LOAD CARRYING CAPACITY OF A DOUBLE-WALL CYLINDRICAL SHELL WITHOUT CONSIDERATION OF AXIAL FORCES

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Abstract

This paper presents a method and the calculation algorithm used to estimate the load carrying capacity of cylindrical part of a double-wall shell of combustion chamber of a regeneratively cooled liquid rocket engine, without taking into account the influence of axial forces. The basic method was developed by Feodosiev, and it was implemented here with small modifications. As an example, a typical case is thoroughly analyzed.

Keywords: Rocket engine, Limit analysis, Combustion chamber, Structural analysis.

1. INTRODUCTION

The short operational time is on of the characteristics of liquid rocket engines, mainly of the expendable ones. Thus it is believed to be reasonable for part of their components to work in a plastic state or for the entire units to approach the plastic state.

The design of a liquid rocket engine (LRE) according to this viewpoint will permit to fix the potential load resistance of the structure to the maximum degree possible. Consequently, the weight/strength ratio of the engine can be rationally designed (Anon., 1983).

The load carrying capacity of a structure at plastic state of the material represents its capacity to resist to loadings with preservation of its dimensions and shapes within allowable limits. Thus the maximum load carrying capacity is considered as the structure loading after which an essential change of dimensions occurs without substantial growth of loading, i.e., there comes a quick developing deformation (Feodosiev, 1980).

The load carrying capacity of a LRE chamber is estimated by the value of limiting gas pressure in the combustion chamber. In order to find the limiting gas pressure is necessary to construct the curve that expresses the relationship between the gas pressure in the combustion chamber and its radial elongation under action of combined pressure and temperature loads.

The limiting gas pressure is the value of pressure such that a small increment on it corresponds to a large increment on the radial elongation (or equivalently on the circumferential total strain) of shells, caused by development of plastic deformations in both shells of the chamber.

This paper is concerned with the calculation of global load carrying capacity of a double layer shell of revolution, a typical structure of combustion chamber case of LRE with regenerative cooling.

The fundamentals of the calculation method, originally devised for manual calculation, were stated by Feodosiev (1963), and are based on the nonbending theory of shells. Here, it was adopted a similar approach, a little more convenient for use in computers. The basic equations of this method and the pertinent computational procedures will be described in next sections. Finally, a practical case is analyzed in order to illustrate the application of the method.

2. THE PROBLEM AND ITS SOLUTION

Considers a cylindrical combustion chamber composed of two shells jointed by arbitrary radially rigid connections. The inner shell will be designed by the index 1 and the outer one by index 2.

Assume as known the basic geometric parameters of this chamber, i.e., their curvature radii (R_1 and R_2) and thicknesses (h_1 and h_2). Denote by R its average radius of curvature.

Are known also the working value of the gas pressure in the chamber (p_g); the pressure in cooling channels (p_l); the average temperatures in each shell (T_1 and T_2), the coefficients of linear expansion of shell materials (α_1 and α_2), the stress-strain curves of these materials, and the reference temperature (T_o) for thermal deformations. The thermo-mechanical properties are evaluated at the average temperature in each shell.

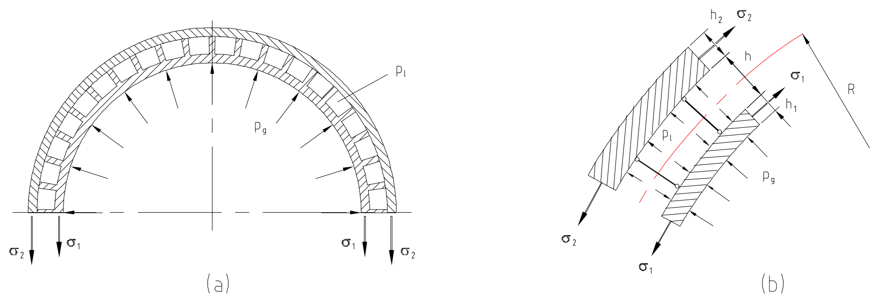

Figure 1: Mechanical model of the combustion chamber.

2.1 Simplifications of the mechanical model

The equations of the simulation model and the calculation procedures are based on the following hypotheses and assumptions (see calculation model in Fig. 1):

- the materials of shells are elasto-plastic, and work equally on tension and compression;
- shells are cylindrical and thin, and their radii are very close ($R_2 \approx R_1 \approx R$, $h_1 \ll R$, $h_2 \ll R$);
- the connections are considered rigid in the radial direction;
- the influence of local effects on the stress state of shells is not taken into account (infinitely long shell);

- the gas pressure p_g in the given section is considered equally distributed on the shell perimeter;
- the temperature field in each shell is axisymmetric, and the temperature value is defined as the average between the temperatures on internal and external surfaces of shell, corresponding to the engine nominal working mode.

2.2 Model equations

Examine the equilibrium of the shell element shown in Fig. 1-a. The inner shell is affected by the current gas pressure p_g, the pressure in the coolant passage p_l, and the contact pressure p_k, which appears as an averaged pressure of coupling forces between the shells (Fig. 1-b).

Only pressures p_k and p_l affect the outer shell, since the pressure on its outer surface is disregarded (if it is substantial, then it should be considered).

The equations of force balance of an element of shells in circumferential direction expressed in terms of stresses (Fig. 1) are:

$$\sigma_1 \, h_1 = (p_g - p_l - p_k) \; R \, , \tag{1}$$

$$\sigma_2 \, h_2 = (p_l + p_k) \; R \, . \tag{2}$$

Adding member by member the two previous equations, the sum of pressures p_l and p_k will be excluded from them. The result is:

$$\sigma_1 \, h_1 + \sigma_2 \, h_2 = p_g \, R \, . \tag{3}$$

In this equation there are two unknown: σ_1 and σ_2. Therefore, for its solution it is necessary to supplement it with the equation of total deformation.

As shown in (Oliveira, 1998), in the process of deformation of an axially symmetrical body, its points are displaced radially. So, there is a simple relationship between the deformations in radial and circumferential directions:

$$\varepsilon_t = \frac{\Delta R}{R} \, , \tag{4}$$

where: ε_t is the total strain in the circumferential direction, ΔR is the radial elongation, and R is the original radius of the body.

This total hoop strain of each shell consists of mechanical and temperature strains:

$$\varepsilon_{1,t} = \varepsilon_1 + \alpha_1 \, \Delta T_1 \, , \qquad \varepsilon_{2,t} = \varepsilon_2 + \alpha_2 \, \Delta T_2 \, . \tag{5}$$

Here ε_1 and ε_2 are only shell mechanical strains in circumferential direction; $\alpha_1 \, \Delta T_1$ and $\alpha_2 \, \Delta T_2$ are their temperature strains; and $\varepsilon_{1,t}$ and $\varepsilon_{2,t}$ are the total strains in each shell.

Since has been considered that $R_1 = R_2 = R$, the total circumferential strains in both shells are equals:

$$\varepsilon_{1,t} = \varepsilon_{2,t} = \varepsilon_t \, . \tag{6}$$

The relation $\sigma = \sigma(\varepsilon)$ for a given material is described by its tensile diagram. So, having this diagram and knowing the value ε it is possible to determine the value of σ.

The system formed by Eqs. (3), (4), (5) and (6) is closed by the stress-strain relations of each shell material, and its direct solution is very easy, and can be accomplished by using simple numerical calculation process, as will be indicated the in next section.

3. DETERMINATION OF GAS PRESSURE CURVE

In this step, the main objective is the determination of the diagram relating the gas pressure p_g with the radial elongation ΔR of the chamber.

For this purpose, the geometrical parameters of the shells (R, h_1, h_2), the average temperatures of the shells (T_1, T_2), the thermo-mechanical properties of the shell materials (α_1, α_2 and their stress-strain curves) corresponding to their average temperatures, and the reference temperature T_o are given.

According to the present method, for definition of each point of the calculation diagram is necessary to be set values of the radial elongation ΔR. Then the corresponding total strain ε_t, the mechanical strains ε_1 and ε_2, the hoop stresses σ_1 and σ_2, and finally the internal pressure p_g that satisfies to the equation of equilibrium, Eq. (3), are calculated.

The calculation will be carried out in the following sequence:

1. Set the value of radial elongation ΔR;
2. Calculate the total hoop strain ε_t:

$$\varepsilon_t = \frac{\Delta R}{R} \; ;$$

3. Calculate the mechanical hoop strains ε_1 and ε_2 using Eqs. (4) and (5):

$$\varepsilon_1 = \varepsilon_t - \alpha_1 \, \Delta T_1 \, , \qquad \varepsilon_2 = \varepsilon_t - \alpha_2 \, \Delta T_2 \; ;$$

4. Find the stresses σ_1 and σ_2 using the stress-strain curves:

$$\sigma_1 = \sigma_1(\varepsilon_1) \, , \qquad \sigma_2 = \sigma_2(\varepsilon_2) \; ;$$

5. Calculate the gas pressure value p_g from Eq. (3):

$$p_g = \frac{\sigma_1 h_1 + \sigma_2 h_2}{R} \; .$$

Repeating this procedure an enough number of times, one can obtain a series of values p_g, corresponding to the prescribed values of ΔR, which allows to construct the appropriate diagram where p_g is put along the axis of ordinates and ΔR or ε_t along the axis of abscissas. This diagram will be used for the determination of limiting pressure $p_{g,u}$ and corresponding safety factor n, which are the subject of the next section.

4. LIMITING PRESSURE AND SAFETY FACTOR

After the conclusion of the operational process described above, will result the calculated diagram of the relation $p_g = p_g(\Delta R)$, whose a typical representation is given in Fig. 2, where the points A, B and C are points in which the curve presents the greatest local curvatures.

On this curve one must find the points of working pressure p_c and of limiting pressure $p_{g,u}$, and determine the safety factor by the load carrying capacity of the shells.

The point C on curve $p_g = p_g(\Delta R)$ of Fig. 2-a, where the straight line drawn from the origin of coordinates is tangent to the curve, must be pointed out. It determines the beginning of the larger shell deformations, which are dangerous with respect to the

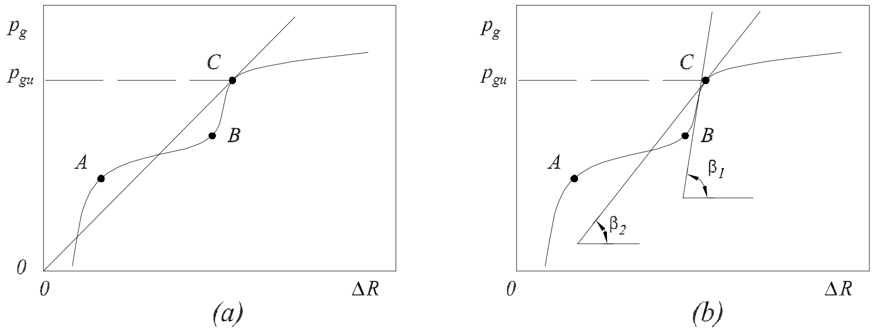

Figure 2: Typical gas pressure curve and limiting pressure determination: (a) first method; (b) second method.

change in geometrical dimensions of the shell, as well as to its strength. The pressure corresponding to point C will be the so called limit pressure $p_{g,u}$.

Alternatively, the limiting point can be determined based on value of the angle of inclination of the tangent to the curve $p_g = p_g(\Delta R)$. If this angle is small, the deformations accrue quickly, assuming inadmissible values. According to Feodosiev (1963), the limiting point is that one where the tangent of the angle of inclination of the line tangent to the curve is equal to a half from its greatest value on the segment BC (see Fig. 2-b).

The values given by these two methods can differ slightly, but the first method is simpler and easier to introduce in a computer program.

The safety factor by the load carrying capacity of shells is the ratio of the limit gas pressure $p_{g,u}$ to the working pressure p_c:

$$n = \frac{p_{g,u}}{p_c} \,.$$

Usually is adopted a value of safety factor on load carrying capacity within the range $n = 1.2 \cdots 1.5$. In case of unsatisfactory strength, it is necessary to change the thickness or the material of the outer shell.

5. STRESS-STRAIN CURVES

In order to introduce the relation $\sigma = \sigma(\varepsilon)$ in the calculation formulas, approximating analytic representations of stress-strain diagram, as described below, will be used.

It is supposed that the material presents an elastic-plastic behavior, i.e., it is a material in which both elastic and plastic strains are present; strain-hardening may or may not be assumed negligible. Besides, some models can present diagram with no step of yielding.

Elastic-plastic material with linear hardening

$$\sigma = \begin{cases} E\,\varepsilon & \text{for } \varepsilon \leq \varepsilon_y \\ \sigma_y + D\,(\varepsilon - \varepsilon_y) & \text{for } \varepsilon > \varepsilon_y \end{cases} ,$$

where E and D are the angular coefficients of the straight segments (in general, D is rather lesser than E, the modulus of elasticity); σ_y and ε_y are the yielding limit and its corresponding strain. The coefficient D can be estimated by:

$$D = \frac{\sigma_u - \sigma_y}{\delta} \,, \qquad \delta = \varepsilon_u - \varepsilon_y \,,$$

where σ_y usually is the material yielding limit $\sigma_{0.2}$; σ_u is the material strength; ε_y and ε_u are the corresponding strains; and δ is the residual strain at rupture.

Elastic-plastic material with nonlinear hardening

$$\sigma = \begin{cases} E\,\varepsilon & \text{for } \varepsilon \leq \varepsilon_y \\ a + \frac{b}{c+\varepsilon} & \text{for } \varepsilon > \varepsilon_y \end{cases} ,$$

where a, b and c are parameters obtained by curve fitting process of experimental data.

Elastic-plastic nonlinear material

$$\sigma = A\,\varepsilon^m ,$$

where A and m are constants chosen in such way to get a good representation of experimental data.

Linear elastic-plastic with nonlinear-hardening material

$$\sigma = \begin{cases} E\,\varepsilon & \text{for } \varepsilon \leq \varepsilon_y \\ A\,\varepsilon^m & \text{for } \varepsilon > \varepsilon_y \end{cases} ,$$

where E is the modulus of elasticity in the elastic part; ε_y is the yielding strain; A and m are constants for the nonlinear part of the curve.

Elastic-plastic nonlinear material with rational relationship

$$\sigma = \frac{a + d\,\varepsilon}{1 + c\,\varepsilon + b\,\varepsilon^2} ,$$

where a, b, c and d are constants obtained of a rational function fitting of experimental data.

Note that other idealizations can be made and incorporated in the analysis, but these models are those of most practical use, from the standpoint of mathematical simplicity, for the present task. Finally, on should take into consideration that most of steels have stress-strain diagrams that can be represented by two curve segments.

6. PROGRAM FOR CALCULATION OF LOAD CARRYING CAPACITY

The calculation of the load carrying capacity of the chamber begins after conclusion of the design and calculation of the cooling system, from where all set of the initial data used in the calculation are known.

A computer program for estimate the load carrying capacity of cylindrical part of a combustion chamber, without taking into account the axial forces, was developed in (Oliveira, 1998), based on the above described method. It was called **LCCCC-R**, and can be used for realize two different tasks:

1. **Verification Analysis**. With the thickness of the walls and the working internal pressure, the safety factor for load carrying capacity can be calculated.

2. **Design Analysis**. Using the prescribed safety factor and the working internal pressure, one must calculate the thickness of the outer shell. (Note that the thickness of inner shell results from its thermal possibilities, being determined by heat flow and temperature calculations at the cooling system design.)

Description of the computer program, instructions on preparation of the initial data, running operation and processing of calculation results are given in the following.

The LCCCC-R program was developed in Pascal language. Its code and detailed description are presented in (Oliveira, 1998). It consists of a **main module**, which coordinates the information flux and the calculation sequence, and three basics modules for specific tasks:

1. **ReadWriteInitData Module**. This procedure reads the input data, prepares additional parameters, and saves these information on specified file of results.

2. **Solver Module**. This procedure calculates the gas pressure corresponding to elongations in the prescribed range, for a given value of the outer shell thickness. It is a practical implementation of the computational procedure given in Section 3.

3. **PostProcessor Module**. This procedure does the calculation of the limit pressure and the safety factor for given outer shell thickness, in accordance with the methods discussed on Section 4.

The data of thermo-mechanical properties of constructional materials of inner and outer shells, average values of their temperatures, average radius of the chamber R and thickness of the inner shell h_1, all at the calculation section of the chamber, constitutes the initial data.

Within the limits of one calculation, the variable parameters of the chamber are outer shell thickness h_2, total circumferential strain of shells ε_t, and the gas pressure p_g.

The range for change of thickness h_2 and also the incremental step Δh_2 results from engineering sense of the analyst.

In order to get appropriate description of curve $p_g = p_g(\Delta R)$, the limits and steps of change of the radial elongation must be selected in such way that are calculated values of p_g in at least eight points.

All the initial data must be input in SI units, and its preparation must consider the following:

1. Thermo-mechanical behavior of materials of shells is taken from special technical literature and, in case of lack of data for the working temperatures of shells, the method of linear interpolation can be used. Anyway, the diagrams of materials used for the description of the relations $\sigma = \sigma(\varepsilon)$ must be given in form of analytical approximations, as will be detailed later.

2. Shell radius R is determined during the calculation of basics dimensions of chamber, and represents the average radius of the cylindrical chamber.

3. The thickness of inner shell h_1 is determined during the design of the chamber cooling system, from the condition of adequate cooling.

4. Initial and final thicknesses of outer shell can be roughly determined as follows:

$$h_{2,i} = \frac{p_g\, R - \sigma_{u1}\, h_1}{\sigma_{u2}} \, , \qquad h_{2,f} = \frac{p_g\, R}{\sigma_{y2}}\, n \, ,$$

where σ_u and σ_y denote ultimate and yielding stresses, respectively, and n is a factor that can be taken as $n = 1.5$.

The obtained values of $h_{2,i}$ and $h_{2,f}$ must be seen only as rough information, and can be conveniently changed, if necessary.

5. It is recommended to choose the thickness increment Δh_2 in such way to get at least two or three intermediate values of thickness h_2 between the extreme values $h_{2,i}$ and $h_{2,f}$.

For each shell material must be supplied by the analyst a set of parameters corresponding to the chosen model of stress-strain curve, which depend on the temperature of the material.

The **main module** works in the following manner:

1. Read from keyboard the name of input data file, and assign the name of file of results.
2. Call **ReadWriteInitData** module.
3. $h_2 = h_{2,i}$
4. $\Delta R = \Delta R_{\text{initial}}$
5. $\varepsilon_t = \frac{\Delta R}{R}$
6. Call **Solver** module to calculate p_g for the current data.
7. $\Delta R = \Delta R + \delta(\Delta R)$
8. Repeat steps from 5 to 7 until $\Delta R > \Delta R_{\max}$
9. Call **PostProcessor** module. (Draw the curve $p_g = p_g(\Delta R)$ and calculate n.)
10. $h_2 = h_2 + \Delta h_2$
11. Repeat steps from 5 to 10 until $h_2 = h_{2,f}$.

7. APPLICATION OF THE METHOD

As an example of application of the above method, it is taken a double layer shell with dimensions $R = 9.2$ cm, $h_1 = 1.5$ mm, and h_2 variable between 1.5 and 4.5 mm, in steps of 0.5 mm.

Average temperatures of inner and outer walls are equal to 575°C and 100°C, respectively. Both shells are made of the same material: Russian stainless steel 08KP. The thermal expansion coefficients, at the corresponding average temperatures of shells, are $\alpha_1 = 14.5 \times 10^{-6}\,^\circ\text{C}^{-1}$ and $\alpha_2 = 10.75 \times 10^{-6}\,^\circ\text{C}^{-1}$. Assume $T_o = 0$°C.

The analytical expressions that represent the stress-strain diagrams of 08KP steel at temperatures of 575 and 100°C were obtained by means of curve fitting, in (Oliveira, 1998). For $T_1 = 575$°C,

$$\sigma = 4.3831 \times 10^8\,\varepsilon^{0.322435}\,.$$

For $T_2 = 100°\text{C}$,

$$\sigma = \begin{cases} 1.421 \times 10^{11}\, \varepsilon & \text{for } \varepsilon \le 0.0012 \\ 3.4567 \times 10^{8}\, \varepsilon^{0.10572} & \text{for } \varepsilon > 0.0012 \end{cases} .$$

Whereby, as already stated, it is assumed that the stress diagrams are the same in compression or in tension, changing uniquely the signs.

The working pressure is the nominal combustion chamber pressure $p_c = 35$ bar. The analysis will be accomplished taking up 50 values of ΔR from 0 to 1.8 mm.

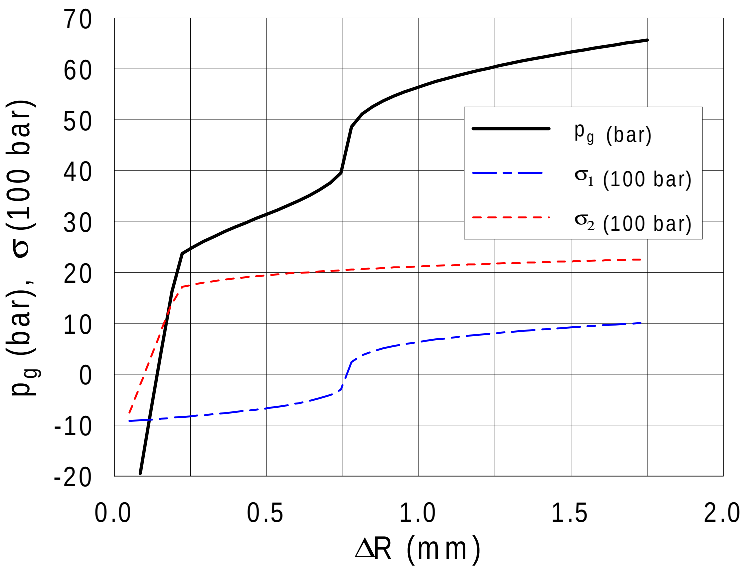

Figure 3: Gas pressure and shell stresses for $h_2 = 2$ mm.

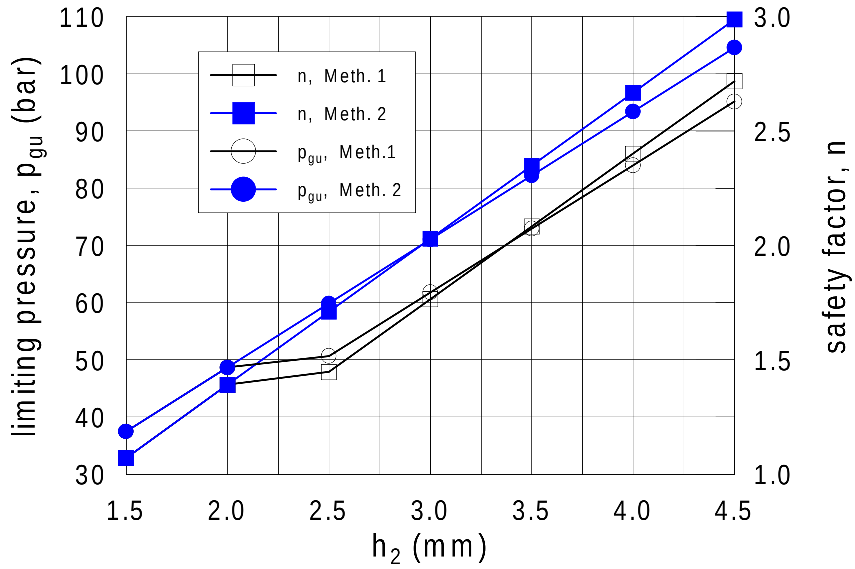

Figure 4: Limiting pressures and safety factors for different outer shell thickness.

Calculation results for $h_2 = 2$ mm are represented in form of curves on Fig. 3. On it is shown the behavior of p_g, σ_1 and σ_2 as functions of ΔR. The point corresponding to chamber pressure $p_g = p_c = 35$ bar could be indicated on the pressure curve.

As is evident from Fig. 3, the increase in cylinder radius at the working pressure constitutes 0.64 mm. The stress in the outer shell will be approximately $\sigma_2 = 200$ MPa, and in the internal shell, $\sigma_1 = -52$ MPa. The latter stress appears to be compressible, which is explained by greater temperature elongation on the internal shell. If the working pressure were greater, then the stress in the internal shell would pass from compressive stress to tensile.

The curves given in Fig. 3 show that at $p_g = 0$ appears a certain deformation state due to temperature expansions within the shell.

It is important to mention that on the pressure curve are observed two quick growth segments. The first segment is at small ΔR and the second begins at approximately $\Delta R = 0.75$ mm. This is explained by the following. If the pressure is low, the outer shell works elastically and with an increase in the pressure, the deformation rises slowly.

If the pressure becomes greater than a definite value then in the outer shell originates plastic deformations. Whereby the outer shell will be loaded not only by pressure forces, but also by forces from the internal side of the more strongly heated shell. A sharp rise in ΔR does take place at an insignificant pressure increase.

At greater pressures the shell resistance again rises and a sharp rise in the pressure curve does take place. This happens when the mechanical elongation of the internal shell covers temperature elongation. The internal shell then begins stretching, becoming a supporting element.

Finally, as it is evident from the curve given in Fig. 3, at pressures greater than $p_g = 48$ bar a sharp rise in the plastic deformation of the shells takes place.

The safety factor of load carrying capacity of the shell will be equal to

$$n = \frac{48.7}{35} = 1.39\,,$$

where $p_{g,u} = 48.7$ bar was determined by program LCCCC-R, according to the above indicated methods (both of them gave the same result).

The curves of limiting pressure and safety factor as functions of the outer shell thickness are given in Fig. 4. These curves permit to determine adequate value for h_2, when possible, or give indication of the necessity to change the material of the outer shell.

This application example shows how the present method can be used. In addition, becomes clear that it can be useful at preliminary study phase of LRE design, where simple and quick tools are particularly attractive. This methodology could be easily implemented in computer programs for preliminary and optimization studies. Note that in preliminary design and/or optimization studies of configurations of a LRE, normally there is no enough data to do a detailed study using a Finite Element computer program for elastoplastic structural analysis.

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