MAXIMUM AND AVERAGE VELOCITIES IN PIPE FLOWS - AN EXPERIMENTAL STUDY

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Summary
This work deals with an experimental study of fully developed isothermal turbulent gas flows inside a 47 mm diameter pipe, with Reynolds numbers in the range 1,000 < Re < 100,000 and Mach number M < 0.13. This Reynolds number range encompasses the laminar and turbulent flow regimes. Instead of just correlating traditional pipe flow variables with Reynolds number, as usually found in the literature, the present work correlates the average velocity with the velocity in the center of the pipe cross section. These velocities have been both obtained experimentally, by using flow meters and Pitot tubes. It has been observed that, for each pair temperature-pressure, there is a one-to-one relation between these velocities, provided we have fully developed flows.

Keywords: Pipe Flows, Isothermal Flows, Laminar and Turbulent Flows, Experimental Correlation.

1. INTRODUCTION

The flow inside pipes is a physical problem that inspires high economic interest from different industrial branches. Among applications of this problem, we can quote, for example, the transportation of petroleum and its derivatives through pipe systems covering extensions of hundreds of kilometers. Another similar example is the transportation of natural gas through pipe systems of international range. We can also mention several segments of the oil industry, the supply of natural gas to consumers, and general flows of gases or liquids in refrigeration systems.

In this work, we consider an experiment where air passes through a pipe in steady state and at low velocities. Within this framework, the isothermal and incompressible hypotheses are adequate representative physical models for the flows. For this purpose, after laying down theoretical considerations for the laminar and turbulent flow of incompressible fluids, the methodology for the experiment is described. In the sequel, the collected data is displayed and analyzed. From this analysis, it can be concluded that a Pitot tube can be used with advantages with respect to traditional laminar flow meters by providing essentially the most valuable information with lower head losses and at lower costs.

2. THEORETICAL BACKGROUND

2.1. Incompressible fluid theory

The fully developed isothermal flow of an incompressible fluid inside a horizontal tube of constant cross section may be described by a simplified form of the Navier-Stokes equations. Let us consider, for the purpose of analysis, a Cartesian coordinate system whose *x*-axis is aligned with the tube centerline. In this one-dimensional flow problem, the fluid has no velocity in the *y* and *z* directions. Then, the *v* and *w* fluid velocity components are considered as non-existing. Only the velocity component *u*, in the *x*-direction, is allowed to vary as a function of the flow itself. Furthermore, the flow is considered here steady. Thus, all derivatives of fluid variables with respect to time vanish. Finally, the flow is fully developed, meaning that the *u* velocity component does not change along the *x* direction.

Under the hypothetical conditions just described, if we use another set of cylindrical coordinates (x, r, θ), where x is the direction of the tube centerline – coincident with the x rectangular coordinate – the continuity equation becomes

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}=0 \qquad \text{or} \qquad \frac{1}{r}\frac{\partial}{\partial r}(r\,\dot{r}) + \frac{1}{r}\frac{\partial\dot{\theta}}{\partial\theta} + \frac{\partial u}{\partial x} = 0 \tag{1}$$

From the analysis of this equation, we reach the conclusion that in this fully developed isothermal flow there is only one velocity component, $u = u\,(r)$, not existing any flow in the radial direction. Such flow is said to be axisymmetric.

The momentum differential equation in cylindrical coordinates now reduces to

$$\rho\; u\;\frac{\partial u}{\partial x} = -\frac{dp}{dx} + \rho\; g\; x + \frac{1}{r}\frac{\partial}{\partial r}(r\,\tau) \tag{2}$$

where τ can represent either laminar or turbulent shear, ρ is the air specific mass, p is the static pressure, and g is the local gravity acceleration.

With a straightforward procedure, we can integrate equation (2) to find out the shear distribution across the pipe, by using the fact that $\tau = 0$ at r = 0. The result of this procedure is

$$\tau = \frac{1}{2}r\frac{d}{dx}(p+\rho\;g\;z) = (\text{constant})\;(r) \tag{3}$$

Thus, this result shows that the shear varies linearly from the centerline to the wall, for either laminar or turbulent flows.

For internal flows in ducts, another important non-dimensional number, obtained through dimensional analysis considerations, is the Darcy friction factor f . This factor can be represented in many ways. In this work it will assume the following form:

$$\frac{8\;\tau_w}{\rho\,V^2} = f = F\left(\text{ Re },\ \frac{\varepsilon}{d}\right) \qquad\qquad \text{Re} = \frac{\rho\,V\,d}{\mu} \tag{4}$$

where d is the diameter of the tube cross section, V is the average velocity of the flow, τ_w is the shear at the tube wall, and Re stands for the flow Reynolds number.

2.2. Laminar flow case

If one solves this problem for the fully developed pipe flow in the laminar case, one obtains an exact expression for the velocity profile, which is given by

$$u = \frac{-1}{4\mu} \frac{dp}{dx} (R^2 - r^2) \tag{5}$$

In this result, R is the radius of the pipe and μ is the fluid viscosity coefficient.

If one solves the problem for the average and the maximum velocities, we obtain that the first is half of the second, i.e.

$$V = \frac{1}{2} u_{\max} \tag{6}$$

There is an exact theoretical relation for the laminar Darcy friction factor, which is given by

$$f = \frac{64 \ \mu}{\rho \ V \ d} = \frac{64}{\mathrm{Re}} \tag{7}$$

2.3. Turbulent flow case

For turbulent flows, it is much harder to solve the differential equations displayed in section 2.1. This higher difficulty is an intrinsic characteristic of the very nature of turbulent flows. In a certain moment of the calculation process, we obtain two indefinite constants, whose values should be defined empirically. A first reasonable proposal for their values has been made by Nikuradse (1933). In this work we have used the values k = 0.41 and B = 5.0 for these constants, as suggested by Knudsen (1958):

$$\frac{u(r)}{u^*} \cong \frac{1}{k} \ln \frac{(R-r)\ u^*}{\nu} + B \qquad\qquad u^* = \sqrt{\frac{\tau_w}{\rho}} \tag{8}$$

In consequence, we obtain

$$V = \frac{u_{\max}}{1 + 1.33\sqrt{f}} \tag{9}$$

The turbulent Darcy friction factor cannot be obtained theoretically in this case. Moody, in 1944, as described by Shames (1973) and by White (1994), plotted, over several experiments, what is now known as the Moody chart for pipe friction. This chart can be obtained through the following formula:

$$\frac{1}{f^{1/2}} = -2.0 \ \log \left(\frac{\varepsilon/d}{3.7} + \frac{2.55}{\mathrm{Re} \ f^{1/2}} \right) \tag{10}$$

This result is an accepted design formula for turbulent friction. Its accuracy is known to be ± 15 percent for design calculations of flows where the Reynolds numbers $\mathrm{Re} < 10^8$ and $0.05 < (\varepsilon/d) < 10^{-6}$. Here, ε is a parameter associated to the roughness of the wall.

There is an alternate formula, given in an explicit way, that yields results less than 2 percent different from those provided by equation (10). This formula reads as follows:

$$\frac{1}{f^{1/2}} \approx -1.8 \ \log \left(\frac{6.9}{\mathrm{Re}} + \left(\frac{\varepsilon/d}{3.7} \right)^{1.11} \right) \tag{11}$$

3. EXPERIMENTAL METHODOLOGY

For the present experimental investigation, a continuous atmospheric airflow generator, designed and built in the Aeronautical Systems Division at the Aeronautics and Space Institute (ASA-IAE), has been used. This generator channels the flow through an aeronautic tube that is 30 m long, until the flow gets to the outflow regulator valve. This valve is responsible for the manual control of the flow in the test line. This apparatus makes the flow to be continuous and facilitates the attainment of the steady state. Permanent regimes are quickly obtained, in a matter of few seconds.

The next step consists in making the flow to pass through the element being tested. The test specimen is a tube smooth in the interior, with constant cross section, where a Pitot probe is conveniently mounted in the central longitudinal axis. This Pitot probe provides us with the proper means of measuring the flow maximum velocity. After this, the flow passes through a laminar flow element, called Meriam. Figure 1 presents photos of these main experimental devices. Further details of the laminar flow meters and of the entire set-up used in the experiment are given in Figures 2 and 3.

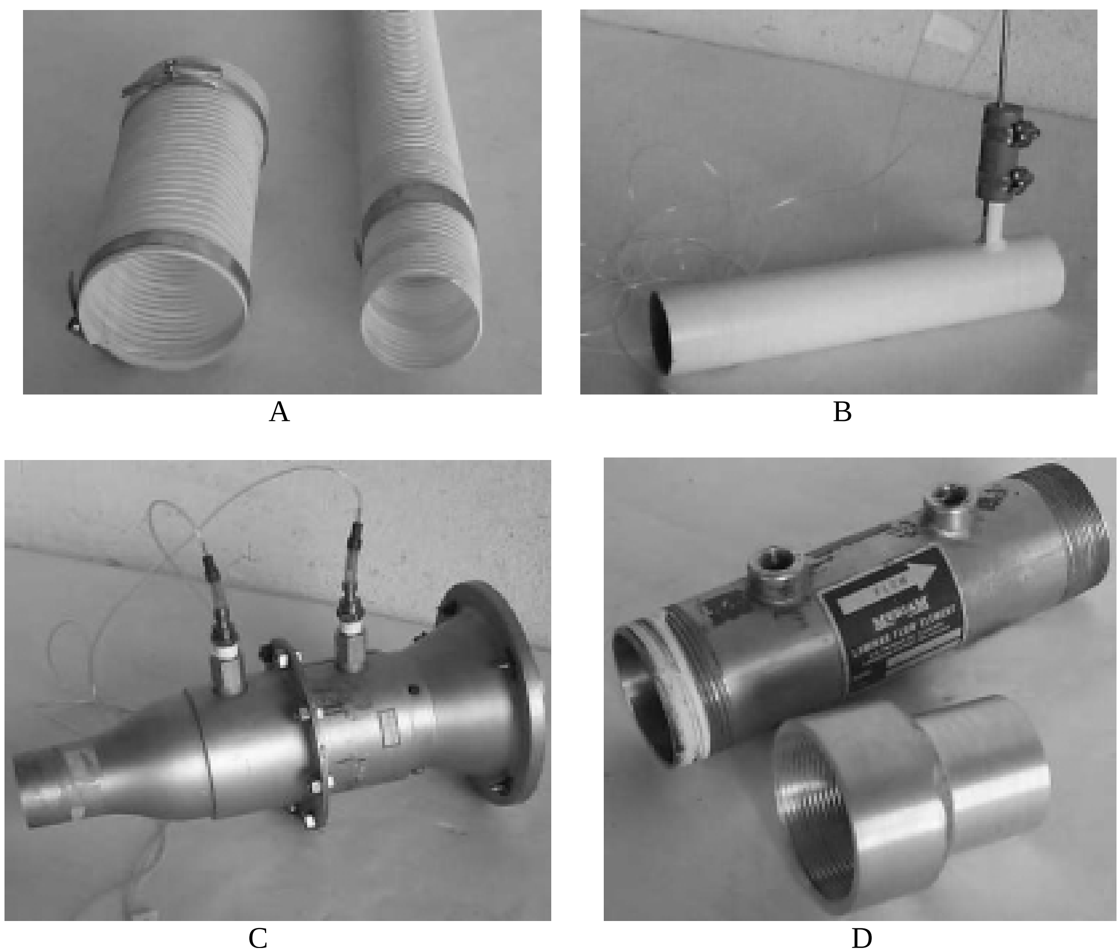

Figure 1. Main devices used in the experiment: A) aeronautical hoses; B) Pitot probe installed in a rigid tube; C) Laminar flow meter Meriam 50MC2-2SF; D) Laminar flow meter Meriam 50MH10-2.

Since there is no source or sink of air in the way between the valve and the laminar element, and since the flow is steady, the airflow rate is the same at each cross section along the test line. This airflow rate divided by the area of the cross section gives the average velocity of the flow in the section.

Therefore, by knowing the dimensions of the pipe cross section where the Pitot probe is located, we can get the maximum air velocity from the Pitot data, as well as the average velocity from the airflow data in that cross section.

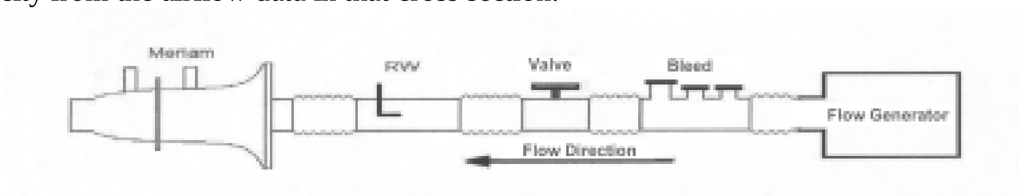

Figure 2. Experimental set-up without honeycomb and with the laminar flow meter Meriam 50MC2-2SF.

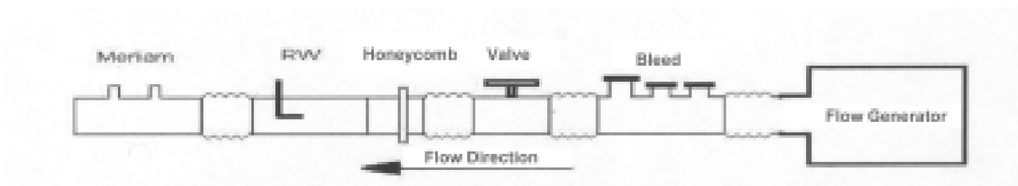

Figure 3. Experimental set-up with honeycomb and with the laminar flow meter Meriam 50MH10-2.

The measurement techniques used in this experiment have followed the recommendations given by Doebelin (1983) and Figliola (1995). More details about the experimental set-up can be obtained from a report by Pan (1999).

Four different experiments have been conducted, as described in Table 1:

Table 1 : Scheme of the experiments conducted.

Experiments	Laminar Element	Use of honeycomb?
1 and 2	50MH10-2	no / yes
3 and 4	50MC2-2SF	no / yes

Honeycomb may be used to turn more homogeneous the flow inside the pipe. In order to determine the influence of this option, Table 1 shows that the use of two different laminar elements has been repeated with and without honeycomb. The results obtained in the four experiments are described in the next section` .

4. DATA AND ITS CALCULATION

Both the Pitot tube and the laminar elements collect data and furnish as output, in each case, a pressure difference Δp. This difference is evaluated by a pressure transducer together with a voltmeter, both coupled to a stabilized current generator.

The pressure transducer has been calibrated using a Betz manometer of 0.05 mm H_2O precision as a standard reference. The laminar elements have their own calibration curves provided by the manufacturer

In summary, the steps taken to obtain the desired measurements are:

1. read, from the laminar element, in the voltmeter, a voltage in mV, deducting the zero value;
2. with the calibration curve of the transducer, the associated pressure difference Δp is obtained;
3. with p and T (ambient values) and the Meriam calibration curve, we get the flow rate in cubic meters per minute;
4. if we divide the previous result by the tube cross sectional area, we get the flow average velocity;
5. the steps 1 and 2 are repeated, but now taking readings from the Pitot tube;
6. with p and T (ambient values again) we obtain the maximum velocity; and
7. a correction for low Reynolds numbers following Anderson (1991) is applied and the final maximum velocity value is obtained.

5. RESULTS

The velocity range that has been used in the experiment was from 0 to 43 m/s, corresponding to Mach numbers from 0 to 0.13. The tube had a diameter *d* of 0.047 m. The associated Reynolds number range was $10^3 < Re < 10^5$. Within this framework, the incompressible flow theory can be used with propriety.

According to White (1994), the roughness ε inside the tube is about 0.15 mm. So, the relation ε/d becomes 0.003 (0.3%). If we apply this parameter value in equation (10), and, then, subsequently in equation (8), we obtain the theoretical relation between Re and V/u_{max}, for this experimentation.

The relation between Δp for the Pitot tube (called ΔR) and Δp for the laminar elements (called ΔM) can be shown in Figure 4. Notice that the relation between these pressures yields a well-defined curve, and that the adherence of the experimental measurements to this curve varies from 98% to 99.5%. These observations serve to validate the present experimental method.

The relation between V and u_{max} obtained experimentally is shown in Figure 5. It can be noted that again both curves are well defined from the experimental points, something that reinforces the validity of the experimental method.

Regardless of Reynolds number variations, a one-to-one relation between V and u_{max} has been obtained and the usage of the Pitot tube has been shown to have high accuracy for the desired use. Therefore, it can replace laminar elements with advantages, keeping a good dependability, low deviation, and smaller costs, by a factor of ten.

The difference between the curves implies probably a discrepancy in the calibration of at least one of the laminar elements. The use of both elements at the same time was not imagined in the beginning of the experiment planning, but can be easily implemented in order to find a correction for the deviation between them.

The relation between the Reynolds number and the velocity ratio V/u_{max} obtained in the experiments is shown in Figure 6, where the value obtained from theory is also plotted. The agreement between experiment and theory starts at Re = 8,000, and the variations, according to the Reynolds number, are up to 6% (for the 50MH10-2 Meriam laminar element) and 11% (for the 50MC2-2SF Meriam laminar element).

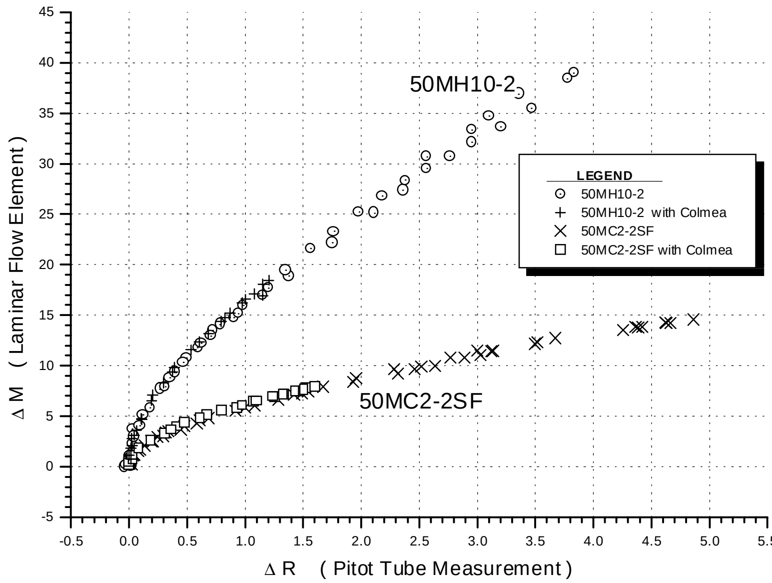

Figure 4 – Pressure differences at the Pitot and the flow meters.

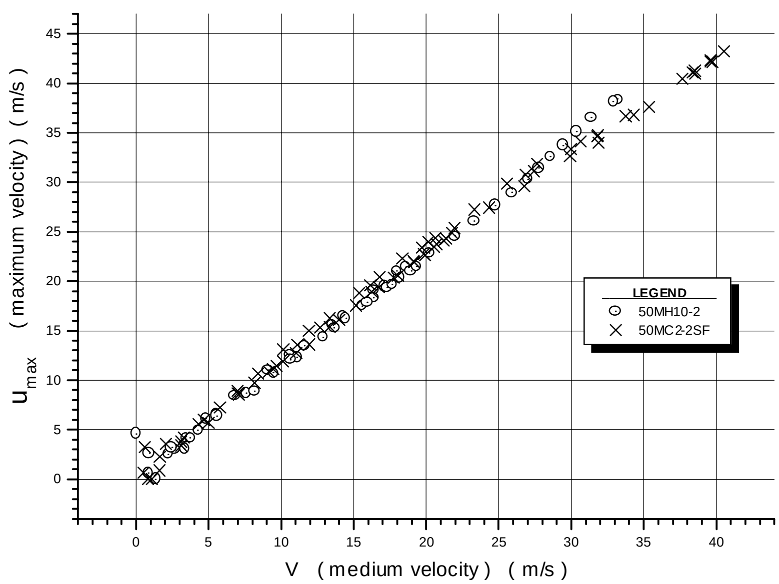

Figure 5 – Relation between the average and maximum velocities.

The data sharpness obtained by the sequencing of the experimental points suggests that its response is good, with 1 until 2% precision, since the standard instruments used give the reality of the phenomena involved.

6. CONCLUDING REMARKS

The relation between the average velocity V and the maximum velocity u_{max} obtained experimentally in this work has shown good alignment and precision of about 1% in the Pitot's data reading. It seems to have a little deviation error in the calibration of the laminar element 50MC2-2SF for Reynolds numbers greater than 70,000.

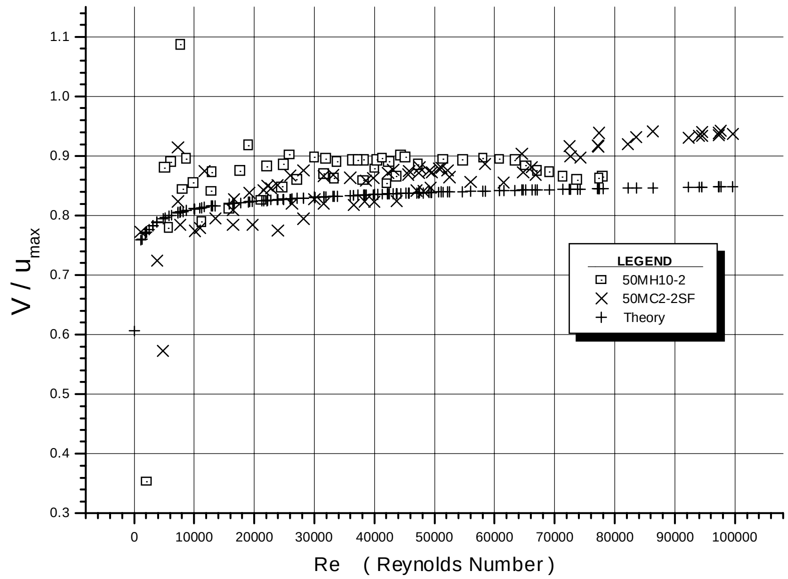

Figure 6 – The influence of the Reynolds number on the average to maximum velocity ratio.

The main conclusion is that the Pitot tube can be used as an alternative equipment for flow measurements of this type with advantages, the major of them being the lower costs and the decrease in head losses. The utility range for this method starts at Reynolds number 8,000 and goes up indefinitely, since a superior limit has not been found in the present experiments. On the other hand, no valid results have been obtained for the laminar flow range.

For future works, it is suggested a new calibration of the laminar elements (Meriam), used here, with a primary standard, in order to couple both experimental curves. Furthermore, it is suggested the expansion of this experiment for fluids of low and high densities, as well as a deeper investigation into the laminar flow range.

7. ACKNOWLEDGEMENTS

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8. BIBLIOGRAPHY

- Anderson Jr., J.D., 1991, “Fundamentals of Aerodynamics”, 2nd edition, McGraw-Hill, New York.
- Doebelin, E.O., 1994, “Measurement Systems”, 4th edition, McGraw-Hill, New York.
- Figliola, R.S. and Beasley, D.E., 1995, “Theory and Design for Mechanical Measurements”, 2nd Edition, John Wiley & Sons, New York.
- Knudsen, J.G., 1958, “Fluid Dynamics and Heat Transfer”, McGraw-Hill, New York.
- Nikuradse, J., 1933, “Strömungsgesetze in Rauhen Rohren”, VDI Forschungsh. 361; English translation, NACA Tech. Mem. 1292.
- Pan, A.G.B., 1999, “Estudo Experimental de um Mensurador de Fluxo de Ar”, Internal Report, ITA-CTA, São José dos Campos, Brazil.
- Shames, I.H., 1973, “Mechanics of Fluids”, McGraw-Hill, New York.
- White, F.M., 1994, “Fluid Mechanics”, 3rd Edition, McGraw-Hill, New York.