TIME ALIASING INDUCED ERRORS IN DIGITAL DOUBLE INTEGRATION

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Abstract

Displacement measurements using the analog double integration of an accelerometer signal presents severe problems, as recently discussed by Ribeiro *et al.* (97). A digital technique introduced by Ribeiro *et al.* in 99, based on a high-pass FIR filter, can solve most of those problems, which are associated with the low frequency signal components, as confirmed by a series of displacement measurements with frequencies components between 1 and 10Hz in laboratory tests. The only important error of this new method occurs in the first seconds of the measurements due to a phenomenon called time aliasing. This paper discusses this error.

Keywords: Displacement Measurements, Accelerometer Signal Integration, Time Aliasing.

1. INTRODUCTION

The actual set of displacements of a huge structure is very important information in structural integrity studies, since displacements are proportional to the elastic stresses. Therefore, if the time history of the displacement is correctly measured, it can be used to determine the peaks and valleys of the loading and the correspondent accumulated damage. However, the direct measure of the displacements history is a difficult task, since all displacement transducers are differential, i.e., require a fixed reference to work properly.

In the other hand, accelerometers are inertial, and can properly work without a fixed reference. However, the accelerometer signal must be double integrated in order to measure displacements. Many commercial charge amplifiers have an analog double integrator amplifier, but they present so many significant problems that in most cases the errors induced by the integration can spoil the experimental results, as shown in Ribeiro *et al.* (1997).

Digital techniques can solve most of those problems, since the digital integration can be easily made after sampling the acceleration signal and do not introduce errors due to the non-linear phase response of the analog double integrator. The problem of this type of integration is due to the signal low frequency components, which introduce a time-increasing error in the displacement signal, known as drift or zero shift. A low pass FIR filter can be used to remove this error. This process is shown in Ribeiro *et al.* (1999). This digital method solves the problem present in the analog double integration but introduces another one called time aliasing. This effect is cited in Brigham (1988) and is called end effect.

This effect is cited in Thornhill *et al.* (1983, pp. 227-231) as an error in the measurement of force using the measurement of accelerations and in Thornhill *et al.* (1983, pp. 227-231) it is explained. In Smith *et al.* (1996) the duality of aliasing and leakage is shown.

The purpose of this paper is to study this error, and to show its effect on the accuracy of the digital double integration process.

2. THE INSTRUMENTATION SYSTEM

The analog instrumentation system used to measure displacements was a piezoelectric accelerometer model 4371 and a charge pre-amplifier model 2635, both manufactured by Bruel and Kjaer (B&K). This system has the advantages of being precise, compact, and relatively easy to use. Besides, it has a tradition of good service in general-purpose vibration measurements. Figure 1 shows the block diagram of this system.

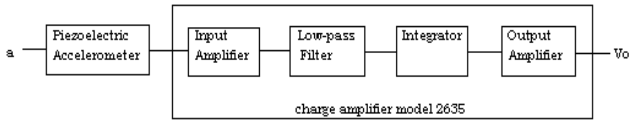

Figure 1. Diagram block of charge amplifier.

However, as discussed in the papers already mentioned above, the use of this system integrator block is not a good choice to measure displacements. Therefore, a laptop with an A/D card has been added to the system, with the purpose of sampling the acceleration signal and executes the necessary mathematical tasks to obtain the displacement signal.

3. THE INTEGRATION METHOD

Figure 2 can be used to obtain the recursive equation that relates the *F(t)*, the integral of the function *f(t)*, by the trapezoidal rule. The Z-transform of this equation, which can be easily implemented in MATLAB or in most mathematical software, is given by (Ribeiro *et al.*, 99)

$$F(z) = \frac{\Delta t}{2}\frac{z^{-1}+1}{z^{-1}-1} f(z) \qquad (1)$$

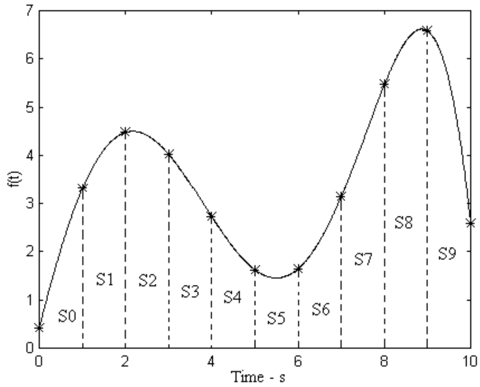

Figure 2. Representation of the trapezoidal rule to make integration of the function *f(t).*

The problem of this integration is the low frequency error it introduces. Figure 3 shows the double integration of a simple 5Hz sinusoidal acceleration signal. The internal algorithm available in the Spectrum Analyzer HP model 35670A realized this integration.

An increasing error due to the zero shift can be seen. This error is always present in this type of numerical double integration and to reduce it, it is necessary to remove the low frequency components of the signal.

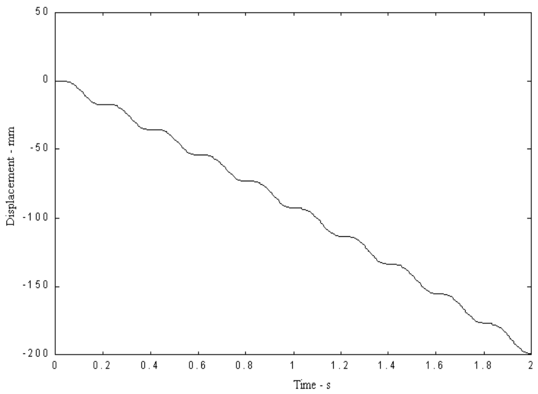

Figure 3. The double integration of a periodic acceleration signal measured using the HP 3570A algorithm.

Figure 4 shows the low pass filter proposed to remove this effect. This filter has a sampling rate of 512Hz and 8192 points. The cut-off frequency is at about 0.9Hz.

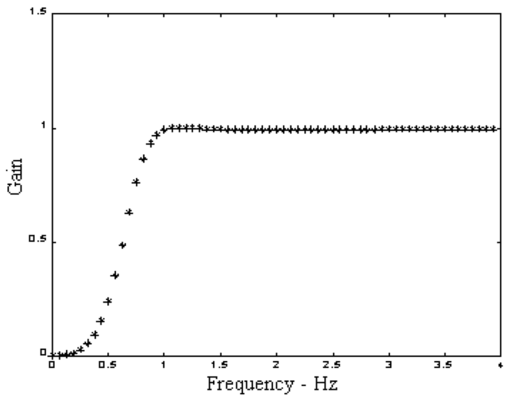

Figure 4. Frequency response of the FIR filter proposed.

4. EXPERIMENTAL RESULTS

Figure 5 shows the measurement of a relatively complex displacement signal using this digital technique. In full line is the accelerometer signal double integration and in dotted line is the signal from an LVDT used as a reference for comparison.

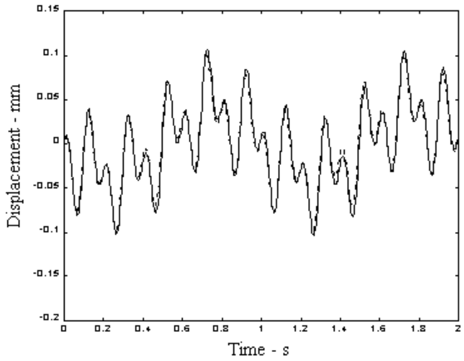

Figure 5. Displacement measured using the digital double integration, signal from LVDT in dotted line and signal from acceleration signal in full line.

However, depending on the frequency components of the acceleration signal, the digital integration can distort the resulting displacement measurement by an error called time aliasing, as shown in Figure 6.

In frequency domain, the frequency aliasing is the effect of frequencies of higher values is summed to lower frequencies values due to low sample rate. Similarly, in time domain the time aliasing is the effect of future values of the signal is summed to past values, causing distortions like the one shown in Figure 6.

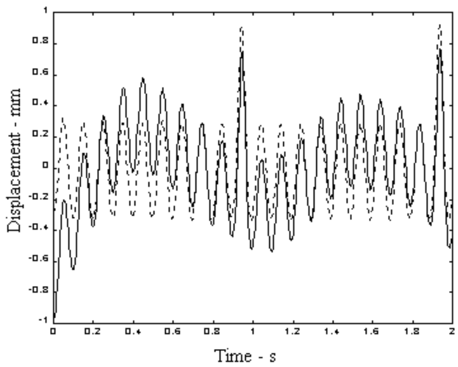

Figure 6. Time aliasing occurring during displacement measurement, signal from LVDT in dotted line and signal from acceleration signal in full line.

This signal was used since it is a good example of error induced by time aliasing. The peaks have the effect of inducing more frequency components, especially low frequency components. The presence of these low frequency components makes the signal be distorted when it is filtered.

5. THE PROBLEM OF TIME ALIASING

The FIR filter used introduces this error. To better understand this problem, Figure 7 shows the filtering of a rectangular signal with a fundamental frequency of 2.0156Hz. This filtering process is a simulation; in full line is the signal being filtered and in dotted line is the filtered signal. The error introduced by the filtering is evident. Since the signal has no frequencies below 2Hz, the filtering would not introduce any modification in the signal, however the time aliasing is present. Figure 8 shows the time response of the filter in full line and the difference between the curves shown in Figure 7, that is the error introduced, in dotted line. The similarity between the curves can be seen.

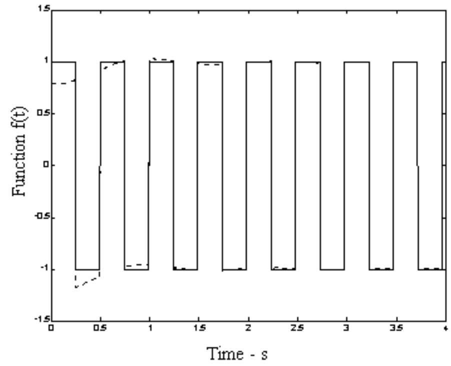

Figure 7. Filtering of a rectangular signal using the FIR filter with a 0.9Hz cut-off frequency, original signal in full line and filtered signal in dotted line.

The error was induced this time since the frequency of the signal is such that its power spectrum has leakage. This leakage induces low frequency components that are distorted when the signal is filtered, as shown in the example of Figure 6.

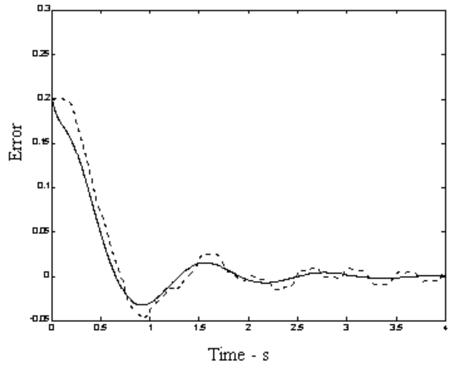

Figure 8. Time aliasing error compared to the time response of the FIR filter, time response of the filter in full line and error introduced in dotted line.

This interesting result, the similarity between the curves, is not a coincidence and deserves more study, since it introduces an error in the displacement measured during its first seconds.

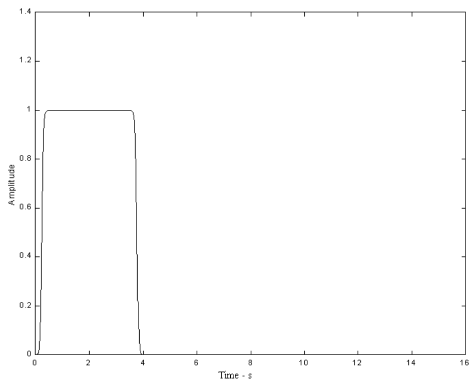

Figure 9. Window used to truncate the signal in time domain.

This error can be reduced by the truncation of the signal in the time domain by the window shown in Figure 9. This window nulls the signal in its first 128 points. Figure 10 shows the measurement of the same signal shown in Figure 6 using this truncation.

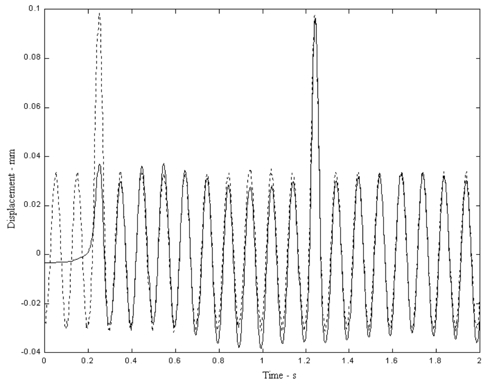

Figure 10. Measurement of displacements with time truncation used to reduce the time aliasing, signal from LVDT in dotted line and signal from accelerometer in full line.

As can be seen, this truncation does not remove all the error, but certainly minimizes it.

6. CONCLUSIONS

Time aliasing is inherent to the process of filtering and cannot be avoided or predicted, but it can be minimized by the proposed window technique. However, to achieve optimum control of this type of error, it is very important to carefully study this phenomenon in order to better understand its behavior.

7. REFERENCES

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