# ROBOT CALIBRATION USING A 3D VISION-BASED MEASUREMENT SYSTEM 

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#### Abstract

This work presents techniques for modeling and performing robot calibration processes under off-line programming using a 3-D vision-based measurement system. Kinematic modeling follows a singularity-free concept. The measurement system consists of a single CCD camera mounted on the robot tool flange and uses space resection models to measure the robot endeffector pose relative to a world coordinate system. A wide-angle lens is used and lens radial distortions are included in the model. Experimentation is performed on a PUMA-500 robot to test its accuracy improvement using the calibration system proposed. The robot was calibrated in many different regions and volumes within its workspace, achieving accuracy between three and six times better when comparing errors before and after calibration. The proposed off-line robot calibration system is fast, accurate and ease to setup.


Keywords: Kinematic model, Robot calibration, Absolute accuracy, Camera calibration.

## 1. INTRODUCTION

Most currently used industrial robots are still programmed by a teach pendant, especially in the automotive industry. However, the importance of off-line programming in industry as an alternative to teach-in programming is steadily increasing. The main reason for this trend is the need to minimize machine downtime and thus to improve the rate of robot utilization. A typical welding line with 30 robots and 40 welding spots per robot takes about 400 hours for robot teaching (Bernhardt, 1997). Nonetheless, for a successful accomplishment of off-line programming the robots need to be not only repeatable but also accurate.

Robot repeatability is unaffected by the method of programming since it is due to random errors (e.g. due to the finite resolution of joint encoders). In contrast, the systematic errors in absolute position are due almost entirely to programming the robot off-line. One of the leading sources of lack of accuracy is the mismatch between the prediction made by the kinematic model and the actual system. Robot constant pose errors are attributed to several sources, including errors in geometrical parameters (e.g. link lengths and joint offsets) and deviations which vary predictably with position (e.g. compliance or gear transmission errors). Robot calibration is an integrated process of modeling, measurement, numeric identification of actual physical characteristics of a robot, and implementation of a new model. By calibration, an improved kinematic model of the physical robot is generated and used in conjunction with simulation and off-line programming systems. This eliminates most of the
systematic errors in position and allows accurate off-line programs to be generated and task completion to be reliably achieved without on-line position editing.

The calibration procedure first involves the development of a kinematic model whose parameters represent accurately the actual robot. Next, robot characteristics specifically chosen are measured with measurement instruments with known accuracy. Then a parameter identification procedure is used to compute the parameter values set which, when introduced in the robot nominal model, accurately represents the measured robot behavior. Finally, the model in the position control software is corrected.

In spite of the industrial needs cited above, no calibration system has been accepted and used generally in industry so far. Notwithstanding many calibration systems being available currently in the market, none of them combine low price, accuracy, ease of use, and speed of setup and implementation (Schröer, 1994).

The objectives of this research are to investigate theoretical aspects involved in robot calibration methods and systems, to develop a feasible low cost vision-based measurement system using a single camera and, finally, to construct a prototype of a robot calibration system. More specific goals can be listed as: to achieve robot position accuracy below 1 mm after calibration; to investigate practical aspects and achievable accuracy using common off-the-shelf CCD cameras as a 3-D measurement system using only a single camera; to build an off-line robot calibration system aiming at low cost, ease of use, flexibility within robot environments, and with an acceptable accuracy. The robot used to test the system was a PUMA-500.

## 2. KINEMATIC MODELING AND PARAMETER IDENTIFICATION

The first step to kinematic modeling is the proper assignment of coordinate frames to each link. Coordinate frames to joints are assigned such that the z -axis is coincident with the joint axis. This convention is used by many authors and in many robot controllers (McKerrow, 1995, Paul, 1981). The x-axis or the $y$-axis have their direction according to the convention used to parameterize the transformations between links. For either perpendicular or parallel joint axes the Denavit-Hartenberg or Hayati modeling convention were used respectively. The requirements of a singularity-free parameter identification model prevents the use of a single minimal modeling convention that can be applied uniformly to all possible robot geometries (Schröer et al., 1997, Baker, 1990). At this point the homogeneous transformations between joints must have been already determined. The other axis (x or y) can be determined using the right-hand rule.

The kinematic equation of the robot manipulator is obtained by consecutive homogeneous transformations from the base frame to the last frame. Thus,

$$
\begin{equation*}
\hat{\mathrm{T}}^{0}{ }_{\mathrm{N}}=\hat{\mathrm{T}}^{0}{ }_{\mathrm{N}}(\mathrm{k})=\mathrm{T}^{0}{ }_{1} \cdot \mathrm{~T}^{1}{ }_{2} \ldots \mathrm{~T}^{\mathrm{N}-1}{ }_{\mathrm{N}}=\prod_{\mathrm{i}=1}^{\mathrm{N}} \mathrm{~T}^{\mathrm{i}-1_{i}} \tag{1}
\end{equation*}
$$

where N is the number of joints (or coordinate frames), $\mathrm{p}=\left[\begin{array}{lll}\mathrm{p}_{1}{ }^{\mathrm{T}} & \mathrm{p}_{2}{ }^{\mathrm{T}} \ldots \mathrm{p}_{\mathrm{n}}{ }^{\mathrm{T}}\end{array}\right]^{\mathrm{T}}$ is the parameter vector for the manipulator, and $p_{i}$ is the link parameter vector for the joint $i$, including the joint errors. The exact link transformation $\mathrm{A}^{\mathrm{i}-1}{ }_{\mathrm{i}}$ is (Driels \& Pathre, 1990):

$$
\begin{equation*}
\mathrm{A}_{\mathrm{i}}^{\mathrm{i}-1}=\mathrm{T}_{\mathrm{i}}^{\mathrm{i}-1}+\Delta \mathrm{T}_{\mathrm{i}} \quad, \quad \Delta \mathrm{~T}_{\mathrm{i}}=\Delta \mathrm{T}_{\mathrm{i}}\left(\Delta \mathrm{p}_{\mathrm{i}}\right) \tag{2}
\end{equation*}
$$

where $\Delta p_{i}$ is the link parameter error vector for the joint $i$.
The exact manipulator transformation $\hat{\mathrm{A}}^{0}{ }_{\mathrm{N}-1}$ is

$$
\begin{equation*}
\hat{A}^{0}{ }_{N}=\prod_{i=1}^{N}\left(T^{i-1}{ }_{i}+\Delta T_{i}\right)=\prod_{i=1}^{N} A^{i-1}{ }_{i} \tag{3}
\end{equation*}
$$

Thus,

$$
\begin{equation*}
\hat{\mathrm{A}}^{0}{ }_{\mathrm{N}}=\hat{\mathrm{T}}^{0}{ }_{\mathrm{N}}+\Delta \hat{\mathrm{T}} \quad, \quad \Delta \hat{\mathrm{~T}}=\Delta \hat{\mathrm{T}}(\mathrm{q}, \Delta \mathrm{p}) \tag{4}
\end{equation*}
$$

where $\Delta \mathrm{p}=\left[\Delta \mathrm{p}_{1}{ }^{\mathrm{T}} \Delta \mathrm{p}_{2}{ }^{\mathrm{T}} \ldots \Delta \mathrm{p}_{\mathrm{n}}{ }^{\mathrm{T}}\right]^{\mathrm{T}}$ is the manipulator parameter error vector and $q$ is the vector of joint variables $\left[\theta_{1}{ }^{\mathrm{T}}, \theta_{2}{ }^{\mathrm{T}} \theta_{\mathrm{N}}{ }^{\mathrm{T}}\right]^{\mathrm{T}}$. It must be stated here that $\Delta \hat{\mathrm{T}}$ is a non-linear function of the manipulator parameter error vector $\Delta \mathrm{p}$.

Considering $m$ the number of measure positions it can be stated that

$$
\begin{equation*}
\hat{\mathbf{A}}=\hat{\mathbf{A}}_{\mathrm{N}}=\hat{\mathbf{A}}(\mathbf{q}, \mathrm{p})=\left(\hat{\mathrm{A}}\left(\mathrm{q}_{1}, \mathrm{p}\right), \ldots, \hat{\mathrm{A}}\left(\mathrm{q}_{\mathrm{m}}, \mathrm{p}\right)\right)^{\mathrm{T}} \quad: \mathfrak{R}^{\mathrm{n}} \times \mathfrak{R}^{\mathrm{mN}} \tag{5}
\end{equation*}
$$

where $\hat{\mathbf{A}}: \mathfrak{R}^{\mathrm{n}} \mathrm{x} \mathfrak{R}^{\mathrm{mN}}$ is function of two vectors with $n$ and mN dimensions, $n$ is the number of parameters and N is the number of joints (including the tool), and

$$
\begin{equation*}
\Delta \hat{\mathbf{T}}=\Delta \hat{\mathbf{T}}(\mathbf{q}, \Delta \mathrm{p})=\left(\Delta \hat{\mathrm{T}}\left(\mathrm{q}_{1}, \Delta \mathrm{p}\right), \ldots, \Delta \hat{\mathrm{T}}\left(\mathrm{q}_{\mathrm{m}}, \Delta \mathrm{p}\right)^{\mathrm{T}} \quad: \mathfrak{R}^{\mathrm{n}} \times \mathfrak{R}^{\mathrm{mN}}\right. \tag{6}
\end{equation*}
$$

All matrices or vectors in bold are functions of $m$. The identification itself is the computation of those model parameter values $p^{*}=p+\Delta p$ which result in an optimal fit between the actual measured positions and those computed by the model, i.e., the solution of the nonlinear equation system (Motta \& McMaster, 1999a)

$$
\begin{equation*}
\mathbf{J} \cdot x=\mathbf{b} \tag{7}
\end{equation*}
$$

where the following notation can be used

$$
\begin{array}{ll}
\mathbf{b}=\mathbf{M}(\mathbf{q})-\mathbf{B}(\mathbf{q}, \mathrm{p}) & \in \mathfrak{R}^{\phi \mathrm{m}} \\
\mathbf{J}=\mathbf{J}(\mathbf{q}, \Delta \mathrm{p}) & \in \mathfrak{R}^{\phi \mathrm{mxn}} \\
\mathrm{x}=\Delta \mathrm{p} & \in \mathfrak{R}^{\mathrm{n}} \\
\mathbf{r}=\mathbf{J} \cdot \mathrm{x}-\mathbf{b} & \in \mathfrak{R}^{\phi \mathrm{m}} \tag{11}
\end{array}
$$

where $\mathbf{B}$ is a vector formed with position and orientation components of $\hat{\mathbf{A}}, \mathbf{M}(\mathbf{q})$ are all measured components and $\phi$ is the number of measurement equations provided by each measured pose. $\mathbf{J}$ is the identification Jacobean and $\mathbf{r}$ is the residue to be minimized. If orientation measurement can be provided by the measurement system then 6 measurement equations can be formulated per each pose. If the measurement system can only measure position, each pose measurement can supply data for 3 measurement equations per pose and then $\mathbf{B}$ includes only the position components of $\hat{\mathbf{A}}$.

One method to solve non-linear least-square problems proved to be very successful in practice and then recommended for general solutions is the algorithm proposed by LevenbergMarquardt (Dennis \& Schnabel, 1983). Several algorithms versions of the L.M. algorithm have been proved to be successful (globally convergent). From eq. (7) the method can be formulated as

$$
\begin{equation*}
\mathrm{x}_{\mathrm{j}+1}=\mathrm{x}_{\mathrm{j}}-\left[\mathbf{J}\left(\mathrm{x}_{\mathrm{j}}\right)^{\mathrm{T}} \cdot \mathbf{J}\left(\mathrm{x}_{\mathrm{j}}\right)+\mu_{\mathrm{j}} \cdot \mathbf{I}\right]^{-1} \cdot \mathbf{J}\left(\mathrm{x}_{\mathrm{j}}\right) \cdot \mathbf{b}\left(\mathrm{x}_{\mathrm{j}}\right) \tag{12}
\end{equation*}
$$

where, according to Marquardt suggestion, $\mu_{\mathrm{j}}=0.001$ if $\mathrm{x}_{\mathrm{j}}$ is the initial guess, $\mu_{\mathrm{j}}=\lambda(0.001)$ if $\left\|\mathbf{b}\left(\mathrm{x}_{\mathrm{j}+1}\right)\right\| \geq\left\|\mathbf{b}\left(\mathrm{x}_{\mathrm{j}}\right)\right\|, \mu_{\mathrm{j}}=0.001 / \lambda$ if $\left\|\mathbf{b}\left(\mathrm{x}_{\mathrm{j}+1}\right)\right\| \leq\left\|\mathbf{b}\left(\mathrm{x}_{\mathrm{j}}\right)\right\|$ and $\lambda$ is a constant valid in the range of $2.5<\lambda<10$ (Press et al, 1994).

## 3. VISION-BASED MEASUREMENT SYSTEM

Robot Calibration using a camera system is potentially fast, automated and easy to use. Cameras can also provide full pose measuring capability (position and orientation). The system configurations can vary from stationary cameras to stereo or single moving cameras. Due to the needs of accuracy and large range of motion during robot calibration procedures, stationary cameras cannot fulfill the requirements, and stereo moving cameras are restricted to local calibration within small volumes since the field-of-view is limited by the distance between the cameras. A single moving camera presents the advantages of a large field-of-view with a potential large depth of field, and a considerably reduced hardware and software complexity of the system. One disadvantage is the need of camera re-calibration at each pose (position and orientation).

The camera model is at first assumed to be as the standard distortion-free "pin-hole" by which every real object point is connected to its corresponding image point through a straight line that passes through the focal point of the lens (Fig.1). The transformation from the world coordinates ( $\mathrm{xw}, \mathrm{yw}, \mathrm{zw}$ ) to the camera coordinates ( $\mathrm{x}, \mathrm{y}, \mathrm{z}$ ) is:

$$
\left[\begin{array}{l}
x  \tag{13}\\
y \\
z
\end{array}\right]=R \cdot\left[\begin{array}{l}
x w \\
y w \\
z w
\end{array}\right]+T
$$

where the rotation matrix R and translation vector T can be written as:

$$
\mathrm{R}=\left[\begin{array}{ccc}
\mathrm{r} 1 & \mathrm{r} 2 & \mathrm{r} 3  \tag{14}\\
\mathrm{r} 4 & \mathrm{r} 5 & \mathrm{r} 6 \\
\mathrm{r} 7 & \mathrm{r} 8 & \mathrm{r} 9
\end{array}\right] \quad \text { and } \quad \mathrm{T}=\left[\begin{array}{lll}
\mathrm{Tx} & \mathrm{Ty} & \mathrm{Tz}
\end{array}\right]^{\mathrm{T}}
$$

The transformation from the 3-D camera coordinates to the distorted or true image coordinates (X,Y), Pd in Fig. 1, can be achieved using the Radial Alignment Constraint algorithm (RAC) (Tsai, 1987, Zhuang \& Roth, 1996, Lenz \& Tsai, 1987).

$$
\begin{align*}
& \frac{X}{1-k \cdot r^{2}} \cong f \cdot \frac{r 1 \cdot x w+r 2 \cdot y w+r 3 \cdot z w+T x}{r 7 \cdot x w+r 8 \cdot y w+r 9 \cdot z w+T z}  \tag{15}\\
& \frac{Y}{1-k \cdot r^{2}} \cong f \cdot \frac{r 4 \cdot x w+r 5 \cdot y w+r 6 \cdot z w+T y}{r 7 \cdot x w+r 8 \cdot y w+r 9 \cdot z w+T z} \tag{16}
\end{align*}
$$

where $k$ is the radial distortion coefficient and $r=\left(X^{2}+Y^{2}\right)^{1 / 2}$. This system can be solved using the Singular Value Decomposition (SVD) method (Zhuang \& Roth, 1996) through the system of linear equations below once R , Tx and Ty are taking to be known and zw is null (coplanar points):


Figure 1 - Camera "pin-hole" model.

$$
\left[\begin{array}{lll}
-X_{i} & x_{i} & -x_{i} \cdot r_{i}{ }^{2}
\end{array}\right]\left[\begin{array}{c}
\mathrm{Tz}  \tag{17}\\
f \\
\mathrm{k} . \mathrm{f}
\end{array}\right]=X_{i} \cdot \mathrm{~W}_{\mathrm{i}}
$$

where $x_{i}=r 1 . \mathrm{xw}_{\mathrm{i}}+\mathrm{r} 2 . \mathrm{yw}_{\mathrm{i}}+\mathrm{Tx}, \mathrm{w}_{\mathrm{i}}=\mathrm{r} 7 . \mathrm{xw}_{\mathrm{i}}+\mathrm{r} 8 . \mathrm{yw}_{\mathrm{i}}$, and $i$ is the index correspondent to each calibration point in a grid of points. Details about the photogrammetric model, scale factor and image center calibration can be seen in the paper published by Motta \& McMaster (1999b). The vision system used consists of a small CCD camera, $752 \times 582$ pixels, a 12.5 mm focus length lens, a software to process images to sub-pixel accuracy, and a target of calibration points as shown in Fig. 2. The disposition of two planes in 45 degrees from the horizontal plane avoids angles smaller than 20 degrees between the camera optical axis and the target plane, which may produce ill-conditioned solutions. The measurement system accuracy was assessed experimentally (Motta \& McMaster, 1999b) showing to vary from 0.2 mm to 0.4 mm at distances from the target from 600 mm to 1000 mm .


Figure 2 - Calibration board and Robot with camera.

## 4. Experimental Results

Within the PUMA-500 robot workspace three calibration Regions were defined to collect data. In each Region several Volumes were defined with different dimensions. Once two different Volumes had the same volume they had also the same dimensions, whatever Region they were in.

Figure 3 represents graphically all Regions and Volumes within the PUMA-500 workspace. It can be observed that in Region 1 there were two Volumes equal in volume and dimensions, but at different locations (V1 \& V1a). The reason for that was to observe the influence of the distance from the camera optical center to the target center point in the calibration overall accuracy, and also how different manipulator configurations within the same workspace Region (different joint motion ranges) could affect results, keeping constant volumes of motion.

### 4.1 Comparison between the Error Before and After Calibration

The average errors of the PUMA-500 calculated in each of the Volumes within the three measurement Regions are shown in this section. For calculated and measured data in different coordinate systems to be compared to each other, the robot base coordinate frame was moved to coincide with the world coordinate system at the measurement target plate. This procedure was carried out through a recalibration of the robot base in each Volume.

The results from the calculation of average errors and their standard deviation in each Volume of a Region can be seen in the graphs shown in Figures 4, 5 and 6, calculated before and after the calibration.

The results show that the error before and after the calibration tended to increase with the increase of the calibration volume in all Regions. The exception is the calibration volume V1a in Region 1, which presented larger errors than Volume V2 and V3.

The results suggest that V1a may have been placed in a workspace region where the robot arm has larger position errors due to the arm geometry configuration. This observation can be strengthened by the fact that after the calibration all Volumes in Region 1 had little differences in accuracy between each other.


Figure 3 - Side and top view of the PUMA-500 Robot workspace showing Regions, Volumes and their dimensions and locations.

Another observation that stands out is the high average and standard deviation of the errors before the calibration in V4, Region 1, changing to be very small after the calibration.

That breaks the tendency observed in other Volumes. However, the average values of the errors for all Volumes are within an equivalent range of reliability shown by the standard deviations.


Figure 4 - Average Error and Standard Deviation calculated before and after calibration in each Volume in Region 1.


Figure 5 - Average Error and Standard Deviation calculated before and after calibration in each Volume in Region 2.

## 5. CONCLUSIONS

The calibration system proposed showed to improve the robot accuracy to well below 1 mm . The system allows a large variation in robot configurations, which is essential to proper calibration. The robot calibration system approach proposed here stood out to be a feasible alternative to the expensive and complex systems available today in the market, using a single camera and showing good accuracy and ease of use and setup.


Figure 6 - Average Error and Standard Deviation calculated before and after calibration in each Volume in Region 3.

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