DAMAGE DETECTION WITH DIAGNOSTIC MODELS BASED ON FRF-MEASUREMENTS

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ABSTRACT

The damage detection procedure presented here uses a diagnostic model. This diagnostic model must be substantially reduced and highly dynamical correct and sensitive for local parameter changes. The generation of a diagnostic model can be formulated as a multicriteria optimization problem. In different steps of the adaptation procedure different physical properties of the real elasto-mechanical structure are adapted. A high sensitive diagnostic model concerning structural damage can be found by adaptation of the calculated FRF's of the diagnostic model to the measured FRF's of the real undamaged structure. The damage detection is solved by an optimization procedure which is based on measurements only. The final result is the correct position of damage. The precision of the damage parameters is a function of the reduction of the diagnostic model. The generation of the diagnostic model and the FRF based damage detection is shown by an experimental example of a crankshaft.

Keywords: damage detection, diagnostic model, FRF measurement, multicriteria optimization

1 INTRODUCTION

Nowadays it is of high economic interest to avoid maintenance work on machines at work. During the past few years many papers have been written focussing on damage detection based on vibration measurements. More than 150 papers with this subject can be found in the review paper [Doebling et al, 1998]. All these papers are based on the fact that the modal properties of a system are functions of the real physical system parameters. The model based damage detection process can be divided in two main tasks. The first task is to create a diagnostic model *D* that represents the real system behaviour. The second task is to identify the damage with help of this diagnostic model. In both tasks we have to deal with an inverse vibration problem. This inverse vibration problem is an ill conditioned mathematical problem. We can solve this problem by creating the diagnostic model and the damage detection on the base of frequency response function (FRF) measurements and experimental modal analysis results.

The requirements on the diagnostic model are the high dynamical correctness and the sensitivity of the model. At the same time it should be substantially reduced concerning the DOF's. The main difficulty is the increasing loss of information due to discretization. The process of model based damage detection is shown in figure 1.

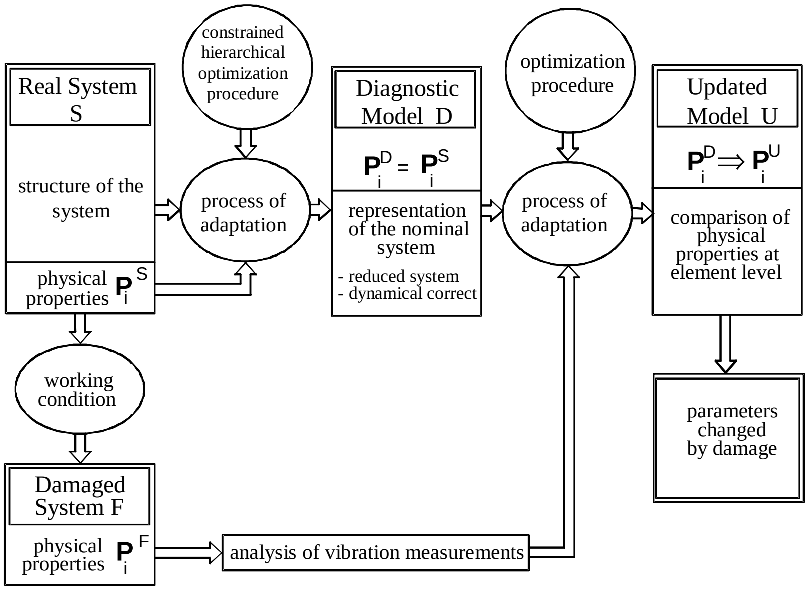

Figure 1: Model based damage detection procedure

2 THE DIAGNOSTIC MODEL

The diagnostic model creation task is to determine the design parameters of the diagnostic model from the vibration behaviour of the real system. In other words it is necessary to solve the inverse problem based on vibration measurements of the real system. It is not possible to find an unequivocal solution for this problem. In the shown procedure the problem of model generation is solved by an optimization process to determine the design parameters.

The diagnostic model is a highly condensed FE-model of the real system. The FE model consists of mass- and beam-elements. The parameters of these elements are determined by a constrained hierarchical optimization procedure. The diagnostic model is the final design of this optimization procedure. It is sensitive for the damage detection task.

2.1 The dynamical properties of the diagnostic model

The diagnostic model represents the real system and should be sensitive for the detection task. For this goal it is necessary to specify qualified dynamical properties. The properties which are used here for this task are:

- mass geometrical properties (total mass, centre of mass and tensor of inertia),
- natural frequencies,
- natural modes and
- FRF's of the system.

2.2 The multicriteria optimization procedure

The dynamical behaviour of the diagnostic model is defined by mass and beam elements of a FE-structure. The properties of the mass elements and beam elements are described by a design vector $\boldsymbol{x} = [\, x_1, \ldots, x_n]^T$ with n variables. The design variables are the mass and the stiffness of a beam element and the coordinates of both element nodes. The design variables of a pure mass element are the mass and the coordinates of the position of the mass.

The task of generation of the diagnostic model can be formulated as a constraint vector-optimization problem [Müller-Slany, 1992]:

$$\min_{x \in \Sigma} \left\{ \boldsymbol{f}\left[\boldsymbol{\varepsilon}(\boldsymbol{x})\right] \middle| \boldsymbol{h}(\boldsymbol{x}) = 0 \right\}, \Sigma := \left\{ \boldsymbol{x} \in \Re^n \middle| \boldsymbol{x}_L \le \boldsymbol{x} \le \boldsymbol{x}_U \right\}, \tag{1}$$

with: $\boldsymbol{x}$: design variables, $\boldsymbol{f}(\boldsymbol{x})$: objective function, $\varepsilon(\boldsymbol{x})$: error expressions, $\boldsymbol{h}(\boldsymbol{x})$:equality constraints, Σ: feasible range, ($\boldsymbol{x}_L$, $\boldsymbol{x}_U$): lower and upper bounds.

The elements of the vector objective function $\boldsymbol{f}(\boldsymbol{x})$ are error expressions $\varepsilon_i(\boldsymbol{x})$ between the dynamical properties of the diagnostic model $P_i^D(\boldsymbol{x})$ and the real system $P_i^S(\boldsymbol{x})$. Usually we can create the vector objective function $\boldsymbol{f}(\boldsymbol{x})$ in the following way:

$$\boldsymbol{f}(\boldsymbol{x}) = \begin{bmatrix} \varepsilon_1(\boldsymbol{x}) \\ \varepsilon_2(\boldsymbol{x}) \\ \varepsilon_3(\boldsymbol{x}) \\ \varepsilon_4(\boldsymbol{x}) \\ \varepsilon_5(\boldsymbol{x}) \\ \varepsilon_6(\boldsymbol{x}) \end{bmatrix} = \begin{bmatrix} \text{error expression of: complete mass} \\ \text{...position of centre of mass} \\ \text{...elements of inertia tensor} \\ \text{...considered natural frequencies} \\ \text{...considered natural modes} \\ \text{...considered FRF adaptation points} \end{bmatrix}. \tag{2}$$

The error expressions $\varepsilon_i(\boldsymbol{x})$ $i = 1, \ldots, 6$ of the objective vector are functions of the design variables $\boldsymbol{x} = 1, \ldots, n$. The first four and the sixth error expressions of the dynamical properties can be formulated by relative error equations. The error expressions $\varepsilon_5(\boldsymbol{x})$ of the natural modes must be calculated in a different form. To achieve better results in the optimization procedure we use the expression (3) which is similar to the Modal Assurance Criterion (MAC) for each mode k:

$$\varepsilon_5(\boldsymbol{x}) = \sum_1^k \left(1 - \frac{\boldsymbol{q}^D(\omega_k, \boldsymbol{x})^T \boldsymbol{q}^S(\omega_k)}{\left|\boldsymbol{q}^D(\omega_k, \boldsymbol{x})\right| \left|\boldsymbol{q}^S(\omega_k)\right|} \right), \tag{3}$$

with: ω_k natural frequency, k: number of considered modes, ($\boldsymbol{q}^D$, $\boldsymbol{q}^S$) mode vector for the diagnostic model D and real system S of mode k.

The optimization problem (1) will be solved by a numerical iteration process with a sequential quadratic programming (SQP) optimization algorithms. The used SQP algorithms E04UCF is taken from [NAG17] library. The structure of this optimization process is shown in figure 2. During the optimization the design vector $\boldsymbol{x}$ will be modified until the final design fulfills the minimization criterion (1). Finally a dynamical highly accurate diagnostic model has been created.

Usually the vector optimization problem is solved by scalarization of the vector objective function $\boldsymbol{f}(\boldsymbol{x})$ [Eschenauer et al, 1990]. Generally the superposition of all weighted components $\varepsilon_i(\boldsymbol{x})$ of the vector objective function $\boldsymbol{f}(\boldsymbol{x})$ to a scalar objective function $s(\boldsymbol{x})$ leads to good results. For the generation of a diagnostic model this method behaves poor. It was necessary to find a better way to solve the optimization task. The developed method is a hierarchical scalarization strategy [Müller-Slany, 1993]. In this method the error expressions $\varepsilon_i(\boldsymbol{x})$ are combined in different preference groups:

1. all mass-geometrical physical properties,
2. all natural frequencies,
3. all natural modes and
4. selected points of the FRF's of the system.

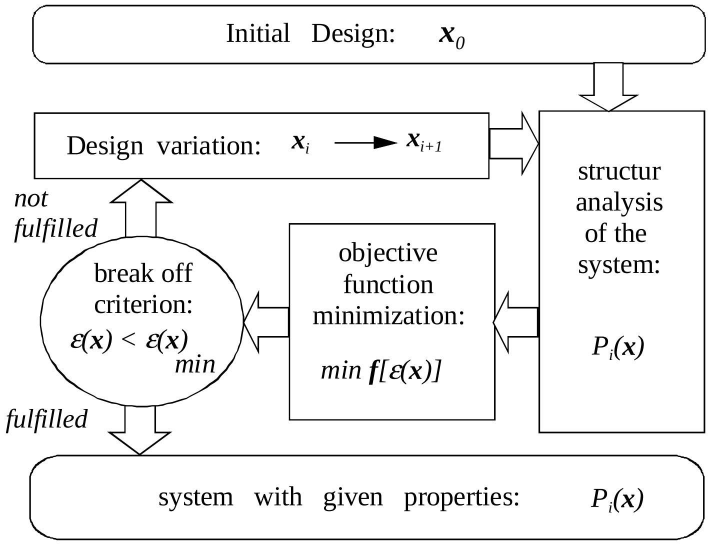

Figure 2: Diagnostic model generation

The diagnostic model creation task can now be done in three or four steps. The number of steps depends on the complexity of the real structure. For a simple structure the steps one, two and four are used to create the diagnostic model. For a complex structure all shown steps are necessary. The optimization will be done following the sequence of steps. At first the mass geometrical properties will be adapted then the natural frequencies. If necessary the natural modes are adapted next and at last the selected points of the FRF's. As an important point of the hierarchical scalarization strategy the physical properties which are adapted must be fixed by additional constraints $\boldsymbol{h}(\boldsymbol{x})$ in the next steps. The result after the last optimization step is a dynamical highly correct model.

2.3 The FRF adaptation process

A very important adaptation step for the diagnostic model is the adaptation of selected points of the FRF's. This adaptation makes the diagnostic model sensitive for detection of a damage of the real structure. Tests have shown that it is not possible to detect damage without FRF adaptation.

We have to compare measured FRF values of the real system and calculated FRF points of the diagnostic model [Pereira, 1996]. The i^{th} column of the calculated FRF matrix or receptance matrix $\boldsymbol{H}^D$ by using the frequency ω_i is given by:

$$\boldsymbol{h}^D{}_i = \left(\boldsymbol{K}^D - \omega_i^2 \boldsymbol{M}^D\right)^{-1} \boldsymbol{f} \ , \tag{4}$$

with: $\boldsymbol{M}$: mass matrix, $\boldsymbol{K}$: stiffness matrix of the diagnostic model and, the force vector $\boldsymbol{f} = [0, \ldots, 0, 1, 0, \ldots, 0]^T$, where the excitation is in the same direction as the measurement. The next step is to build the error function ε_6 for the FRF adaptation process:

$$\varepsilon_6 = \sum_i \sum_j \left| h_{i,j}^S - h_{i,j}^D \right| . \tag{5}$$

Here the number of used DOF's of the diagnostic model is j and the number of updated FRF points is i. The number of selected points of the FRF depends on the complexity of the structure of the system.

3 THE DAMAGE DETECTION PROCEDURE

The goal of the damage detection procedure is to identify the damage of a real system by vibration measurements. The damage detection process is based on adaptation of the diagnostic model to the dynamical properties of the real damaged structure, see figure 1. The damage can be described by the difference of the design parameters of the diagnostic model D: $\boldsymbol{x}^D$ and the adapted diagnostic model U: $\boldsymbol{x}^U$.

For complex systems the damage detection task is an adaptation procedure in two steps. In the first step the natural frequencies of the diagnostic model D will be adapted to those of the damaged system F. In the second step the FRF's of the adapted diagnostic model will be adapted to the real measured FRF's of the damaged system F. The design vector $\boldsymbol{x}$ for the damage detection procedure now contains only the stiffness parameters of the diagnostic model. This is because of the expected crack has no influence on the mass and the location of the model elements.

4 EXAMPLE OF THE PROCEDURE WITH A CRANKSHAFT

The diagnostic model creation procedure and the damage detection procedure shall be shown on an example of a crankshaft of a VW engine. The crankshaft is shown in figure 3. It has a mass of 15,74 kg and a length of 430 mm. The diameter of the main bearings is 54 mm and the diameter of the connecting rod bearings is 48 mm. The damage is a cut of 5 mm depth.

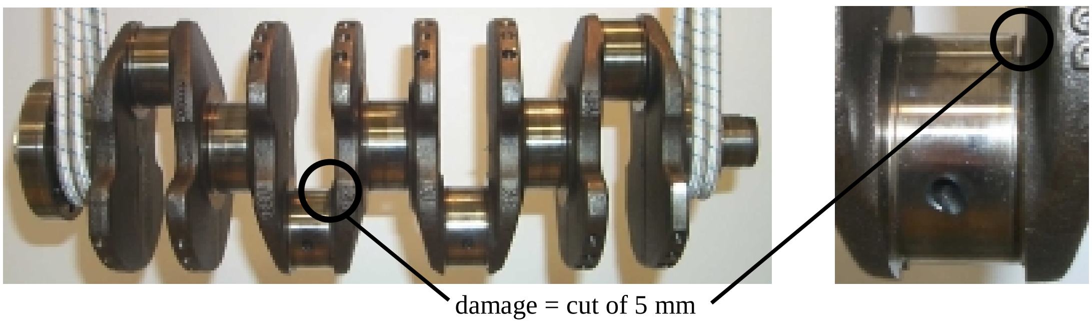

Figure 3: Crankshaft and the damage position

The diagnostic model is built by 25 beam and 8 mass elements. The design vector $\boldsymbol{x}$ for the diagnostic model contains 148 elements. These 148 elements are:

- 8 masses of the 8 mass elements,
- 17 masses of the 25 beam elements (8 masses of the beam elements are set to zero),
- 48 variables to define the position of the 26 nodes (the first and the last node is fixed) and
- 75 variables to define the stiffness of the 25 beam elements.

The layout of the diagnostic model is shown in figure 4.

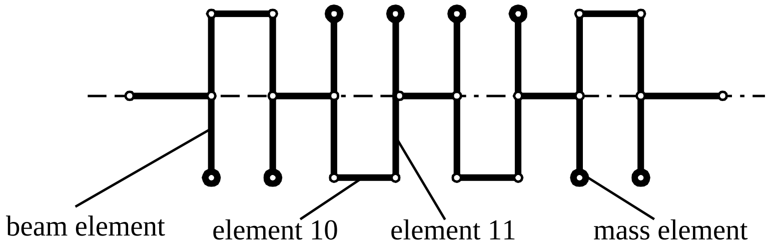

Figure 4: The diagnostic model for the crankshaft

For the crankshaft it is necessary to work with all four steps of the diagnostic model creation procedure. The first problem is to find a qualified initial design for the diagnostic model which has the correct sequence of natural modes and which does not change them during the FRF adaptation process. The final errors of the diagnostic model *D* after the FRF adaptation are shown in table 1.

Table 1: The precision of the diagnostic model after the FRF adaptation

physical properties	error of adaptation
total mass	$\varepsilon_1 < 5,0 \cdot 10^{-4}$
position of centre of mass	$\varepsilon_2 < 5,0 \cdot 10^{-4}$
used elements of tensor of inertia	$\varepsilon_3 < 5,0 \cdot 10^{-4}$
first 6 natural frequencies	$\varepsilon_4 < 5,1 \cdot 10^{-4}$
first 6 natural modes (MAC values %)	$85,2 < \varepsilon_5 < 96,1$
8 selected points of FRF	$\varepsilon_6 < 1,5 \cdot 10^{-5}$

The result of the FRF adaptation is shown in figure 5. The vertical lines in figure 5 represent the frequency points where the FRF's of the diagnostic model are adapted to the measured FRF's of the real system.

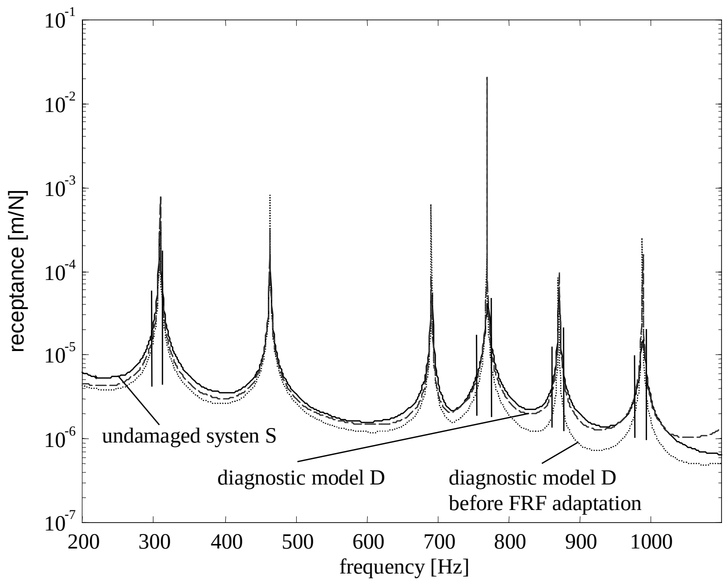

Figure 5: Adaptation of the sum of FRF's of the diagnostic model

The crankshaft is damaged by a cut on the bearing's right side which is represented by the connecting of the elements 10 and 11 of the diagnostic model. The damaged bearing has a diameter of 48 mm and the cut has a depth of 5 mm. The maximum difference for the first six measured natural frequencies is 2 %. The difference of the FRF sum between the undamaged and the damaged crankshaft is shown in figure 6.

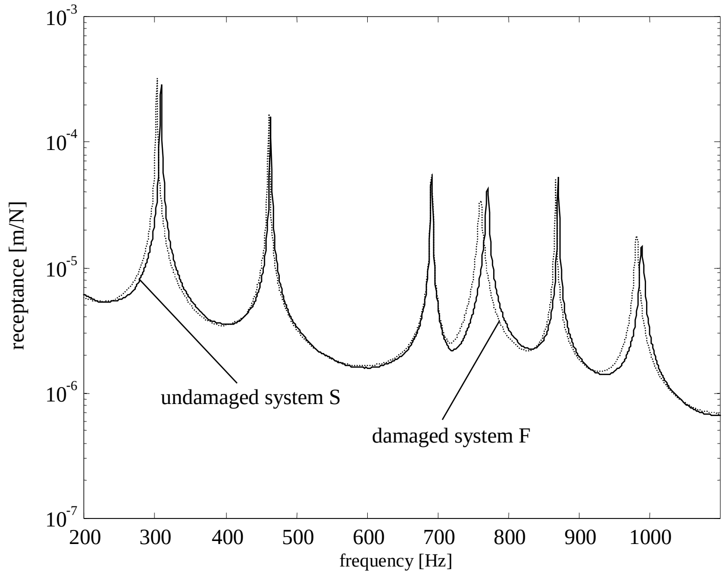

Figure 6: FRF sum difference between the undamaged and damaged system

The damage detection is done in two steps. In the first step the first six natural frequencies of the diagnostic model are adapted to the natural frequencies of the damaged crankshaft. In the second step 12 FRF points of the model are adapted to the measured FRF's. The vertical lines in figure 7 represents the 12 FRF adaptation points. The design variables ***x*** in the damage detection procedure are 75 stiffness parameters.

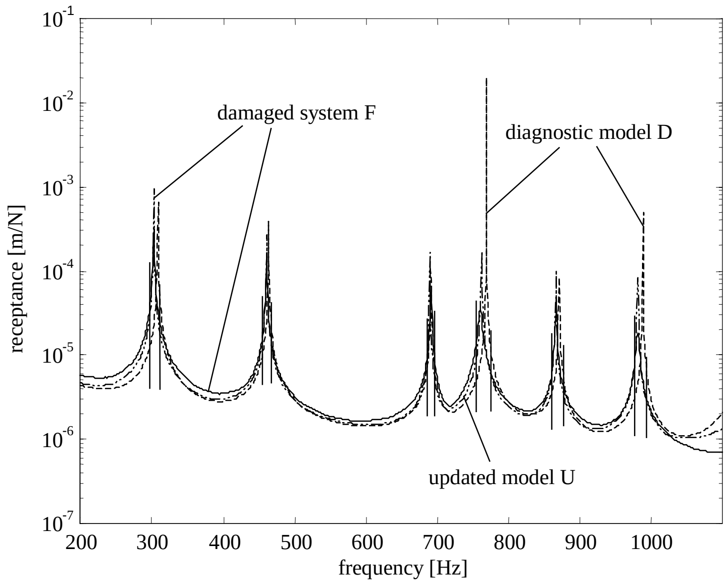

Figure 7: FRF sum adaptation of the damaged model *U*

Comparing the resulting design vector $\boldsymbol{x}^U$ with the design vector $\boldsymbol{x}^D$ of the diagnostic model the damage can be found clearly. on element 11. The most affected stiffness parameter is a bending parameter of element 11 with an amount of -12,7 % comparing the diagnostic model *D* with the updated diagnostic model *U*. The next highest difference is –2,8 %. The quality of the FRF adaptation is shown in figure 7.

5 CONCLUSION

The introduced diagnostic model is both, a dynamical highly correct model and a highly condensed one. The diagnostic model will be found in a hierarchical optimization procedure in which the model behaviour will be adapted to chosen dynamical properties of the real system. The basis for this adaptation are real measurements of the original system. The damage detection is based on this diagnostic model.

The procedure is shown by using the measurements of a crankshaft from a VW engine. It is possible to adapt the diagnostic model to the crankshaft's behaviour with a very small error concerning dynamical properties. After the generation of a diagnostic model a cut with a depth of 5 mm was made on a connecting rod bearing of the real crankshaft. The damage detection procedure is able to identify the introduced damage very clearly.

Acknowledgement

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