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NONLINEAR, ADAPTIVE CONTROL SYSTEM FOR PAYLOAD EXTRACTION OPERATIONS.

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Abstract. *The development of a nonlinear, adaptive control system for payload extraction operations is presented. Load extraction in cargo aircraft during flight is considered as a dangerous maneuver since it normally affects the longitudinal stability of the aircraft, commonly rendering it unstable. The online adpatation control strategy seems an adequate approach for solving the problem, since it can deal with the drift in the plant parameters caused by the movement of the load inside the aircraft. In this work, the effects of a continously varying C.G. position on the longitudinal flight dynamics are modeled in detail. A controller is then proposed based on the derived equations of motion, applying the technique of dynamic inversion coupled with a model reference adaptive controller to deal with the varying parameters.The dynamics system considered for the control problem consists of the modeled aircraft dynamics augmented by the unkown parameters - whose dynamics are controlled by the chosen adaptation laws. The demonstration of stability for the complete system is done via Lyapunov's stability theorem for nonlinear dynamic systems. A suitable Lyapunov Function Candidate is proposed. Simulation results are presented and discussed based on the theory and the advantages and drawbacks for the application of such a control law on real aircraft control systems are listed and critically analysed.*

Keywords: *LAPES, Payload Extraction Systems, Control Design, Adaptive Control.*

1. INTRODUCTION

Several kind of payload extraction manouvers are performed as tatical delivery methods in places where there is no runway for an aircraft to land or the terrain is not adequate for aircraft operations, when precise location of the delivery is important.

This kind of manouvers can be done with the aid of a LAPES (low altitude payload extraction system) which involves several systems inside the aircraft - as the parachutes for pulling the load, the pallets and rail system inside the fuselage and of course some stability augmentation system to help the pilot acomplish this normally dificult task.

Figure 1: A C-130 performing a Low-Altitude Payload Extraction manouver. (U.S. Air Force).

Load extractions are a critical and dangerous manouver, not only they are done in low altitudes and relatively low velocity, but also, as it shall be shown in section 4, as the load moves inside the aircraft towards its rear, the aircraft's center of gravity (CG) gets dislocated, severely affecting the aircraft's dynamics, sometimes even unstabilizing it. This maneuver was already extensively studied from the aircraft control point of view during the decade of the 1960 by works like Rutan and Stroup (1967).

Since one of the promissing applications of adaptive control is to be able to maintain a controlled plant following a desired trajectory even in the presence of unpredictable parameter change (as discussed in Astrom and Wittenmark (2008)), it seems apropriate to study the development of a LAPES with this control technique.

2. DYNAMICS

2.1 LONGITUDINAL AIRCRAFT DYNAMICS

Following the works of Etkin (1972); Roskam (2001); Duke *et al.* (1988) one can write the nonlinear equations for the an aircraft's longitudinal dynamics.

Considering the state variable given by $X = [\theta \quad q \quad \alpha \quad V \quad H]^T$ and the control variable is given by $U = [\mathrm{d}_p \ \mathrm{d}_\pi]^T$ the diferential equations defining the longitudinal dynamics is given by:

$$\dot{q} = \frac{\boxed{\mathcal{M}} + z_{F/CG} F \cos(\alpha_f) - \boxed{\dot{I_{yy}}}q}{\boxed{I_{yy}}} \tag{1a}$$

$$\dot{V} = \frac{1}{\boxed{m}}\left(\boxed{-D} + F\cos(\alpha_f - \alpha)\right) - g\sin(\theta - \alpha) \tag{1b}$$

$$\dot{\alpha} = q - \frac{1}{\boxed{m}V}\left(\boxed{L} - \boxed{m}g\cos(\theta-\alpha) + F\sin(\alpha_f - \alpha)\right) \tag{1c}$$

$$\dot{\theta} = q \tag{1d}$$

$$\dot{x} = V\cos(\theta - \alpha) \tag{1e}$$

$$\dot{H} = V\sin(\theta - \alpha) \tag{1f}$$

Where the aerodynamic forces and moment are given by:

$$L = \tfrac{1}{2}\rho V^2 C_L \quad D = \tfrac{1}{2}\rho V^2 C_D \quad \mathcal{M} = \tfrac{1}{2}\rho V^2 \bar{c} C_{\mathcal{M}}$$

The aerodynamic coeficients $(C_L, C_{\mathcal{M}})$, can be reasonably well aproximated to depend linearly on the state variables. The drag coefficient C_D is normally aproximated as a parabolic function of the lift coefficient:

$$\begin{aligned} C_L &= C_{L_0} + C_{L_\alpha}\alpha + \boxed{C_{L_{\hat{q}}}}\frac{q\bar{c}}{2V_{ref}} + C_{L_{\mathrm{d}_p}}\mathrm{d}_p \\ C_D &= C_{D_0} + k_1 C_L + kC_L^2 \\ C_{\mathcal{M}} &= C_{\mathcal{M}_0} + \boxed{C_{\mathcal{M}_\alpha}}\alpha + \boxed{C_{\mathcal{M}_{\hat{\dot{\alpha}}}}}\frac{\dot{\alpha}\bar{c}}{2V_{ref}} + \boxed{C_{\mathcal{M}_{\hat{q}}}}\frac{q\bar{c}}{2V_{ref}} + \boxed{C_{\mathcal{M}_{\mathrm{d}_p}}}\mathrm{d}_p \end{aligned}$$

All the boxed terms is the above equations are dependent on the aircraft's $C.G.$ location as varied by the displacement of the cargo load. In the next section the origin and form of this variations is seen in detail.

2.2 EFFECTS OF INTERNAL MASS DISPLACEMENT ON AIRCRAFT DYNAMICS

The dynamics of the aircraft are affected by the displacement of an internal load as it changes the center of gravity location from its original position. These corrections are presented below.

The moment of inertia of the airplane carrying a load can be expressed as the sum of both the airplane and load inertias with respect to the current $C.G.$ location:

$$I_{yy}^{Total/X_{cg}} = I_{yy}^{A/X_{cg}} + I_{yy}^{C/X_{cg}} \tag{2}$$

Each of the terms on the right-hand side of equation 2 can be further expressed as the sum of the moment of inertia of each component with respect to its own $C.G.$ with the product of its mass to the square of the distance from its $C.G.$ to the $C.G.$ of the complete system. Defining X_{cg0} as the center of mass of the unloaded aircraft and X_{cgC} as the position of the load's center of mass we can write:

$$I_{yy}^{Total/X_{cg}} = I_{yy}^{A/X_{cg0}} + (X_{cg} - X_{cg0})^2 m_A + I_{yy}^{C/X_{cgC}} + (X_{cg} - X_{cgC})^2 m_C \tag{3}$$

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The term $I_{yy}^{C/X_{cgC}}$ in equation 3 is considered negligible with relation to the others. The displacement of the center of gravity of the loaded airplane with respect to its original location $(X_{cg} - X_{cg0})$ will appear in other expressions as well, which motivates the definition:

$$\Delta X_{cg} \triangleq X_{cg} - X_{cg0} \tag{4}$$

The current position of center of gravity of the aircraft at a given time is expressed as:

$$X_{cg} = \frac{X_{cg0}m_A + X_{cgC}m_C}{m_A + m_C} \tag{5}$$

Defining the *aircraft load-mass ratio* as $\mu_A \triangleq m_A/(m_A + m_C)$, the relation $(X_{cg} - X_{cgC})$ can be expressed as:

$$(X_{cg} - X_{cgC}) = \frac{-\mu_A}{(1-\mu_A)}(X_{cg} - X_{cg0}) = \frac{-\mu_A}{(1-\mu_A)}\Delta X_{cg} \tag{6}$$

Substituting equations 4 and 6 in equation 3 we find:

$$\boxed{I_{yy} = I_{yy0} + \frac{m_A}{1-\mu_A}\Delta X_{cg}^2} \tag{7}$$

Where the inertia of the empty plane $I_{yy}^{A/X_{cg0}}$ was represented as I_{yy0}. The derivative of the inertia with respect to time is obtained directly from equation 7 and is given by:

$$\boxed{\dot{I_{yy}} = \frac{2m_A}{1-\mu_A}\Delta X_{cg}\Delta\dot{X}_{cg}} \tag{8}$$

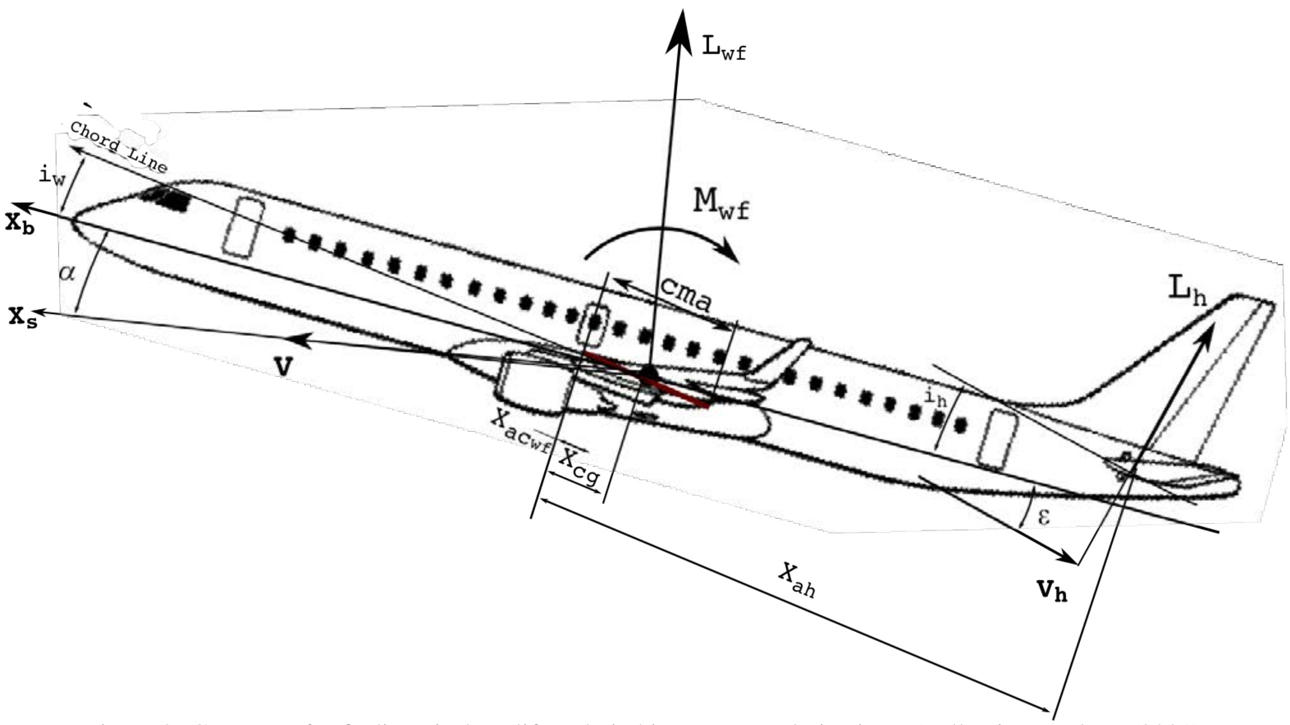

Figure 2: Geometry for finding airplane lift and pitching moment derivatives. (Following Roskam (2001))

Variation of $C_{\mathcal{M}_\alpha}$ Taking the definitions of figure 2 as a base to calculate the balance of longitudinal forces and moments acting on the aircraft, one arrives at equation 9 for the dependence of the pitching moment with the angle of attack Roskam (2001); Etkin (1972).

$$C_{\mathcal{M}_\alpha}|_{X_{cg}} = C_{L_{\alpha wf}}(\bar{X}_{cg} - \bar{X}_{ac_{wf}}) - C_{L_{\alpha h}}\eta_h\frac{S_h}{S}(\bar{X}_{ac_h} - \bar{X}_{cg})(1 - \frac{\mathrm{d}\epsilon}{\mathrm{d}\alpha}) \tag{9}$$

In equation (9), the subscript wf means "the wing-fuselage system" and the subscript h means "the horizontal tail". The term η_h is the ratio of the dynamic pressure acting on the horizontal tail with the dynamic pressure acting on the wing

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(which can be greater than 1 if for example the engines are blowing air right at the tail). Notice also that all distances are normalized by a reference distance - in this case the mean aerodynamic chord - since equation (9) is dimensionless.

The pitching moment calculated in 9 is with respect to the aircraft's center of gravity. There is a point of the aircraft for which the pitching moment does not vary with angle of attack. This point, called the neutral point, can be calculated from the definition by imposing that $C_{\mathcal{M}_\alpha}|_{N.P.} = 0$. So, re-writting 9 for the neutral point one gets:

$$\bar{X}_N = \frac{C_{L_\alpha wf}\bar{X}_{ac_{wf}} + C_{L_\alpha h}\eta_h\frac{S_h}{S}(1-\frac{\mathrm{d}\epsilon}{\mathrm{d}\alpha})\bar{X}_{ac_h}}{C_{L_\alpha wf} + C_{L_\alpha h}\eta_h\frac{S_h}{S}(1-\frac{\mathrm{d}\epsilon}{\mathrm{d}\alpha})} \tag{10}$$

Defining:

$$C_{L_\alpha h}' \triangleq C_{L_\alpha h}\eta_h\frac{S_h}{S}(1-\frac{\mathrm{d}\epsilon}{\mathrm{d}\alpha}) \tag{11}$$

And substituting 10 and 11 in 9 one finds:

$$C_{\mathcal{M}_\alpha}|_{X_{cg}} = \underbrace{(C_{L_\alpha wf} + C_{L_\alpha h}')}_{C_{L_\alpha}}(\bar{X}_{cg} - \bar{X}_N) \tag{12}$$

Finally, substituting $\bar{X}_{cg}$ in 12 for $\bar{X}_{cg0} + \Delta\bar{X}_{cg}$ we find the formula for the correction of the derivative of $C_{\mathcal{M}_\alpha}$ with $C.G.$ displacement:

$$\boxed{C_{\mathcal{M}_\alpha}|_{X_{cg}} = C_{\mathcal{M}_\alpha}|_{X_{cg0}} + C_{L_\alpha}\Delta\bar{X}_{cg}} \tag{13}$$

Variation of $C_{\mathcal{M}_{\mathrm{d}_p}}$ Referring again to figure 2 one expresses the variation of pitching moment with elevator position as Roskam (2001). After substitution the same substitution for $\bar{X}_{cg}$ done before:

$$\boxed{C_{\mathcal{M}_{\mathrm{d}_p}}|_{X_{cg}}} = -\underbrace{C_{L_\alpha h}\eta_h\tau_e\frac{S_h}{S}}_{C_{L_{\mathrm{d}_p}}}(\bar{X}_{ac_h} - \bar{X}_{cg}) = \boxed{C_{\mathcal{M}_{\mathrm{d}_p}}|_{X_{cg0}} + C_{L_{\mathrm{d}_p}}\Delta\bar{X}_{cg}} \tag{14}$$

Variation of $C_{L_{\hat{q}}}$ and $C_{\mathcal{M}_{\hat{q}}}$ When the aeroplane undergoes a sudden pitch variation, the angle of attack of the horizontal tail is modified by Roskam (2001); Etkin (1972):

$$\Delta\alpha = \frac{q(X_{ac_h} - X_{cg})}{V_0} \tag{15}$$

This corresponds to an increase in lift:

$$\Delta L = \frac{1}{2}\rho V_0^2 S_h\eta_h C_{L_\alpha h}\frac{q(X_{ac_h} - X_{cg})}{V_0} \tag{16}$$

Dividing 16 by $\frac{1}{2}\rho V_0^2 S$ one finds:

$$\Delta C_L = \underbrace{2C_{L_\alpha h}\eta_h\frac{S_h}{S}(\bar{X}_{ac_h} - \bar{X}_{cg})}_{C_{L_{\hat{q}}}}\underbrace{\left(\frac{q\bar{c}}{2V_0}\right)}_{\hat{q}} \tag{17}$$

Thus the variation of $C_{L_{\hat{q}}}$ with ΔX_{cg} is given by:

$$C_{L_{\hat{q}}}|_{X_{cg}} = C_{L_{\hat{q}}}|_{X_{cg0}} - \frac{C_{L_{\hat{q}}}|_{X_{cg0}}}{(\bar{X}_{ac_h} - \bar{X}_{cg0})}\Delta\bar{X}_{cg} \tag{18}$$

As for the derivative $C_{\mathcal{M}_{\hat{q}}}$, it is simply the negative of the product of $C_{L_{\hat{q}}}$ with $(\bar{X}_{ac_h} - \bar{X}_{cg})$:

$$C_{\mathcal{M}_{\hat{q}}}|_{X_{cg}} = -2.2C_{L_\alpha h}\eta_h\frac{S_h}{S}(\bar{X}_{ac_h} - \bar{X}_{cg})^2 \tag{19}$$

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Where a factor of 10% was added to take into account the effect of the wing on the pitching moment. This is called the *fudge-factor* Roskam (2001). Using 18 and 19 one can solve for the horizontal tail distance from the center of mass:

$$(\bar{X}_{ac_h} - \bar{X}_{cg0}) = \frac{C_{\mathcal{M}_{\hat{q}}}|_{X_{cg0}}}{-1.1 C_{L_{\hat{q}}}|_{X_{cg0}}} \tag{20}$$

Substituting 20 in equations 18 and 19 one finds:

$$\boxed{C_{L_{\hat{q}}}|_{X_{cg}} = C_{L_{\hat{q}}}|_{X_{cg0}} + \frac{1.1(C_{L_{\hat{q}}}|_{X_{cg0}})^2}{C_{\mathcal{M}_{\hat{q}}}|_{X_{cg0}}}\Delta\bar{X}_{cg}} \tag{21}$$

$$\boxed{C_{\mathcal{M}_{\hat{q}}}|_{X_{cg}} = C_{\mathcal{M}_{\hat{q}}}|_{X_{cg0}} + 2.2 C_{L_{\hat{q}}}|_{X_{cg0}}\Delta\bar{X}_{cg} + \frac{(1.1 C_{L_{\hat{q}}}|_{X_{cg0}})^2}{C_{\mathcal{M}_{\hat{q}}}|_{X_{cg0}}}(\Delta\bar{X}_{cg})^2} \tag{22}$$

Variation of $C_{\mathcal{M}_{\hat{\dot{\alpha}}}}$ As in the case of the $C_{\mathcal{M}_{\hat{q}}}$ derivative, $C_{\mathcal{M}_{\hat{\dot{\alpha}}}}$ is the negative product of $C_{L_{\hat{\dot{\alpha}}}}$ with the distance from the horizontal tail with the $C.G.$.

This effect is caused manly by the variation in intensity of the vortices shed by the wing with instantaneous angle of attack. The vortices shed by the wing at a time t will reach the horizontal tail at a time $t + \Delta t$, where Δt is the time the flow takes to get from the wing to the horizontal tail. This delay is a funcion of the aicraft's speed and geometric properties, as the distance from the aerodynamic centers of the wing and horizontal tail. There is a common practical approximation in the literature Roskam (2001); Etkin (1972) where this distance is approximated by the distance from the $C.G.$ to the horizontal tail. While this is a valid approximation for most flight conditions, this is not true in our case.

One can find in the cited references a more in depth description of the calculations for the derivatives $C_{L_{\hat{\dot{\alpha}}}}$ and $C_{\mathcal{M}_{\hat{\dot{\alpha}}}}$. We only stress here that $C_{L_{\hat{\dot{\alpha}}}}$ does **not** depend on the aircraft's center of mass on a first approximation. It only depends on the geometric properties of the aeroplane.

The dependence of $C_{\mathcal{M}_{\hat{\dot{\alpha}}}}$ with the $C.G.$ position is thus given by:

$$\boxed{C_{\mathcal{M}_{\hat{\dot{\alpha}}}}|_{X_{cg}} = -C_{L_{\hat{\dot{\alpha}}}}(\bar{X}_{ac_h} - \bar{X}_{cg}) = C_{\mathcal{M}_{\hat{\dot{\alpha}}}}|_{X_{cg0}} + C_{L_{\hat{\dot{\alpha}}}}\Delta\bar{X}_{cg}} \tag{23}$$

2.3 PAYLOD EXTRACTION DYNAMICS

It is necessary to establish what will be the dynamics of the aircraft's center of gravity position during the time the load is being ejected from the fuselage as in figure 3.

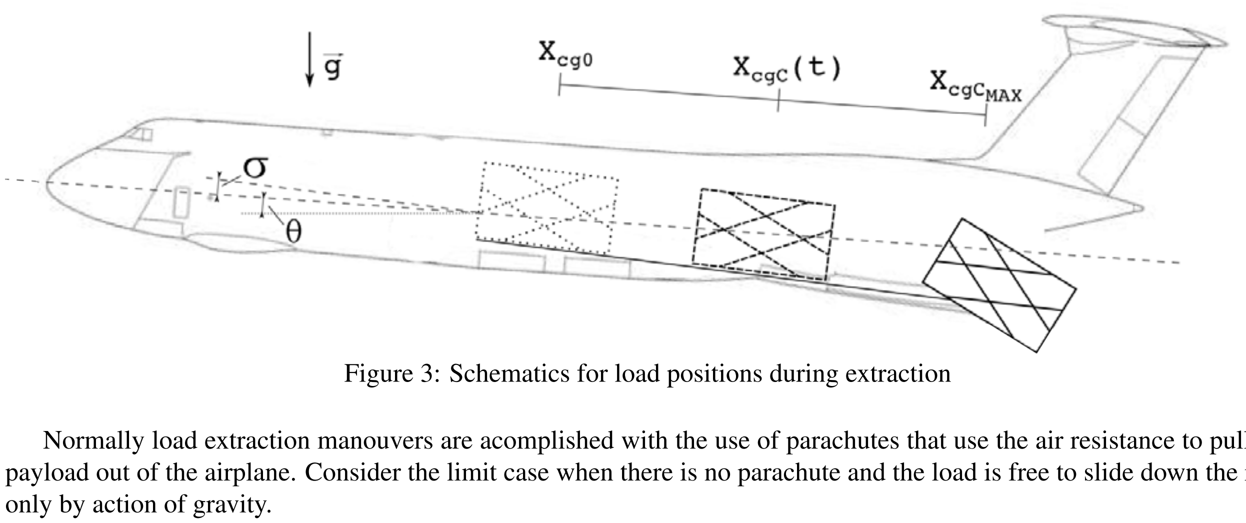

Figure 3: Schematics for load positions during extraction

Normally load extraction manouvers are acomplished with the use of parachutes that use the air resistance to pull the payload out of the airplane. Consider the limit case when there is no parachute and the load is free to slide down the rails only by action of gravity.

As presented in figure 3, for this study, the load's center of gravity position (X_{cgC}) is considered to coincide with the empty aircraft center of gravity location (X_{cg0}) just before the ejection command is issued.

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Then, the load starts sliding down the rails by action of gravity. Notice that the rails (dotted line in 3) have an inclination of σ with respect to the longitudinal axis of the airplane (the dashed line). The dynamics are then described by the classic problem of a mass sliding down an inclined plane with no friction.

Now, back to the dynamic modeling of the sliding load. While inside the aircraft, the load will accelerate according to 24:

$$\ddot{X}_{cgC} = g\sin(\sigma+\theta), \quad X_{cgC} < X_{cgC_{max}} \tag{24}$$

where σ and θ in equation 24 are defined as in figure 3. Note that σ is fixed.

Since the equations are being developed as a funcion of the total CG displacement (ΔX_{cg}) from its original position (X_{cg0}), one can use equations 24, 5 and 6 to find the dynamics for ΔX_{cg}. Substituting 6 for X_{cg} in 5 one finds:

$$\underbrace{\frac{m_A}{m_A+m_C}}_{\mu_A}(X_{cg0}-X_{cgC}) = \frac{-\mu_A}{(1-\mu_A)}\Delta X_{cg} \tag{25}$$

Taking the second time-derivative of 25 and replacing 24 for $\ddot{X}_{cgC}$ we have:

$$\Delta\ddot{X}_{cg} = (1-\mu_a)g\sin(\sigma+\theta), \quad \Delta X_{cgC} < \Delta X_{cgC_{max}} \tag{26}$$

We have thus all the required dynamics necessary to simulate the load extraction from the aircraft.

3. NONLINEAR-ADAPTIVE CONTROLLER DEVELOPMENT

A nonlinear adaptive control law for regulation of the pitch angle using the elevator as the only control input will be developed. It is further assumed the aircraft has the following sensors:

- A rate gyroscope for measuring the pitch angular rate q
- An integrating gyroscope for mearing the pitch angle θ
- Air sensors (pitot, static intake, thermometer) for measuring the dynamic pressure $\bar{q}$ and estimating the aircraft's velocity V
- An accelerometer at the aircraft's center of gravity and aligned with the aeroplane Z body axes.

The derivation of the controller follows Lyapunov's second method as in Singh and Steinberg (1996); Steinberg and Page (1998); Astrom and Wittenmark (2008). Consider the MRAC regulator architecture given by the diagram 4.

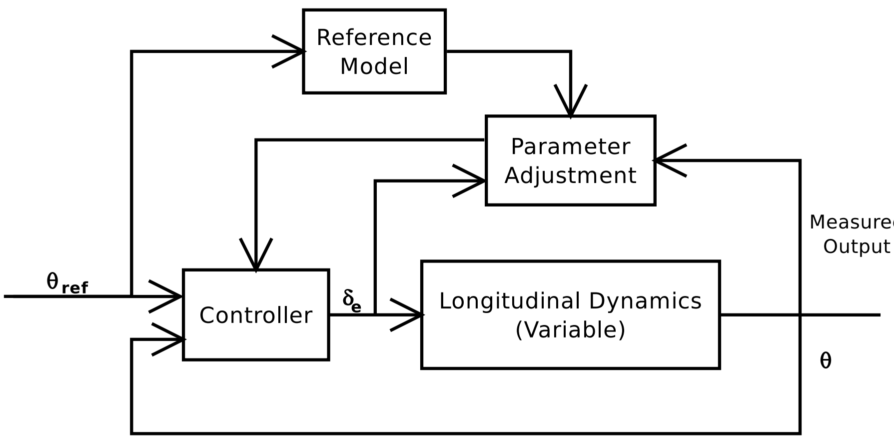

Figure 4: Block diagram for a model-reference adaptive controller (MRAC) for pitch control during load extraction.

The objective is to find a control law $\delta_p(q,\theta,\theta_{ref},\hat{A},\hat{b})$ dependent on the estimated parameters and adaptation schemes $\dot{\hat{A}}$ and $\dot{\hat{b}}$ that will stabilize the trajectory in the reference model's trajectory and adapt the parameters to their real values.

This can be acomplished by defining a Lyapunov candidate function dependent on the trajectory error and the parameter's estimation errors and defining the control and adaptation laws so that its derivative will be negative semi-definite.

First, though, the *underlying control problem* (as described in Astrom and Wittenmark (2008)) will be solved using feedback linearization on the fast dynamics. The obtained ideal control will be after compared with the resulting control dependent on the parameter's estimates.

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Feedback Linearization Law: Centainty Equivalence From equations 1d and 1a one finds for the aircrafts fast longitudinal dynamics:

$$\begin{aligned}
\dot{\theta} &= q \\
\ddot{\theta} = \dot{q} &= \frac{\mathcal{M} + z_{F/CG} F \cos(\alpha_f) - \dot{I_{yy}} q}{I_{yy}}
\end{aligned} \tag{27}$$

Expanding the aerodynamic moment in its components the second equation in 27 can be written as:

$$\begin{aligned}
\ddot{\theta} =& \frac{\bar{q} S \bar{c}}{I_{yy}} \left(C_{\mathcal{M}_0} + C_{\mathcal{M}_\alpha} \alpha + C_{\mathcal{M}_{\hat{\dot{\alpha}}}} \dot{\alpha} \frac{\bar{c}}{2V_0} + C_{\mathcal{M}_{\hat{q}}} q \frac{\bar{c}}{2V_0} + C_{\mathcal{M}_{\mathrm{d}_p}} \mathrm{d}_p \right) \\
&+ \frac{z_{F/CG} F \cos \alpha_f}{I_{yy}} - \frac{\dot{I}_{yy} q}{I_{yy}}
\end{aligned} \tag{28}$$

Now, as seen in section 2, the aerodynamic coefficients in (28) will vary with $C.G.$ position. We thus separate the state variables in one vector and the coefficients in another to get:

$$\ddot{\theta} = A_0 + \phi_{A1}^T A_1 + \phi_b^T b \delta_p \tag{29}$$

Where:

$$\begin{aligned}
A_0 =& \frac{z_{F/CG} F \cos \alpha_f}{I_{yy}} - \frac{\dot{I}_{yy} q}{I_{yy}} \\
A_1 =& \frac{\bar{q}_0 S \bar{c}}{I_{yy}} \begin{bmatrix} C_{\mathcal{M}_0} & C_{\mathcal{M}_\alpha} & C_{\mathcal{M}_{\hat{\dot{\alpha}}}} \frac{\bar{c}}{2V_0} & C_{\mathcal{M}_{\hat{q}}} \frac{\bar{c}}{2V_0} \end{bmatrix}^T \\
b =& \frac{\bar{q}_0 S \bar{c}}{I_{yy}} C_{\mathcal{M}_{\mathrm{d}_p}} \\
\phi_{A1} =& \frac{\bar{q}}{\bar{q}_0} [1 \quad \alpha \quad \dot{\alpha} \quad q]^T \\
\phi_b =& \frac{\bar{q}}{\bar{q}_0} \\
\bar{q}_0 =& \frac{1}{2} \rho_0 V_0^2
\end{aligned} \tag{30}$$

In equation 3, ρ_0 means the air density at sea level and V_0 the airspeed for which the aerodynamic coefficients where estimated.

Let us assume that it is desired to have a dynamics for the pitch angle described by:

$$\ddot{\theta}_m = -k_1 \dot{\theta}_m - k_2 (\theta_m - \theta_{ref}) \tag{31}$$

Where k_1 and k_2 make a stable dynamics the the reference model system.

With perfect knowlegde of the system's parameters and a perfect state measurement one finds by dynamic inversion that the control:

$$\mathrm{d}_p = \frac{1}{\phi_b^T b} \left(-A_0 - \phi_A^T A - k_1 \dot{\theta}_m - k_2 (\theta_m - \theta_{ref}) \right) \tag{32}$$

will make the fast dynamics follow the desired model. Since neither the parameters nor the state are known precisely the control law given by 32 has to be modified.

Adaptive Feedback Linearization Note first that one can group A_0 and A_1 by augumenting the regression vector ϕ_{A1} to get:

$$\ddot{\theta} = \phi_A^T A + \phi_b^T b \mathrm{d}_p \tag{33}$$

$$A = [A_0 \quad A_1]^T \tag{34}$$

$$\phi_A = [1 \quad \phi_{A1}]^T \tag{35}$$

Now consider we only have estimates of the vectors A and b given by $\hat{A}$ and $\hat{b}$ and that the errors in such estimates are given by:

$$\begin{aligned}
\tilde{A} =& A - \hat{A} \\
\tilde{b} =& b - \hat{b}
\end{aligned} \tag{36}$$

Now let us introduce the generalized error s given by:

$$s = \dot{\theta} - \dot{\theta_m} + K(\theta - \theta_m) = q - q_m + K(\theta - \theta_m) \tag{37}$$

Following works such as Singh and Steinberg (1996); Astrom and Wittenmark (2008), this work proposes the following Lyapunov candidate function V:

$$V = \frac{1}{2}s^2 + \frac{1}{2}\tilde{A}^T P \tilde{A} + \frac{1}{\gamma}\tilde{b}^2$$

we find that:

$$\frac{\mathrm{d}V}{\mathrm{d}t} = s\dot{s} + \tilde{A}^T P \dot{\tilde{A}} + \frac{2}{\gamma}\tilde{b}\dot{\tilde{b}} \tag{38}$$

A hypothesis is need to continue the developement of equation 38. From the definition of the parameters errors in equation 36 it follows that their time-derivatives will be a sum of the actual parameters time variation with the adapted parameters time variation. The hypothesis assumed is that the true parameters time variation is small enough so that we can consider the following approximation:

$$\begin{aligned} \dot{\tilde{A}} &= -\dot{\hat{A}} \\ \dot{\tilde{b}} &= -\dot{\hat{b}} \end{aligned} \tag{39}$$

This is the same as considering that the parameters are "slow varying" with respect to both the aircraft dynamic modes and most importantly the controller adaptation dynamics.

Now, the controller being developed is actually able to track a pitch setpoint, as it will follow any desired model dynamics. For simplicity, let us consider the regulation problem, where $q_m = 0$ and $\theta_m = \theta_{ref}$. In those conditions, derivating equation 37 in time gives:

$$\dot{s} = \dot{q} + Kq = \phi_A^T A + \phi_b^T b\mathrm{d}_p + Kq \tag{40}$$

Using 40, 36 and 39 in 38 one finds:

$$\frac{\mathrm{d}V}{\mathrm{d}t} = s\left(\phi_A^T \hat{A} + \phi_b \hat{b}\mathrm{d}_p + Kq + \phi_A^T \tilde{A} + \phi_b \tilde{b}\mathrm{d}_p\right) - \tilde{A}^T P \dot{\hat{A}} - \frac{2}{\gamma}\tilde{b}\dot{\hat{b}} \tag{41}$$

Choosing the control law:

$$\mathrm{d}_p = \frac{1}{\hat{b}\phi_b}\left(-Kq - \phi_A^T \hat{A} - c_1 s\right) \tag{42}$$

Where: $c_1 > 0$

And substituting in equation 41 we find that:

$$\frac{\mathrm{d}V}{\mathrm{d}t} = -c_1 s^2 + s\left(\phi_A^T \tilde{A} + \phi_b \tilde{b}\mathrm{d}_p\right) - \tilde{A}^T P \dot{\hat{A}} - \frac{2}{\gamma}\tilde{b}\dot{\hat{b}} \tag{43}$$

Now we choose the parameter's adaptations to cancel the unknown terms in equation :

$$\dot{\hat{A}} = sP^{-1}\phi_A \tag{44}$$

$$\dot{\hat{b}} = \frac{\gamma}{2}s\phi_b \mathrm{d}_p \tag{45}$$

and the derivative of the Lyapunov function becomes:

$$\frac{\mathrm{d}V}{\mathrm{d}t} = -c_1 s^2 \tag{46}$$

This function is negative as long as the measured trajectory is not equal the refence trajectory. In the case of regulating the pitch angle, this will happen as long as the measured pitch angle is not equal to the reference signal or if there is any pitch rate measured by the rate gyro. With the control law in 42 and adaptations 44 and 45 the traking error will thus always go to zero.

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Controller implementation Comparing equations 32 and 42, together with de definition of s in 37, it is found that the adaptive control law corresponds to the ideal control law, only by exchanging the real parameter's by the estimated ones.

Notice, however, that according to the definition of ϕ_{A1} in equations 3, it would be necessary to measure α and $\dot{\alpha}$ to apply the control law deduced in 42. As those measures are not normally available in standard aircraft, they shall be replaced by the considered measured variables described in the beggining of this section.

Keeping in mind the regulation problem of keeping the pitch for leveled flight, one finds that for such flight conditions:

$$\alpha \approx \theta \tag{47}$$

Moreover, considering ouput from the accelerometer placed in the aircraft's CG and aligned with its vertical axis (see Duke *et al.* (1988)), and the expression for $\dot{\alpha}$ as in 1c one finds that for small angular rates q it is reasonable to assume:

$$\dot{\alpha} \approx q + \frac{g}{V}(1 - n_z) \tag{48}$$

Where n_z is the normalized measure of the accelerometer. Finally, substituing equations 47 and 48 for the expression of ϕ_A one finds the implementation of the control law and parameter adaptations as:

$$\mathrm{d}_p = \frac{1}{\hat{b}\phi_b}\left(-Kq - \phi_{Am}^T \hat{A} - c_1 s\right) \tag{49}$$

$$\dot{\hat{A}} = sP^{-1}\phi_{Am} \tag{50}$$

$$\dot{\hat{b}} = \frac{2}{\gamma} s\phi_b \mathrm{d}_p \tag{51}$$

Where: $\phi_{Am} = \begin{bmatrix} 1 & \frac{\bar{q}}{\bar{q}_0} & \frac{\bar{q}}{\bar{q}_0}\theta & \frac{\bar{q}}{\bar{q}_0}\left(q + \frac{g}{V}(1-n_z)\right) & \frac{\bar{q}}{\bar{q}_0}q \end{bmatrix}$

$K, c_1, P,$ and γ are parameters chosen by the designer

Equations 49 to 51, together with definitions and 37 form the adaptive control laws for pitch tracking. Which shall be used for pitch regulation with the choice of $q_m = 0$ and $\theta_m = \theta_{ref}$, where θ_{ref} will the the initial trimmed pitch angle in the following simulations.

4. SIMULATION

The airplane chosen for this study was the Lockheed Martin C-5A Galaxy. A scaled two-sided view of the aircraft and the parameters for its dynamic model are given in figure **??**.

From the dimensions described the schematics, it is possible to infer the length of the path described by the load inside the aircraft. The relevant parameters for this case study are presented in table 1.

Mass of unloaded aircraft	m_A	$248416kg$
Mass of the load	m_C	$15000kg$
Length of the rails	$X_{cgC_{max}} - X_{cg0}$	$26.2m$
Maximum $C.G.$ displacement	$\Delta X_{cg_{max}}$	$1.49m$
Initial $C.G.$ position	X_{cg0}	$0.30\bar{c}$
$C.G.$ position at extraction	$X_{cg_{max}}$	$0.46\bar{c}$
Inclination of cargo ramp	σ	$5°$

Table 1: Parameters of load extraction for the C-5A Galaxy.

The aircraft's parameters used in this work belongs to the Lockheed Martin's C-5A Galaxy in "Power Approach Configuration" as presented in Heffley and Wayne (1972).

The simulations consisted in implementing the dynamic equations for longitudinal flight with varying center of gravity developed in section 2. The $C.G.$ dynamics was implemented as described in subsection 2.3 First the trim commands and states were found for a nominal flight condition as described in table 2.

During the simulation of the aircraft's flight, equation 26 can be used together with initial conditions $\Delta X_{cg}(TE^-) = \Delta X_{cg0}$ and $\Delta \dot{X}_{cg}(TE^-) = 0$ to calculate the current position of the CG at each time step after the extraction comand instant (TE).

Once the load is extracted ($X_{cgC} = X_{cgC_{max}}$), the simulation must restart from its current state but considering the coefficients, mass and inertia for the unloaded aircraft. This "restart" is accomplished in the software Matlab® by the use of the "Events" option of the `ode45` option set Inc. (2000-2012) in the following manner:

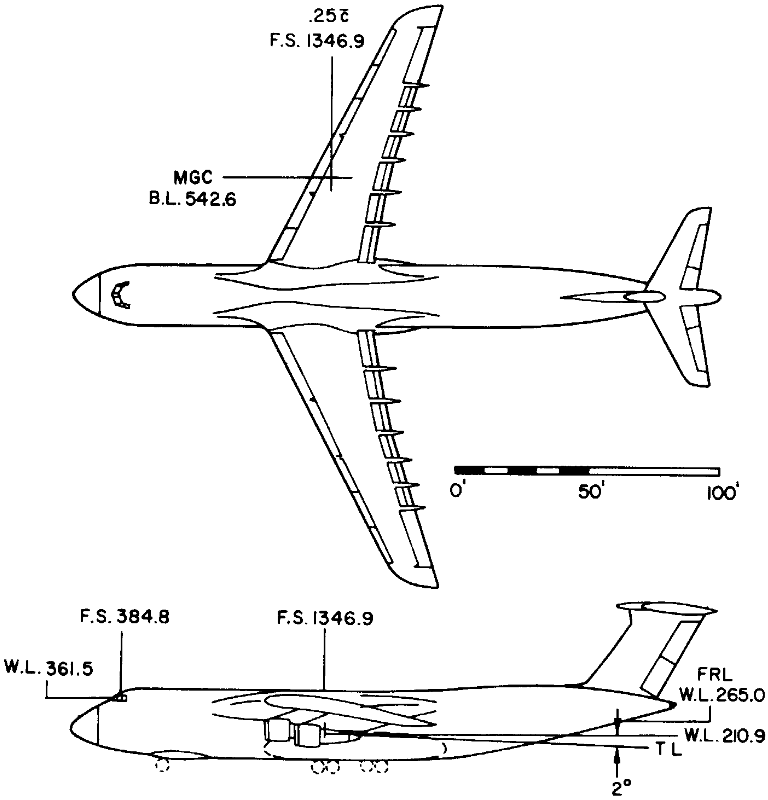

Figure 5: Top and side view of the C-5A. (Heffley and Wayne (1972))

Leveled Flight Conditions		Trimmed State and Commands	
H	500 m	$\alpha = \theta$	0.68°
V	75 m/s	d_p	0.06°
ΔX_{cg}	0 m	Throttle	39.8%

Table 2: Nominal flight conditions before load extraction

- The simulation starts with the load's $C.G.$ at the same position as the aircraft's $C.G.$
- At time TE, an extraction command is issued and the load starts sliding down the rails following the dynamics 24
- At each integration step, an event function is called to check the load's $C.G.$ position and compare it to its maximum limit $X_{cgC_{max}}$
- When this limit is reached, the integration is stoped. The current time and state are saved and a "load dropped flag" is set to True.
- If the current time is less then the final integration time, which was previously chosen, the integration is then restarted with initial conditions equal to the saved simulation state just before the drop.

5. RESULTS

Two cases were studied. One with fixed controls at the trimmed positions for later reference and one with the adaptive controller in action. Three gain sets were applied on the second case. Simulations were run starting from nominal flight conditions and "activating" the load dynamics at "$T = 5$s" (for the fixed controls) or "$T = 7s$" (with the adaptation scheme).

Fixed commands Figure 6 shows the simulation results on the states for the load extraction with fixed controls. The red dot in the graphics shows the instant when the load was extracted. The influence of the CG variation is seen clearly by the large fluctuations of angle of attack during and after the load movement.

The natural, fixed control variation of the parameters which will be adapted is shown in figure 7. Notice the sharp change in the parameters values at the moment of extraction.

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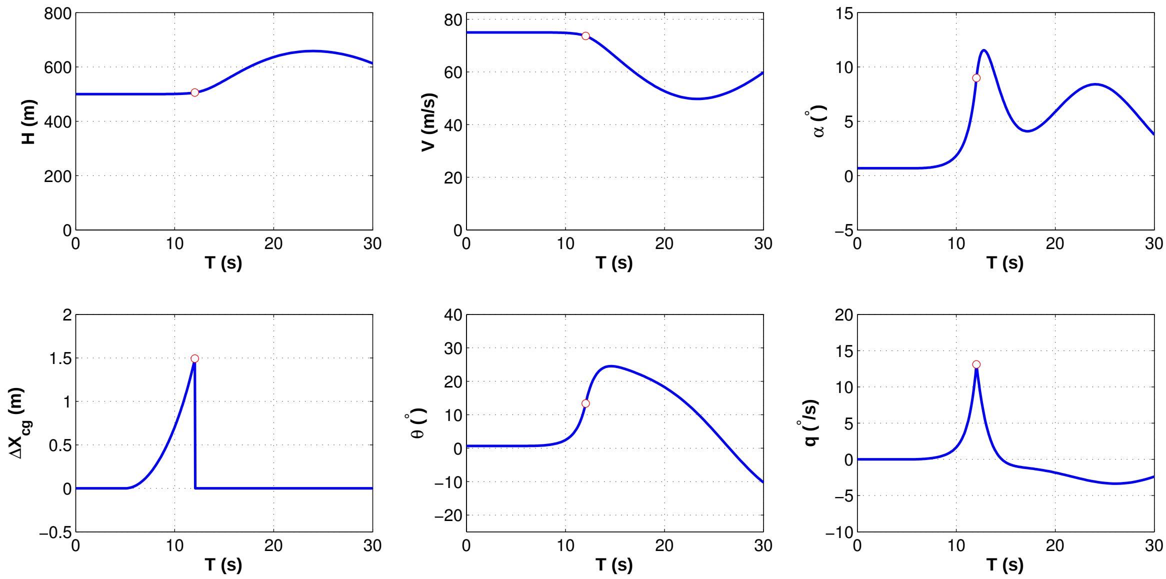

Figure 6: Load extraction simulation with fixed commands. Evolution of the states.

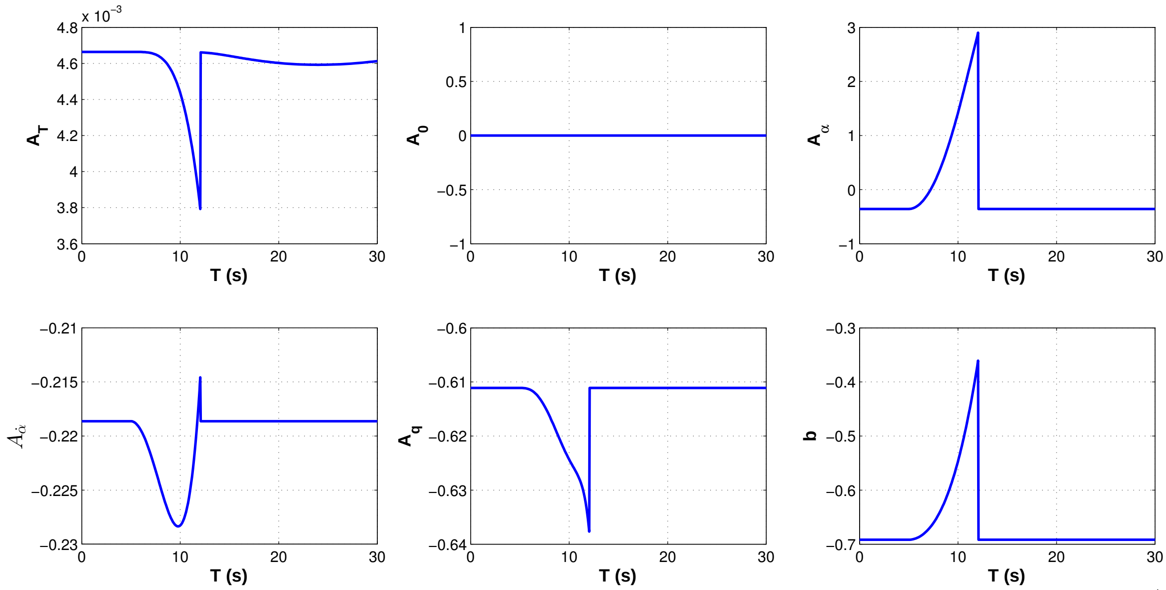

Figure 7: Load extraction simulation with fixed commands. Variation of the parameters in parameter vector A_1

Also notice that only the fast dynamics parameters (α, θ, and q) change noticeably due to CG movement. The extraction time is was $TE = 12.04s$ meaning it took approximately $7s$ for the load to travel from the CG to the rear of the aircraft.

Adaptive controller - nominal case Figures 8 to 10 show the simulation results for the adaptive controller. The controller parameters for this cenario are listed in table 3.

K	2
c_1	1
P	diag([1 1 1 1 1])
γ	1

Table 3: Controller parameters. Case 1.

It is seen the controller is effective in mantaining the pitch of the aircraft during movement of the load and after that to reestablish the pitch after the load drop.

In figure 9 it is noted that the control effort does not impose any saturation problems, although it is certain that a more realistic simulation would impose saturations in the elevator rate.

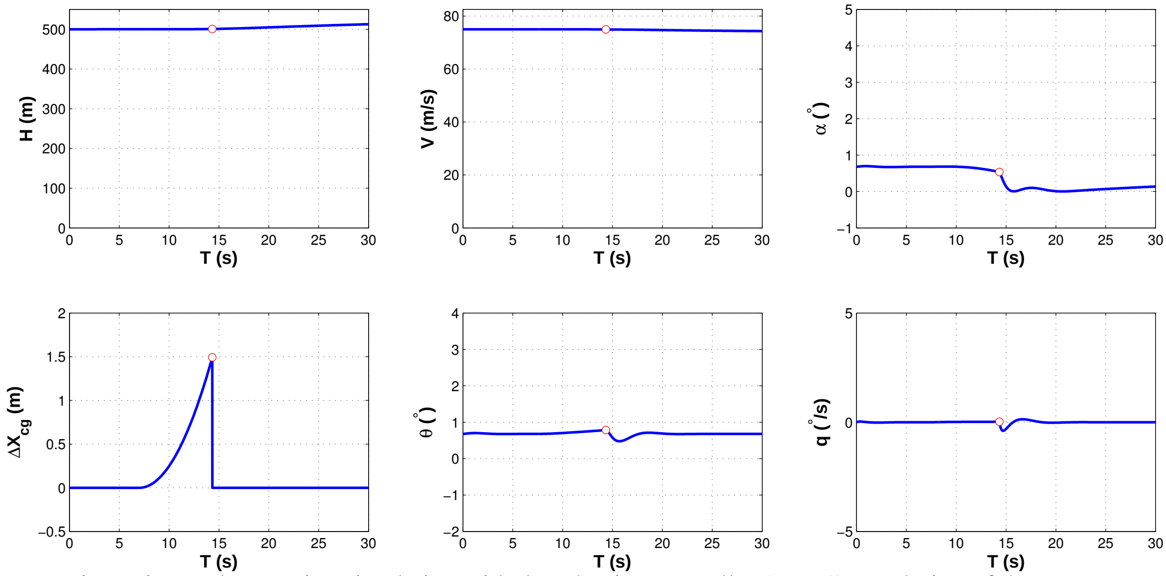

Figure 8: Load extraction simulation with the adaptive controller (case 1). Evolution of the states.

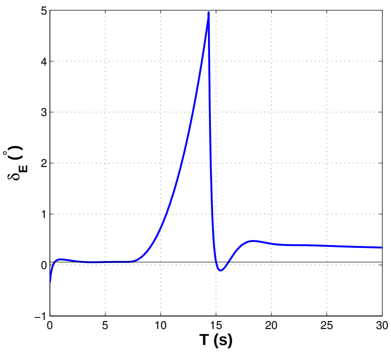

Figure 9: Load extraction simulation with the adaptive controller (case 1). Elevator Command.

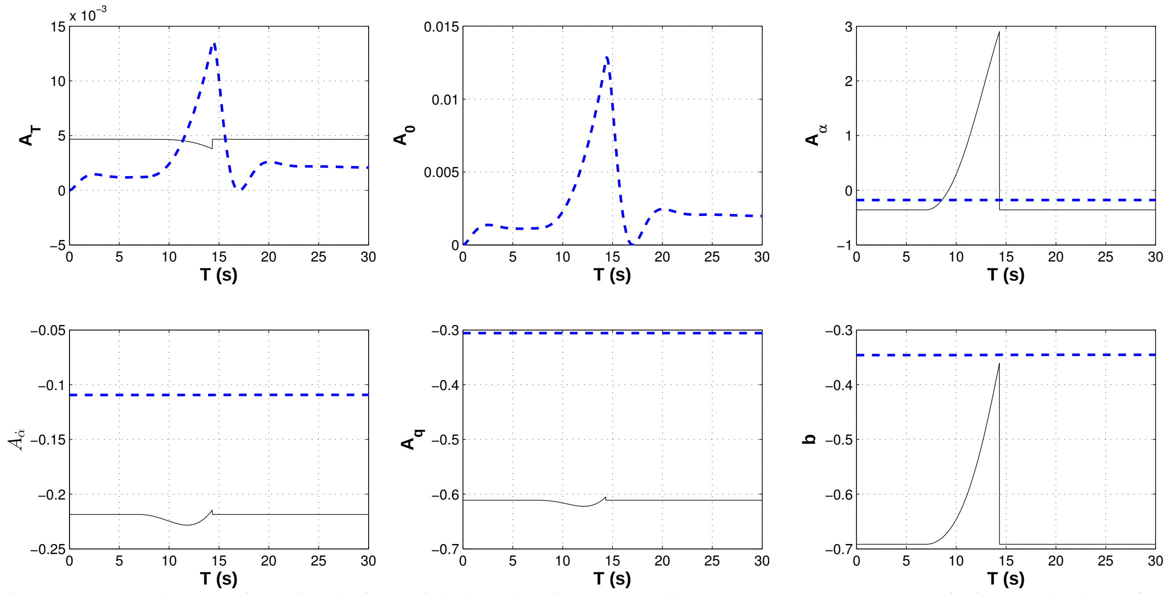

Figure 10: Load extraction simulation with the adaptive controller (case 1). Parameter variation and adaptation.

Yet perhaps the most surprising fact from the adaptation simulation is seen in figure 10. The results show a much more effective adaptation of the paramters A_T and A_0 when compared to the adaptations of the other parameters. This suggests that the gains in matrix P should be calibrated if one wants to favour the adaptation of all parameters.

Nevertheless, it can be considered that all the unmodelled dynamics were “absorbed” by the variation of those terms. It is worth studying what would happen if those terms adaptations were shut down and the adaptations of the other terms

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amplified. This is done in case 2.

6. CONCLUSIONS

This work has shown the feasibility of applying an adaptive controller to the problem of pitch regulation during a payload extraction manouver. The controller design is nonlinear and based on the feedback linearization theory. The adaptive architecture is a Model-Reference Adaptive System (MRAS or MRAC) and the resulting nonlinear-adaptive controller was developed by direct application of Lyapunov's second method.

Such a control strategy offer some advantages as the operation in more extreme deviations from the nominal flight condition - thanks to the underlying nonlinear controller - and savings on the modelling effort, since a simpler model can be used for the adpative case, as long as it takes into account all the main dynamic of the system.

There are also drawbacks to be pointed. The complexity of the controller is higher, as given by the augmented state which encompasses the parameter's estimates and the controller parameters such as the model reference and the controller gains. Proofs of stability for the stochastic (real) case are still somewhat limited. Finally, as discussed in Astrom and Wittenmark (2008), a robust linear controller may give better responses when fast parameter variation occurs.

Some sugestions for future work on the problem presented here are:

- The development of a least-square based, self-tuning controller for payload extraction , both in it's direct and inderct form, and a comparison of their performances with the MRAC studied in this work.
- Extending the application to the stochastic case, including sensor noise and other random disturbances on the system and using the maximum likelyhood adaptive self-tuning controller for a LAPES (Low-Altitude Payload Extraction System).
- Using the adaptive controller architectures to build a "auto-tunner" for simple UAV applications: Suppose an UAV that works on a wide enough flight envelope to request different controller gains to keep the performance paremeters under the various flight conditions. An adaptive controller could be used during the development phase in flight tests to find the appropriate system gains, reducing the number of test flights, the control team workload and the overall cost of the flight test campaign.

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8. RESPONSIBILITY NOTICE

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