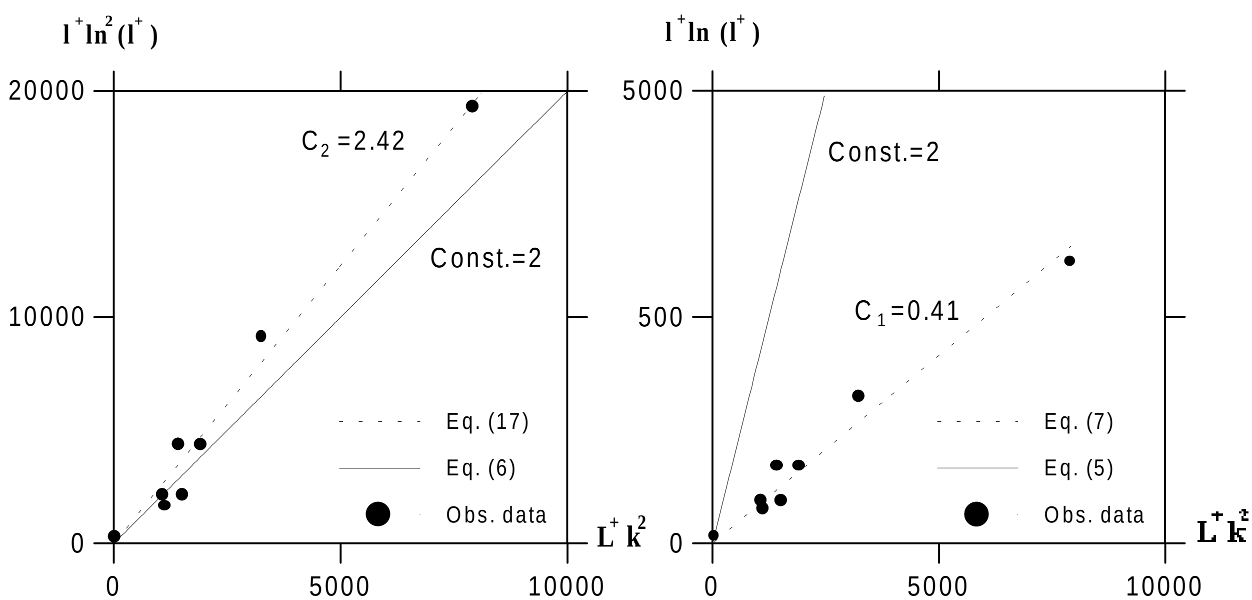
Recalling that $u_0 >> \Delta u$ and returning to the streamline co-ordinate system, where $z_h + l_{max}$ is simply equal to *l*, we finally get

$$\frac{l}{L_h} = C_3 \frac{u_*^{\,2}}{\overline{u}_0^{\,2} (l_{max})} . \tag{23}$$

Eq. (23) is identical to eq. (16), except for the constant to be determined. As the constant value is entirely arbitrary, we can set $C_2(x) = C_3(x)$, therefore proving that $l = l_{max}$, indeed. In the next item, we calculate the value of $C_2(x)$ through comparison with field data and test the overall capability of expression (17).

4. COMPARISON WITH OBSERVATIONAL DATA

Many field studies provide the observational data needed for our purposes. The most popular data can be obtained from the work of Taylor et al. (1987) and Copin et al. (1994). The available measurements for *l* are represented in fig. (2) together with the results of eqs. (17), (5), (6) and (7).



(2.a) (2.b)

Fig.2. Non-dimensional height of maximum speed-up.

Equation (17) was tested for some values of $C_2(x)$ and we verified that good agreement is obtained with $C_2(x) = 2.0$ for the HT. All measurements were made at the HT and, therefore, it was not possible to assess the x dependence of $C_2(x)$. The result, however, confirms JEN's equation, expression (6). We also obtained the best fit value of $C_2(x) = 2.42$, for eq. (17), which has never been proposed before. We believe, however, that more field data is necessary before we can state that this value is definitive. Our tests also showed that the agreement between observation and JH's expression, eq. (5), is acceptable only for the Bungendore Ridge (BR) results (Bradley, 1983), not represented in fig. (2.b). This conclusion is confirmed by other workers, e.g. Mickle et al. (1988) and Taylor and Walmesley (1996). The BR results deserve some attention, nevertheless.

According to Taylor et al. (1987), the z_0 value varied between 0.002 and 0.005 m during the BR experiment. Those limits, with *l* estimated as 5m, (following Taylor et al., 1987) correspond to the values that were well represented by eq. (5). In addition, the speed-up vertical profile presented a very broad maximum, from the first measurement point up to the height of 8m. The point represented in figs. (2.a) and (2.b) were calculated supposing that *l*=1 m and adopting z_0=0.0035 m as an average value. This point agrees very well with eq. (17).

Fig. (2.b) shows a plot of the field data against eqs. (5) and (7), for the same experimental data used on fig. (2.a). Agreement is good in this case too, except in the BR case. Comparison of eqs. (5) and (7) shows that CL's expression differs from JH's only by a constant. In fact, before CL proposed his expression, Teunissen et al. (1987) had suggested that a different value for the constant could correct its prediction ability. CL proposed $C_1 = 0.59$ based on one field result (210^{o} wind direction case of ASK). Based on fig. (2.b), we propose a value of $C_1 = 0.41$, which seems to fit the observational data better, as a whole. To our present knowledge, this result is also new.

5. CONCLUSIONS

In this paper, we present a study on the height of maximum speed-up for flows over low hills under neutral atmosphere. The flow was assumed to be two-dimensional and the upwind velocity profile was considered to be logarithmic. Furthermore, the determination of the function $C_2(x)$ was made from observational data obtained over the HT; so the result is restricted to this site.

In our analysis, we show that *l*, calculated as the inner layer depth, is indeed the height of maximum speed-up, l_{max}. We also propose a value for the function $C_1(x)$ in CL's expression, eq. (6), and confirm that JEN's expression gives better results than JH's. Regarding this fact, we suggest that JH's expression should definitively be substituted for JEN's or CL's (which seem to work just as well). We also believe that the BT's results ratify that the best expression lies between CL's and JEN's, as proposed here.

One thing worth of note is that the simple demonstration we present here, showing that $l=l_{max}$, has apparently passed unnoticed over the years. We speculate that this is probably due the fact that comparison between predictions for l_{max} and observations of *l* has always showed good agreement, in most cases.

It is also worth noting that the demonstration of the equality $l=l_{max}$ could have been used itself as a new form of obtaining *l*. It uses the classical hypothesis about the relative order of inertia and turbulence terms and introduces the requirement that $\partial\Delta\bar{u}/\partial z = 0$, which guarantees that $l=l_{max}$. Furthermore, it makes no use of turbulence closure models and allows to a re-calibration of the function $C_2(x)$ in case it is required by new observational data available.

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