LANDING ACCURACY ANALYSIS FOR BALLISTIC REENTRY VEHICLE AFTER DE-BOOST AT CIRCULAR ORBIT

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Abstract

The paper includes analysis of typical set of disturbances acting on ballistic spacecraft during reentry the Earth atmosphere. It is shown that the most significant disturbing factors are execution errors of de-boost impulse and variation of atmospheric parameters with respect to standard values. Non-nominal aerodynamic characteristics and displacement of vehicle center of mass from symmetry axis are disturbing factors also. There are analytical partial derivatives of reentry parameters and landing point location with respect to de-boost impulse errors. For typical de-orbit conditions the numerical values of derivatives are calculated. The paper contains also investigation results of landing point dispersion due to disturbed atmosphere. Computational Model of the Earth Disturbed Atmosphere-CMEDA (KIAM RAS) is used for reentry simulation. The total number of calculated disturbed trajectories is more than 5000. All presented results are necessary for ballistic design and choice of optimal mission scheme for an orbital type reentry vehicle.

Key words: Ballistic reentry, Landing dispersion, Disturbed atmosphere

1. INTRODUCTION

The problem of delivery experimental and observation results from orbit to the Earth arises very often in the process of space research. The simplest and cheapest solution of the problem is the use of a small ballistic reentry vehicle. Such kind vehicle has no control system for guidance into the atmosphere. So, a dispersion of landing point may be significant, and it complicates the search of vehicle and very often a safe landing also. If the vehicle is reusable, there is a problem of heat flux restriction to retain the aerodynamic shape. Both factors are principal for choice of vehicle shape and optimal mission scheme but the paper considers mainly an accuracy problem.

After mission analysis were obtained following preliminary results (Sikharulidze, 1998). The initial mass of reentry vehicle is of 150...200 kg. The optimal thrust of de-boost engine is of 750 N. The aerodynamic shape is frustum of a cone type. Rational orbit is circular one with altitude of 250...300 km, in the plane of equator almost. Optimal reentry corridor on fight path angle θ_{en} (between velocity vector and conditional boundary of the atmosphere at altitude of 100 km) is of $-3^o...-4^o$. Corresponding value of de-boost velocity impulse is of 250...360 m/s. This corridor provides a good landing accuracy and restricted heat flux also. The optimal direction of de-boost impulse is against to orbital velocity vector. It provides a required reentry angle with minimum propellant consumption, (Sikharulidze, 1982).

An analysis of derivatives (i.e. functions of influence) of reentry parameters or landing point position with respect to errors allows to estimate the sensitivity of trajectory on disturbances. Then it is possible to recognize the most significant factors and take measures for minimization of their effects.

One of the most significant disturbing factors are errors of de-boost impulse ΔV realization on

- time of execution,
- value of de-boost impulse,
- in-plane orientation,
- out-of-plane orientation.

Another important disturbing factor is a difference of real atmosphere from standard one. To obtain statistical characteristics (mathematical expectation, dispersion, maximum and minimum deviation) it is necessary to calculate a few hundreds of reentry trajectories for each set of initial conditions. Clearly that very important is a model of the Earth disturbed atmosphere.

Enough important disturbing factor is a difference of real aerodynamic characteristics of reentry vehicle from nominal values. The difference may arise due to 2 reasons:

- non-correct determination of characteristics,
- change of aerodynamic shape during reentry.

The first reason does not depend on type of reentry vehicle (single-usable or reusable). It is known that at the project phase an error of aerodynamic coefficients determination is of 10...20%. The second reason is essential for reentry vehicle with ablating (collapsed) heat protection material. The aerodynamic shape of reusable reentry vehicle should not change in flight. In the opposite case a reentry vehicle is no reusable one.

The last considered disturbing factor is a displacement (shift) of vehicle center of mass (c.m.) from the symmetry axis. The displacement may arise due to

- error of c.m. position determination,
- movement of c.m. after expenditure of propellant, gas, etc.,
- asymmetric change of aerodynamic shape in flight.

For reusable reentry vehicle only the first and second reasons are essential.

The landing accuracy is very important for reentry vehicle, especially for reusable one. A high landing accuracy allows to restrict a required landing polygon, simplifies tracking at the final phase of trajectory, makes easy the search of vehicle and its recovery after soft landing. It provides also a good condition for capture the vehicle by helicopter at parachute descend phase if the vehicle has no system of soft landing (solid motor or shock absorber).

2. SET OF DISTURBANCES AND ESTIMATION OF LANDING ACCURACY

Landing accuracy significantly depends on given set of disturbing factors, their values and possible combination. The most significant disturbances are: de-boost impulse execution time, disturbed atmosphere, non-nominal ballistic coefficient and c.m. displacement from symmetry axis. Preliminary estimation of landing accuracy we can get in linear approximation by use partial derivatives of downrange and crossrange with respect to disturbing factors. At analysis of disturbances one uses some mathematical models these differ from real physical processes. It is impossible also to take into account all real disturbances due to insufficient understanding of real physical processes. So, calculated landing error should be 20...30% less than given polygon to provide the necessary reserve of landing accuracy.

2.1. De-boost errors

An error of de-boost impulse execution time δt_{db} only shifts the reentry trajectory. Derivative of landing point position with respect to execution time error is (Sikharulidze, 1999, NT-164)

$$\partial L/\partial t_{db} = V_{cir} R_E / r_{cir}. \tag{1}$$

Here V_{cir} is a circular velocity, r_{cir} is a radius of orbit, R_E = 6378 km is the Earth mean radius. For circular orbit with altitude of H_{air} = 300 km (all following results are given for this orbit) there is $\partial L/\partial t_{db}$ = 7380 m/s. It means that only 1s error of de-boost maneuver execution time shifts the landing point on 7380 m.

An error of de-boost impulse value $\delta(\Delta V)$ influences on initial conditions of reentry, i.e., on reentry velocity V_{en} and angle θ_{en} (see Figure 1). Besides, the error changes an angular range of extra-atmospheric trajectory Φ_{en} (from de-boost point to reentry point) and flight time t_{en} at this phase. As a result, geocentric coordinates of reentry point (latitude φ_0 and longitude λ_0) are changed.

Derivative of entry velocity with respect to de-boost impulse value ΔV is

$$\partial V_{en}/\partial \Delta V = -(1-\Delta V/V_{cir}) / (V_{en}/V_{cir}). \tag{2}$$

For reentry angles $\theta_{en} = -3^o$ and -4^o the value of derivative is of $\partial V_{en}/\partial \Delta V = -0.97$.

Derivative of reentry angle with respect to de-boost impulse value is determined by equation

$$\partial \theta_{en}/\partial \Delta V = -360^o (r_{cir}/r_{at}) \{[(r_{cir}/r_{at})-1]/[2-(r_{cir}/r_{at}+1)(1-\Delta V/V_{cir})^2]\}^{1/2} / [\pi V_{cir} (V_{en}/V_{cir})^2]. \tag{3}$$

Here $r_{at} = R_E + h_{at}$ is the radius of atmosphere, h_{at} = 100 km is the altitude of conditional boundary of the atmosphere. There are $\partial \theta_{en}/\partial \Delta V$ = -0.009 degree/(m/s) if $\theta_{en} = -3^o$ and -0.007 degree/(m/s) if $\theta_{en} = -4^o$.

Derivative of extra-atmospheric angular range with respect to de-boost impulse value is

$$\partial \Phi_{en}/\partial \Delta V = -360^o \{[(r_{cir}/r_{at})-1]/[2-(r_{cir}/r_{at}+1)(1-\Delta V/V_{cir})^2]\}^{1/2} / \{\pi V_{cir} [1-(1-(\Delta V/V_{cir})^2)]\}. \tag{4}$$

There are $\partial \Phi_{en}/\partial \Delta V$ = -0.14 degree/(m/s) if $\theta_{en} = -3^o$ and -0.07 degree/(m/s) if $\theta_{en} = -4^o$. In the second case the sensitivity is 2 times less.

Derivative of extra-atmospheric flight time with respect to de-boost impulse value is described by very complicated equation, so below are given only numerical values: $\partial t_{en}/\partial \Delta V$ = -2 s/(m/s) if $\theta_{en} = -3^o$ and -1 s/(m/s) if $\theta_{en} = -4^o$. In the second case the sensitivity is 2 times less also.

Very important property of optimal de-boost maneuver (against to motion direction at the circular orbit) was proved early (Sikharulidze, 1999, NT-164). *The maneuver provides both the maximal value of reentry angle* $|\theta_{en}|$ *and non-sensitivity in linear approximation of total descent trajectory to small errors of de-boost impulse orientation in the motion plane*. It means that all derivatives of motion parameters at extra-atmospheric phase with respect to impulse orientation in the orbit plane (pitch angle ϑ in Figure 1) are zero.

An error of de-boost impulse orientation in the horizontal plane (yaw angle ψ in Figure 1) produces a side component of de-boost impulse ΔV_{sd}. As a result, the motion plane turns around the local vertical (i.e. radius-vector of initial point) by a small angle. In the same time the descent trajectory (in a new plane) does not change. A side displacement of landing point ΔB appears, and its value depends on derivative

$$\partial \Delta B / \partial \Delta V_{sd} = V_{cir} R_E \sin\Phi_\Sigma / [1 - (\Delta V / V_{cir})^2]. \quad (5)$$

Here Φ_Σ is a total angular range from de-boost point to landing one. There are $\partial\Delta B/\partial\Delta V_{sd}$ = -770 m/(m/s) if θ_{en}= -3^o and -670 m/(m/s) if θ_{en}= -4^o.

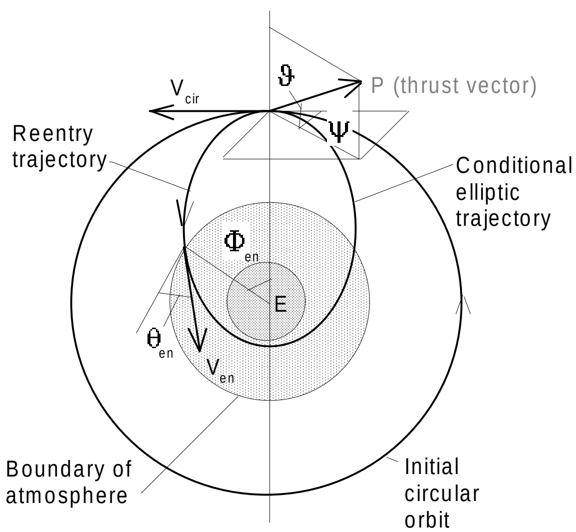

Figure 1. Scheme of de-boost maneuver

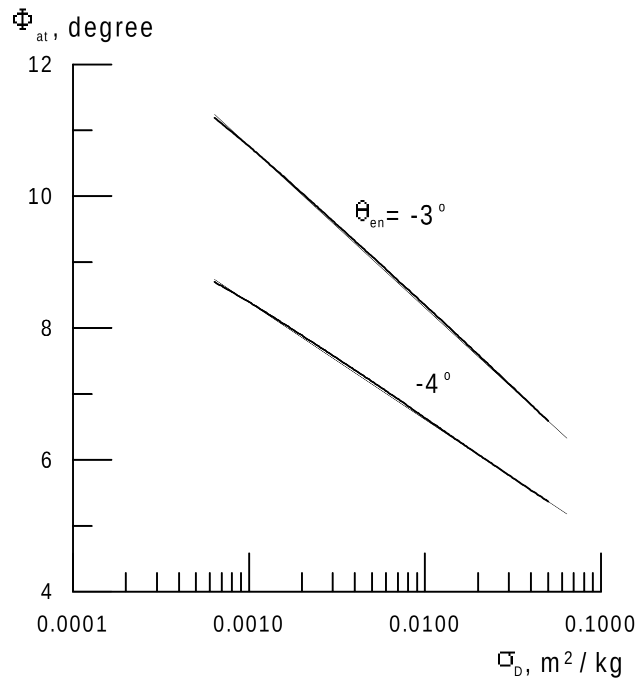

Figure 2. Angular range of atmospheric phase

2.2. Variation of density and wind

The model of disturbed atmosphere is very important for simulation of reentry trajectory. It means more for ballistic reentry when the vehicle has no guidance into the atmosphere.

Computational Model of the Earth Disturbed Atmosphere - CMEDA was developed at the Keldysh Institute of Applied Mathematics (KIAM) in 1968-1998 (Sikharulidze, Korchagin, Kostochko, 1999). The CMEDA is intended for

- development of vehicle guidance algorithms,
- estimation of expected accuracy of maneuver,
- determination of aerodynamic loads, etc.

It is the global model for altitudes from 0 km up to 120 km and includes all 12 months of the year. The CMEDA contains season-latitude, diurnal and random components of density variations and a wind field also. It allows to generate an unlimited number of disturbed atmosphere states for simulation of various flight conditions.

A variation of density $\delta\rho$ is represented as normalized deviation of disturbed density ρ from standard one ρ_{st}:

$$\delta\rho = (\rho - \rho_{st}) / \rho_{st} . \quad (6)$$

The total variation includes season-latitude, diurnal and random components

$$\delta\rho = \delta\rho_{sl}(H, \varphi, N) + \delta\rho_d(H, \varphi, \tau) + \delta\rho_r(H, \lambda, \varphi, N, \xi), \tag{7}$$

where H is an altitude, φ is a latitude, λ is a longitude, N is a month number, τ is a local solar time, ξ is a random vector.

Season-latitude and diurnal variations are systematic and describe a mean or expected state of atmosphere as function of altitude, latitude, month and local time. The random component determines a difference between "actual" state of atmosphere and systematic components. For description of random component the method of normalizing functions was developed. It is based on the analysis of experimental measurement data. Three normalizing functions allow simulating the harmonic density variations as function of altitude, latitude and longitude.

The model of wind contains zonal (along the parallel) and meridian components of a wind velocity. The zonal component U consists of three terms, season-latitude, diurnal and random:

$$U = U_{sl} + U_d + U_r. \tag{8}$$

The meridian component has a random nature.

Software of the CMEDA is compatible with any operational system that contains C-compiler.

While vehicle flights into the atmosphere, dispersion of landing point arises due to variation of density and wind. The bigger is reentry angle $|\theta_{en}|$, the smaller is dispersion. So, for reentry angle of $\theta_{en} = -3^o$ the mean square root of downrange variation is of $\sigma_L = 4.83$ km and crossrange one is of $\sigma_B = 1.20$ km. The limit errors of landing point in assumption of standard (normal) distribution law are of $\Delta L = \pm 3\sigma_L = \pm 14.5$ km and $\Delta B = \pm 3\sigma_B = \pm 3.6$ km . If reentry angle is of $\theta_{en} = -4^o$, there are accordingly $\sigma_L = 4.03$ km, $\sigma_B = 1.15$ km and $\Delta L = \pm 12.1$ km, $\Delta B = \pm 3.4$ km. Landing accuracy after de-boost at the quasi-equatorial orbit does not depend almost on the season (month). Maximum difference is of 10% (Sikharulidze, Korchagin, Moraes, 1999).

2.3. Uncertainty of aerodynamic coefficients

Any variation of aerodynamic coefficient from nominal value is a significant disturbing factor. Really, derivative of velocity on time for motion into the atmosphere is described by equation

$$dV/dt = -\sigma_D \rho V^2/2 - g \sin\theta, \tag{9}$$

where $\sigma_D = C_D S/m$ is a ballistic coefficient of reentry vehicle, C_D is a drag force coefficient, S is a middle are, m is a mass of vehicle, ρ is a density of atmosphere, V is an air velocity (with respect to the atmosphere), g is a gravity acceleration, θ is a flight path angle. Common accuracy of C_D determination is of 10...20%.

Figure 2 shows that angular range of atmospheric trajectory Φ_{at} versus $\lg\sigma_D$ is a linear function almost (with accuracy of 1%). It may be described by equation

$$\Phi_{at}(\sigma_D, \theta_{en} = -3^o) = 10.764^o - 2.457^o (\lg \sigma_D + 3.0). \tag{10}$$

If q is an accuracy of σ_D determination (for example q = 0.1...0.2), then $q\sigma_D$ is a variation of ballistic coefficient from nominal value. The landing point possible downrange displacement due to uncertainty of ballistic coefficient σ_D (or drag coefficient C_D) is of

$$\Delta L\ (\theta_{en} = -3^o) = (110\ \text{km/degree})\ \partial\Phi_{at}/\partial\sigma_D \cdot q\ \sigma_D = -119\ \text{km}\cdot q. \qquad (11)$$

This equation proves very important result (Sikharulidze, 1999, NT-164): *possible landing point displacement due to variation of ballistic coefficient* σ_D *within limit of accuracy depends only on reentry angle* θ_{en} *and given accuracy* q *but does not depend on value of ballistic coefficient*. If q = 0.1 (accuracy of σ_D determination is of 10%) the possible landing point displacement is of ±12 km.

For reentry angle $\theta_{en} = -4^o$ there are following equations:

$$\Phi_{at}(\sigma_D, \theta_{en} = -4^o) = 8.395^o - 1.783^o (\lg \sigma_D + 3.0), \qquad (12)$$

$$\Delta L(\theta_{en} = -4^o) = -85\ \text{km}\cdot q. \qquad (13)$$

If q = 0.1 the possible landing point displacement is of ±8.5 km. This value is 1.4 times less than in the case of reentry angle of $\theta_{en} = -3^o$.

For ballistic reentry vehicle a lift force coefficient is zero in nominal case (C_L=0), and any displacement of c.m. from symmetry axis is one of the most significant disturbing factors. It violates the axial symmetry of vehicle mass distribution while the aerodynamic shape retains the axial symmetry. As a result, a trim angle of attack α_{trim} arises that is almost constant during flight into the atmosphere. The angle of attack produces a lift force L that changes the ballistic reentry trajectory in controlled one with casual guidance law. A disturbing force acts in the orthogonal plane to the air velocity V. A direction of lift force L in this plane is random that produces a casual guidance law.

A disturbance of ballistic trajectory depends on lift-to-drag ratio $k_{trim} = L/D = C_L/C_D$. In linear approximation, derivative of lift-to-drag ratio with respect to c.m. displacement from symmetry axis ($-y_F$) is described by equation (Sikharulidze, 1999, NT-164)

$$dk_{trim}/dy_F = -C_L^{\alpha}/[C_D b(x_F/b)(1 + C_L^{\alpha}/C_D)]. \qquad (14)$$

Here $C_L^{\alpha} = \partial C_L/\partial\alpha$ is a derivative of lift coefficient with respect to the angle of attack, b is a main linear size of vehicle (diameter or length), x_F/b is a static stability margin. For Apollo shape reentry vehicle there are at $\alpha_{trim} = 0$: $C_D \approx 1.2$, $C_L^{\alpha} \approx -1.01$ rad^{-1}. If static stability margin is of x_F/b =-0.1 (typical value) and b=1 m, then $dk_{trim}/dy_F \approx -0.053$ mm^{-1}. It means that c.m. displacement on 1 mm only produces lift-to-drag ratio $k_{trim} \approx 0.053$. If the entry angle is of $\theta_{en} = -3^o$, this value of k_{trim} may generate downrange error of -46 km...+60 km and crossrange error of ±12 km. If $\theta_{en} = -4^o$, there are the downrange error of -33 km...+44 km and crossrange error of ±10 km. Vehicle rotation around symmetry axis with angular velocity of ω_x =10...20 degree/s (roll rate) allows significantly reduce (almost to zero) the effect of c.m. displacement from symmetry axis. Efficient action of arisen lift force is near zero, and descent trajectory is close to ballistic one.

2.4. Estimation of landing accuracy

Analysis of landing point accuracy under considered set of disturbances includes calculation more than 5000 reentry trajectories.

Preliminary estimation of landing accuracy in linear approximation may be obtained by partial derivatives of downrange and crossrange with respect to disturbing factors. There are four groups of errors. Errors of the first group affect on landing point directly (de-boost impulse execution time δt_{db} and side component of de-boost impulse ΔV_{sd}). The total error of execution time takes into account the non-nominal value of thrust ($\delta P = \pm 1\%$) and delay of engine input valve during switch-on and switch-off. An error of the second group [error of de-boost impulse value $\delta(\Delta V)$] disturbs reentry parameters at the boundary of atmosphere and, as a result, produces a dispersion of landing point. The error of de-boost impulse value takes into account the error of integrator (± 0.5%) and dispersion of thrust impulse during switch-off (±5%). In this case the atmospheric phase of trajectory significantly influences on dispersion of landing point. An error of the third group (error of engine orientation in the motion plane) in linear approximation does not influence on the landing accuracy. Errors of the fourth group are not related with de-boost maneuver (disturbed atmosphere, non-nominal ballistic coefficient, c.m. displacement from symmetry axis).

All results of landing accuracy analysis are presented in Table 1. One can see that among downrange errors the biggest component is due to error of de-boost impulse value. The second reason is disturbed atmosphere. Non-nominal drag coefficient and c.m. displacement from symmetry axis generate approximately equal errors. The total downrange error ($\pm 3\sigma_L$) is of $\Delta L_\Sigma = \pm 28.6$ km for reentry angle of $\theta_{en} = -3^o$ and ± 21.8 km for reentry angle of $\theta_{en} = -4^o$.

Table 1. Main components of landing point error, *km*

Group	Reason of error	Reentry angle -3°	Reentry angle -4°
	Downrange errors		
1	Execution time of de-boost impulse ($\delta P = \pm 1\%$)	± 1.67	± 2.55
2	Error of de-boost impulse value (±0.5%)	± 18.71	± 14.03
3	Error of de-boost engine orientation in the motion plane (pitch angle $\vartheta = \pm 1.5^o$)	0	0
4	Disturbed atmosphere (CMEDA)	± 14.5	± 12.1
	Non-nominal drag coefficient ($\delta\sigma_D = \pm 10\%$)	± 12.0	± 8.7
	Displacement of c.m. from symmetry axis	± 10.5	± 7.0
	Total downrange error ($\pm 3\sigma_L$)	± 28.6* ± 26.6**	± 21.8* ± 20.7**
	Crossrange errors		
1	Error of de-boost engine orientation in the horizontal plane (yaw angle $\psi = \pm 1.5^o$)	± 4.77	± 6.3
4	Disturbed atmosphere (CMEDA)	± 3.6	± 3.4
	Displacement of c.m. from symmetry axis	± 2.4	± 2.1
	Total crossrange error ($\pm 3\sigma_B$)	± 6.4* ± 6.0**	± 7.5* ± 7.2**

* Without angular rotation

** With roll rate of ω_x =10...20 degree/s

Among crossrange errors the biggest one is due to error of de-boost impulse orientation out of motion plane. The second reason (on value) is disturbed atmosphere. The total crossrange error ($\pm 3\sigma_B$) is of $\Delta B_\Sigma = \pm 6.4$ km for reentry angle of $\theta_{en} = -3^o$ and ± 7.5 km for reentry angle of $\theta_{en} = -4^o$ (Sikharulidze, 1999, NT-170).

If vehicle rotates around symmetry axis with angular velocity of $\omega_x = 10...20$ degree/s, the total downrange error will be of $\Delta L_\Sigma = \pm 26.6$ km for reentry angle of $\theta_{en} = -3^o$ and ± 20.7 km for reentry angle of $\theta_{en} = -4^o$. The total crossrange error will be of $\Delta B_\Sigma = \pm 6.0$ km for reentry angle $\theta_{en} = -3^o$ and ± 7.2 km for reentry angle of $\theta_{en} = -4^o$. Accuracy increases insignificantly due to assumption about small c.m. displacement from symmetry axis (only ± 0.2 mm). In case of bigger displacement a difference will be more significant.

Obtained accuracy of landing point depends on accepted assumptions about errors of de-orbit maneuver and parameters of vehicle, disturbed atmosphere, etc. May be this accuracy is optimistic one but it illustrates an order of expected landing accuracy. To guarantee the landing within given polygon, it is necessary to have a reserve (about of 20...30%) for compensation of non-considered disturbances and inaccuracy of motion model. Really, it is impossible to provide a high landing accuracy for ballistic reentry vehicle. Very often this task has solution by use the guided parachute system at the final phase of trajectory.

The modern guided parachute can provide lift-to-drag ratio of $k_{par} = L/D = 2$. It means that horizontal transfer may be 2 times bigger than vertical one. If parachute starts operation at altitude of $H_{par} = 15$ km it can compensate landing error about of 30 km. For $H_{par} = 10$ km it can compensate landing error about of 20 km. The tracking ground radar that measures a distance to vehicle (may be, velocity also) and angular position can provide correction of motion at the landing phase. The onboard equipment for parachute control may be very simple.

2.5. Acknowledgements

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