THREE-DIMENSIONAL UNCERTAINTY EVALUATION APPLYING MONTE CARLO METHOD

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Abstract. *Coordinate Measuring Machines (CMMs) are fast, accurate, flexible, versatile and allow a reliable quality control. Nevertheless, their performance has been limited by several factors and consequently, the evaluation of measurement uncertainty is very difficult. This work aims to assess the uncertainty associated with measurements on a CMM using Monte Carlo simulation. During the simulation the software Oracle Crystal Ball and Excel were used. At the same time, a study was performed to determine the influence of the number of iterations in the quality of the simulation results. The work was carried out according to the following steps: i) study and analysis of the Monte Carlo method; ii) planning and carrying out measurements; iii) identification of the variables that influence in the determination of dimensional and geometry characteristics; iv) adoption of a simplified mathematical model to the measurement process; v) simulation and estimation of measurement uncertainty. It was concluded that to evaluate the measurement uncertainties in the CMM, under the conditions of this study, 300,000 iterations were sufficient to obtain reliable results. Furthermore, the number of iterations used during simulation remarkably influences the shape of the distribution and consequently its statistical parameters and the value of the measurement uncertainty.*

Keywords: measurement uncertainty, Monte Carlo method, CMMs

1. INTRODUCTION

The need to quantitatively indicate the quality of a measurement result led to the establishment of criteria to determine the value that represents the measurement uncertainty. Measurement uncertainty is defined as a non-negative parameter that characterizes the dispersion of the values attributed to a measurand based on the information used (BIPM *et al.*, 2008). This parameter can be considered an indicator of the performance of measuring instruments and the reliability of the measurement result. The evaluation and statement of the uncertainty contribute to the traceability of values and the suitability of measurements to the current technical standards.

If uncertainty is not declared, the measurement results cannot be compared, either among themselves or with reference values given in a specification or standard. It is therefore necessary to have a procedure readily implemented, easily understood and widely accepted to characterize the quality of a measurement result, that is, to evaluate and express its uncertainty (ISO TAG 4/WG 3, 2008).

The understanding of ISO TAG 4/WG 3 (2008) is not a simple task, its interpretation and correct application require specialized technical training. The methodology presented in this document can be applied only to those cases where it is possible to formulate a mathematical model to describe the output variable as a function of the input variables. The attainment of such a model is not always an easy task (Nielsen, 2006). The main cause of this is often in the complexity of the measurement system in question, the lack of knowledge about its operating principle, measurement errors and their sources. In this sense, Sommer and Siebert (2006) highlight the importance of identifying the influence quantities for the proper development of the measurement model, since the establishment of this mathematical model is essential for determining the values of the associated uncertainty.

Also it can be added that if the mathematical model, which describes the functional relationship between the input and output variables of a given measuring system is complex, as in measurement with Coordinate Measuring Machines (CMMs), the calculation of partial derivatives that represent the coefficients of sensibility, becomes extremely laborious. This can lead to the appearance of miscalculations and has been one of the main factors for not adopting the methodology proposed in ISO TAG 4/WG 3 (2008).

In the specific case of the CMMs, this can be justified due mainly two factors. Firstly the large number of variables that can contribute to the uncertainty of measurement (Wilhelm *et al.*, 2001) leading to complex mathematical models of measurement such as (Hocken *et al.*, 1977; Guye, 1978; Zhang *et al.*, 1985; Cardoza, 1995; Piratelli, 1997; Di

Giacomo, *et al.,* 1999; Hermann, 2007). Second and no less important, the machine versatility also contributes to the complexity of the model, since it allows the measurement of several metrological features of a workpiece (Balsamo *et al.,* 1999). These factors can increase the difficulty to evaluate measurement uncertainty in CMMs, as shown in (Vieira Sato, 2003; Miguel, *et al.,* 2003; Valdés, *et al.,* 2005 and Abackerli, *et al.,* 2010).

From the foregoing, it can be concluded that the application of the method of propagation of uncertainty becomes too limited to ensure reliable results in many real situations. Thus, in 2008, was published a document entitled "Evaluation of measurement data - Supplement 1 to the Guide to the expression of uncertainty in measurement - Propagation of distributions using a Monte Carlo method" (BIPM *et al.,* (2008b), in which the main objective is to increase the scope of ISO TAG 4/WG 3 (2008).

According to the BIPM *et al.,* (2008b), the method of evaluation of uncertainty proposed by ISO TAG 4/WG 3 (2008) presents some applicability limitations, such as: it is appropriate for explicit models with only one output variable. Models less frequent, as those with multiple outputs, complex variables or implicit functions are not detailed in the ISO TAG 4/WG 3. In addition, the mathematical model describing the measurement process must be linear and the output variable must have a normal distribution.

The document JCGM 101 (2008) is based on Monte Carlo simulation and is free of most of the limitations of ISO TAG 4/WG 3 (2008) and it does not contradict the recommendations presented in it (Cox and Harris, 2003).

The method discussed uses computer simulations where the probability distributions of the input variable are propagated (Balsamo *et al.*, 1999 and Schwenke *et al.,* 2000). The input information for the application of the Monte Carlo method are almost the same as those used in the approach of the ISO TAG 4/WG 3 (2008), in other words, it starts from a mathematical problem describing the measurand from the input variable and their statistical information. The fundamental difference is that the statistical input used in the ISO TAG 4/WG 3 (2008) are basically the mean, standard deviation, and the degrees of freedom of each distribution. In the Monte Carlo method the very Probability Density Functions (PDF) of input distribution are used, providing more complete information on these distributions. With these informations it is possible to simulate the distribution of values of the output quantity where it is possible to obtain the associated uncertainty.

The simulation is performed simply in different software such as Crystal Ball and Mathematica. However some authors prefer to use Excel to this purpose, including: Piratelli and Di Giacomo (2003). Note that Excel 2007 makes it possible to work with up to 1,000,000 lines, overcoming a limitation of Excel 2003, whose limit was 65,000 lines. This create new application possibilities. Therefore, this work aims to apply the Monte Carlo method to evaluate the uncertainty associated with measurements taken on a Coordinate Measuring Machine using the software Oracle Crystal Ball. At the same time a study was performed to determine the influence of the number of iterations in the quality of simulation results, as well as, evaluate the potential of Excel 2007 to perform the calculation of measurement uncertainty through Monte Carlo simulation.

2. METHODOLOGY

Several dimensional and geometric features of a part (diameter and circularity deviation (Hole 1), distance between two points (center of Hole 2 and 3), diameter (Sphere 4) and flatness deviation (Surface 5)) (Fig. 1a) were measured using a manual Coordinate Measuring Machine, type moving bridge (Fig. 1b).

This machine, model BR443, was manufactured by Mitutoyo and it is manually operated. The machine resolution is 0.001 mm and the work volume is 400 mm (Axis X), 400 mm (Axis Y) and 300 mm (Axis Z). The management of the measurement task is carried out by a computer program, the Geopak-Win. According to the calibration certificate, the CMM used in the measurements has linear expanded uncertainties of $(1.2 + L/1300)$ μm and $k = 2.2$ for the X axis; of $(1.2 + L/1300)$ μm for the Y axis; and of $(1.0 + L/1300)$ for the Z axis. The probing error is 2.9 μm.

To obtain the diameter and the circularity deviation of the Hole 1, 13 points were determined in a cross section of the feature evaluated. In turn, the flatness deviation of Surface 5 was determined taking 13 points. In order to find out the distance between the centers of Hole 2 and Hole 3, four points were determined at each one. Finally, to obtain the diameter of the Sphere 4, 5 points were determined. In all cases, five measurement cycles were performed.

The measurement process was conducted with controlled temperature of 20±1°C. To achieve the thermal equilibrium, the machine, the workpiece and the measuring devices were exposed during to the reference temperature for 12 hours. The temperature was monitored using a thermo-hygrometer with nominal range of -20°C to 60°C and resolution of 0.1°C. Afterwards, the Monte Carlo method was applied to the calculation of the associated uncertainty. Therefore, it was used the program Oracle Crystal Ball V.11.1.1.1.00 and the Excel 2007.

2.1 Identification of the influence variables

The following influence variables were considered: variability of the measurand values; CMM resolution; standard uncertainty associated with the CMM calibration, in the directions considered during the measurements; error of the CMM probing system; temperature variation during measurement and distancing of the temperature in relation to 20 °C.
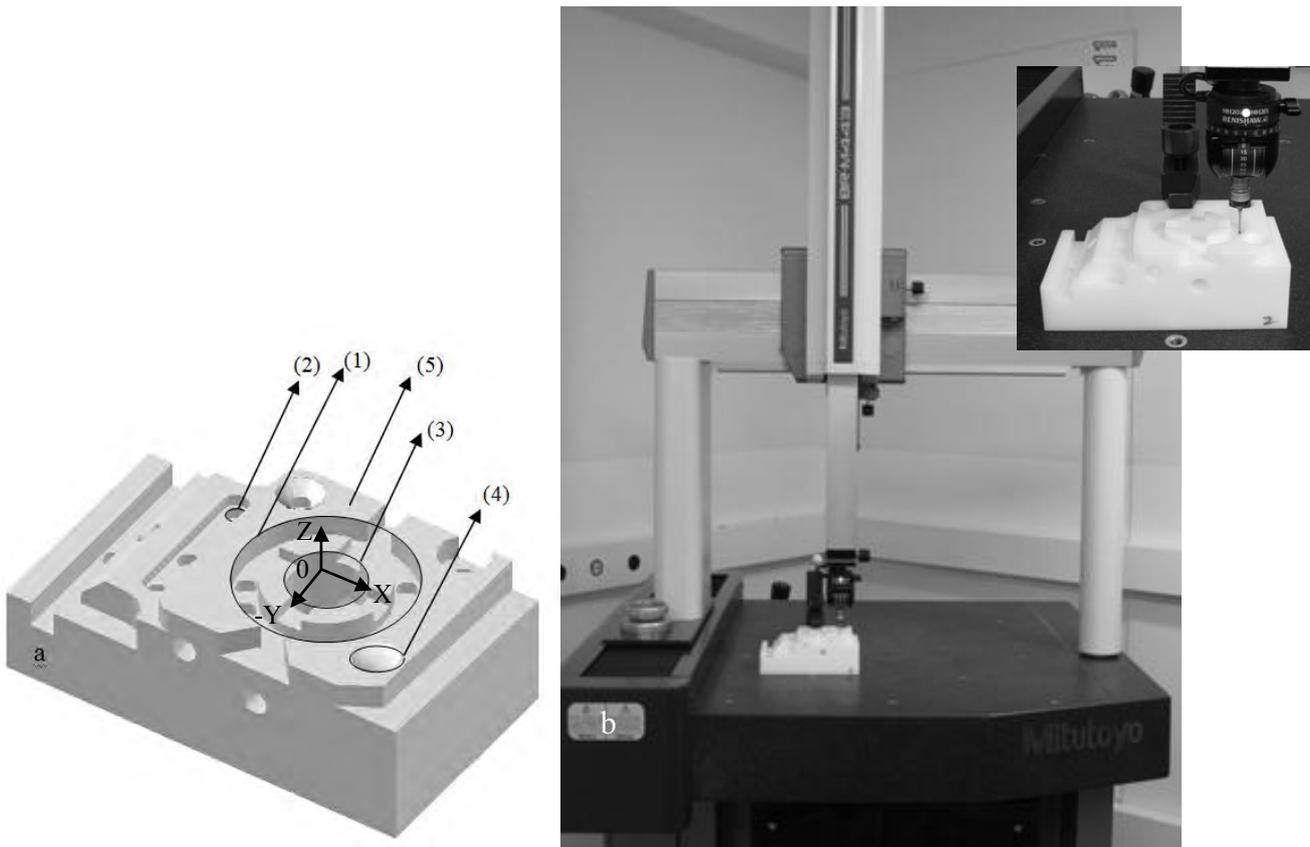
Figure 1 - Measured part (a) and Coordinate Measurement Machine used in the experiments (b)

2.2 The mathematical model

Then it was proposed a mathematical model for each evaluated characteristic, dimensional or geometric:

a) Distance between two points

The distance (D) between two points, $P_1 = (X_1, Y_1, Z_1)$ and $P_2 = (X_2, Y_2, Z_2)$ is given by Eq. (1).

$$D(P_1, P_2) = \sqrt{(X_2 - X_1)^2 + (Y_2 - Y_1)^2 + (Z_2 - Z_1)^2} \tag{1}$$

It was necessary to include in the mathematical model the variability associated with the values of the distance between two points considering the N cycles, according to Eq. (2).

$$D(P_1, P_2) = \Delta s(D) + \sqrt{(X_2 - X_1)^2 + (Y_2 - Y_1)^2 + (Z_2 - Z_1)^2} = \Delta s(D) + \Delta P_1 + \Delta P_2 \tag{2}$$

The Equation (2) takes into account the variability of the distance between two points $(\Delta s(D))$, considering the N measurement cycles and the corrections associated with the coordinates of two point, P_1 and P_2.

b) Flatness Deviation

During the measurement in the used CMM, the value of the flatness deviation is determined from a plan set to the n points probed ($n > 3$) and whose coordinates (X, Y and Z) were collected using the program Geopak-Win. Therefore it is used the Least Squares Method, as shown in Eq. (3) (Vieira Sato, 2003).

$$\pi = \beta_0 + \beta_1 \cdot X + \beta_2 \cdot Y \tag{3}$$

Where π represents the plan set to the n points probed and β_0, β_1 and β_2 are the coefficients of least squares.

Hereafter the program calculates the distance Di for each point $P_i = (X_i, Y_i, Z_i)$ to the plan π adjusted. Since the distance between the points $P_{max} = (X_{max}, Y_{max}, Z_{max})$ and $P_{min} = (X_{min}, Y_{min}, Z_{min})$ to the plan π is given by Eqs (4 and 5) (Vieira Sato, 2003):

ISSN 2176-5480

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$$D_{max}\left(P_{max},\pi\right)=\frac{\left|\beta_0+\beta_1\cdot X_{max}+\beta_2\cdot Y_{max}-Z_{max}\right|}{\sqrt{\beta_1^2+\beta_2^2+1}} \tag{4}$$

$$D_{min}\left(P_{min},\pi\right)=\frac{\left|\beta_0+\beta_1\cdot X_{min}+\beta_2\cdot Y_{min}-Z_{min}\right|}{\sqrt{\beta_1^2+\beta_2^2+1}} \tag{5}$$

Finally, the flatness deviation is determined by Eq. (6).

$$D_{Pl}=P_{max}-P_{min} \tag{6}$$

Where D_{Pl} represents the flatness deviation and P_{max} and P_{min} the furthermost points higher and inferiorly to the plane π. Adding to the mathematical model the variability associated with the flatness deviation values considering the N cycles is obtained using Eq. (7).

$$D_{Pl}=\Delta s(D_{Pl})+\Delta P_{max}+\Delta P_{min} \tag{7}$$

Where $\Delta s(D_{Pl})$ is the variability of the flatness deviations considering the N measurement cycles and the corrections associated with the furthest point superiorly in relation to the plane (P_{max}) and the furthest point inferiorly from the plane (P_{min}).

c) Hole Diameter
The Equation (8) allows to calculate the radius of the circle, considering n points ($n\geq3$).

$$r=\sqrt{\left(X-X_c\right)^2+\left(Y-Y_c\right)^2} \tag{8}$$

where $\left(X_c,Y_c\right)$ represents the coordinates of the center of the circle and r is its radius.
Adding on Equation (8) the variability associated to the values of the diameter considering the N cycles performed $(\Delta s(d))$, the Eq. (9) is obtained.

$$d=\Delta s(d)+2\sqrt{\left(X-X_c\right)^2+\left(Y-Y_c\right)^2} \tag{9}$$

d) Circularity Deviation
To calculate the circularity deviation it is necessary to determine the distance (Di) for each point P_i to the centre of the circle (P_C), identifying which points are most ($Pmax$) and less ($Pmin$) away from the point P_C.
The circularity deviation is given by the difference between the two distances, Eq. (10).

$$D_{CIR}=P_{max}-P_{min} \tag{10}$$

Adding on Equation (10) the variability associated to the values of the circularity deviation to the N cycles performed $(\Delta s(D_{CIR}))$, Eq. (11) is obtained.

$$D_{CIR}=\Delta s(D_{CIR})+P_{max}-P_{min} \tag{11}$$

e) Sphere Diameter
The sphere diameter or similarly the sphere radius can be determined from the Eq. (12).

$$R=\sqrt{X_c^2+Y_c^2+Z_c^2-\rho} \tag{12}$$

Adding on Equation (12) the variability associated to the values of the sphere diameter to the N cycles performed $(\Delta s(D_E))$, Eq. (13) is obtained.

$$D_E=\Delta s(D_E)+2\cdot\sqrt{X_c^2+Y_c^2+Z_c^2-\rho} \tag{13}$$

f) Coordinates X, Y and Z of the probed points

To determine the uncertainty associated with each coordinate (*X, Y* and *Z*) of any point *P,* probed during the measurement is used the following mathematical model.

$$C_i = \Delta R_{MM} + \Delta IC_{Ci} + \Delta E_A + c_i.\Delta T.(\alpha_{Pe} + \alpha_{MM}) + c_i.\delta T.(\alpha_{Pe} + \alpha_{MM}) \tag{14}$$

Where: c_i is the coordinate X, Y or Z considered; ΔR_{MM} is the correction associated to the CMM resolution; ΔIC_{Ci} is the correction due to the standard uncertainty associated with the CMM calibration; ΔE_A is the correction due to the errors of the CMM probing system; δT is the corrections due to temperature variation during measurement; ΔT is the corrections due to the distancing of the temperature in relation to 20°C; α_{MM} is the coefficient of linear thermal expansion of the CMM scales (glass) and α_{Pe} is the coefficient of linear thermal expansion of the material of the workpiece (bakelite).

2.3 Identification of the PDFs for the input variables

An important step for the evaluation of uncertainty using Monte Carlo method is the identification of the PDFs to be assigned to each of the input variables present in the mathematical model of the measurement. Being that, in this work, to the variables variability of the measurand, distancing of the temperature in relation to 20°C and standard uncertainty associated with the CMM calibration, a normal distribution was adopted. To the variables, CMM resolution, thermal expansion coefficients and temperature variation was adopted the rectangular distribution and for the probing error the triangular distribution. From the available information about each input variable, the parameters that characterize each of the distributions were determined.

2.4 Calculation of the standard uncertainty associated with each input variable

The values of the measurand are characterized by a normal distribution and can be calculated by Eq. (15).

$$u(me) = \sqrt{\frac{s^2}{n}} \tag{15}$$

Where me is the measurand considered.

The uncertainty associated to the resolution can be considered as an evaluation of type B, because it cannot be estimated statistically. In this case, a rectangular distribution and the uncertainty is given by Eq. (16) was assumed.

$$u(\Delta R_{MM}) = \frac{CMM\ Resolution}{\sqrt{3}} \tag{16}$$

For linear measurements the standard uncertainties associated to the calibration of the CMM are expressed by Eq. (17) to (19).

$$IC_{MM(x)} = \frac{U_{Px}}{k} \tag{17}$$

$$IC_{MM(y)} = \frac{U_{Py}}{k} \tag{18}$$

$$IC_{MM(z)} = \frac{U_{Pz}}{k} \tag{19}$$

If the measurement is performed in the plane *XY, XZ* or *YZ*, then, the Eq. (20) to (22) must be used, respectively.

$$IC_{MM(x,y)} = \sqrt{IC_{MM(x)}^2 + IC_{MM(y)}^2} \tag{20}$$

$$IC_{MM(x,z)} = \sqrt{IC_{MM(x)}{}^2 + IC_{MM(z)}{}^2} \tag{21}$$

$$IC_{MM(y,z)} = \sqrt{IC_{MM(y)}{}^2 + IC_{MM(z)}{}^2} \tag{22}$$

While for volumetric measurements, the uncertainty associated with the calibration of the machine considers the declared uncertainty for the three coordinate axes, as in Eq. (23).

$$IC_{MM(x,y,z)} = \sqrt{IC_{MM(x)}{}^2 + IC_{MM(y)}{}^2 + IC_{MM(z)}{}^2} \tag{23}$$

The standard uncertainty associated with the error of the CMM probing system is given by Eq. (24), considering a triangular probability distribution.

$$u(\Delta E_A) = \frac{E_A}{\sqrt{6}} \tag{24}$$

The standard uncertainty associated with the difference between coefficients of thermal expansion of the scale and workpiece is given by Eq. (25).

$$u(\Delta\alpha) = \frac{0.01(\alpha_P - \alpha_E)}{\sqrt{3}} \tag{25}$$

Where α_E is the coefficient of thermal expansion of the scale (glass) and α_P is the coefficient of thermal expansion of the workpiece (bakelite).

For the distancing of the temperature in relation to 20 °C (ΔT_{20}), the standard uncertainty is determined using Eq. (26).

$$u(\Delta T_{20}) = \sqrt{\left(\frac{\Delta T}{\sqrt{3}}\right)^2 + \left(\frac{\Delta R_T}{2\cdot\sqrt{3}}\right)^2 + \left(\frac{\Delta I_T}{k_T}\right)^2} \tag{26}$$

where ΔT is the difference between the room temperature and 20 °C; ΔR_T is the correction in relation to the thermometer resolution and ΔI_T is the uncertainty associated with the thermometer indication system.

The uncertainty due to temperature variation during measurement is given by Eq. (27).

$$u(\Delta\delta T) = \sqrt{\left(\frac{Var(T)}{\sqrt{3}}\right)^2 + \left(\frac{\Delta R_T}{2\cdot\sqrt{3}}\right)^2 + \left(\frac{\Delta I_T}{k_T}\right)^2} \tag{27}$$

2.5 Determination of the number of iterations M

Having the necessary data to the simulation, pre-trials aiming to determine the number of iterations for each case. For this, it was used the program Oracle Crystal Ball V.11.1.1.1.00. The value of M was varied assuming the following values: 10,000, 50,000, 100,000, 300,000, 500,000 and 1,000,000. Completed the simulation, the vectors for each input variable were substituted into the corresponding equation and the vector of values of the output variable were obtained. Then, the expanded uncertainty was determined for a range of 95.45 %. As stopping criteria were used the following aspects: obtaining a distribution with normal shape for the output variable and asymmetry coefficient value close to zero.

2.6 Determination of the standard uncertainty

From the PDF of the measurand it is determined the value of uncertainty associated with the output variable. Therefore, the following steps should be followed.

a) Discrete representation of the function distribution for the output variable: The discrete representation of the PDF for the output variable *Y* can be obtained according to the following procedure:

- Classify the M values of Y provided by the Monte Carlo method in ascending order. Denote the classified values of the model by *y(r), r = 1, ..., M*;
- If necessary, do small numerical perturbations for any replicated value so that the overall result of *y(r), r = 1, ..., M*, form a strictly increasing sequence;
- Take the PDF of the output variable as the set *y(r), r = 1, ..., M*.

The function *y(r)*, when in a form of histogram and with appropriate width classes, form a distribution of frequencies that, when normalized to have unit area, provides an approximation of the PDF for *Y*. This histogram can be useful helping in the comprehension of the nature of the PDF, as in the extension of its asymmetry.

b) Estimative of the output variable and the associated standard uncertainty

In this step the arithmetic mean and the standard deviation of the output variable are calculated. The mean is considered as an estimate *y* from *Y* and the standard deviation represents the standard uncertainty *u(y)* associated to *y*.

c) Range of coverage for the output variable

If the final distribution of all calculated values of the measurand is normal and the value of symmetry (skewness) is close to zero, from the concept of a standard normal distribution and for a desired coverage probability, it is possible then to define the upper and lower limits of the PDF of the measurand values, having the values of mean and standard deviation, usually the chosen coverage probability is 95.45%, which expanded uncertainty is defined by Eq. (28).

$$U\left(p = 95,45\%; k = 2\right) = \left(L_s - L_i\right)/2 \qquad (28)$$

where L_s is the upper limit of the interval and L_i is the lower one.

The range of coverage for *Y* can be determined from the discrete representation of the PDF. Then the value of *q* must be determined using Eq. (29).

$$q = pM \qquad (29)$$

where p represents the probability and *M* the number of iterations.

Equation (29) is valid only when *q* is an integer. Otherwise, *q* is given by Eq. (30).

$$q = pM + 1/2 \qquad (30)$$

Thus, the interval defined by *[y$_{initial}$, y$_{final}$]* has a range of 100%.

2.7 Uncertainty evaluation using Excel

Finally proceeded the evaluation of the uncertainty using the Excel 2007. The results were compared with those found previously, using the program Oracle Crystal Ball V.11.1.1.1.00. This comparison considered among other aspects, the number of iterations required for the simulation, simulation time, shape of the distribution of the output variable, degree of asymmetry of the distribution and the value of the expanded uncertainty.

3. RESULTS AND DISCUSSION

The results obtained during the measurement, from which was evaluated the uncertainty using the Monte Carlo method, can be seen in Table 1.

Table 1. Results of measurement.

Characteristic	L1 (mm)	L2 (mm)	L3 (mm)	L4 (mm)	L5 (mm)	Mean (mm)	Standard Deviation (mm)
Hole Diameter	67.915	67.911	67.914	67.916	67.916	67.914	0.002
Circularity Deviation	0.078	0.085	0.089	0.078	0.089	0.084	0.006
Flatness Deviation	0.032	0.030	0.036	0.030	0.032	0.032	0.003
Distance between two points	28.944	28.953	28.950	28.941	28.949	28.947	0.005
Sphere Diameter	20.072	20.160	20.186	20.185	20.113	20.143	0.050

From Table 1 it follows that the values of standard deviation found for the first four characteristics measured are small. However, the standard deviation associated with the measurement of the diameter of the sphere is high, assuming the value of 0.050 mm. This can be explained, firstly, because the measurement strategy was not adequate, specifically, the number of points probed. Secondly, because the measurement of the sphere is more complex when compared to the other measured characteristics, once the probe tip tends to slip when in contact with the surface is evaluated.

Table 2 presents the values of expanded uncertainty associated with the measurement of the diameter of the sphere and obtained for different values of iterations. It is also presented the statistical parameters that describe the shape of the distribution of the output variable.

Table 2. Parameters of the output variable distribution for the simulations (sphere diameter).

Number of iterations	Expanded Uncertainty (mm)	Asymmetry Coefficient (Skewness)	Flattening Coefficient (Kurtosis)
10,000	0.0442	-0.0703	3.06
50,000	0.0444	-0.0119	3.04
100,000	0.0444	-0.0044	3.01
300,000	0.0446	-0.0032	3.00
500,000	0.0446	-0.0036	3.00
1,000,000	0.0446	-0.0024	3.00

In Table 2 was observed that the number of iterations considered during the Monte Carlo simulation influences the value of the expanded uncertainty associated with the measurement insignificantly. For $M \geq 300{,}000$ the expanded uncertainty did no change. Note in Table 2 that for a coverage probability of 95.45 % the value of uncertainty shows a slight increase of 0.4 µm when the number of iterations increases from 10,000 to 1,000,000. It was observed, furthermore, that the values of the asymmetry coefficient decrease significantly with the increasing of the number of iterations, indicating that for lower values of M the shape of the output variable distribution deviates from the normal distribution, as shown in Figure 2. It was observed that for 10,000 iterations the output variable distribution has a flattening coefficient or kurtosis higher than 3, indicating that it is slightly more tuned than the standard normal distribution. With the increase of the number of iterations, the kurtosis tends to 3.00 and from 300,000 it undergoes no change.

Figure 2 shows the graphs that illustrate the output variable distribution considering different values of M during the simulations performed to determine the measurement uncertainty of the diameter of the sphere. The results for the other measured characteristics are not presented here because they are similar.

The influence of the number of iterations in the format of the output distribution is clearly observed in the histograms shown in Figure 2. For 10,000 and 50,000 iterations the shapes of the distributions differ greatly from the normal and, therefore are not adequate for the evaluation of the measurement uncertainty. From 300,000 iterations it is no longer possible to observe significant changes in the distribution shapes, being this number of iterations sufficient to estimate the uncertainty associated with the measurement.

A similar analysis was performed for all measured characteristics. In all cases the distribution of the calculated values of the measurand is normal and the value of the symmetry coefficient (skewness) is close to zero.

Table 3 briefly describes the values of expanded uncertainty obtained for a range of 95.45 % of all measured characteristics, as well as the number of iterations from which this value is stable and reliable.

As expected, the expanded uncertainty values associated with the measurement of the first four characteristics are small and vary between 0.004 mm and 0.006 mm. While the expanded uncertainty associated with the measurement of the diameter of the sphere is 0.045 mm, it is noteworthy that the largest parcel of contribution to the final uncertainty, in this case, was the variability associated with the values of the measurand. Then it was performed the Monte Carlo simulation using Excel 2007. The graphs that illustrate the distribution of the output variable of all characteristics are shown in Figure 3. The number of iterations performed in each is shown in Table 3.
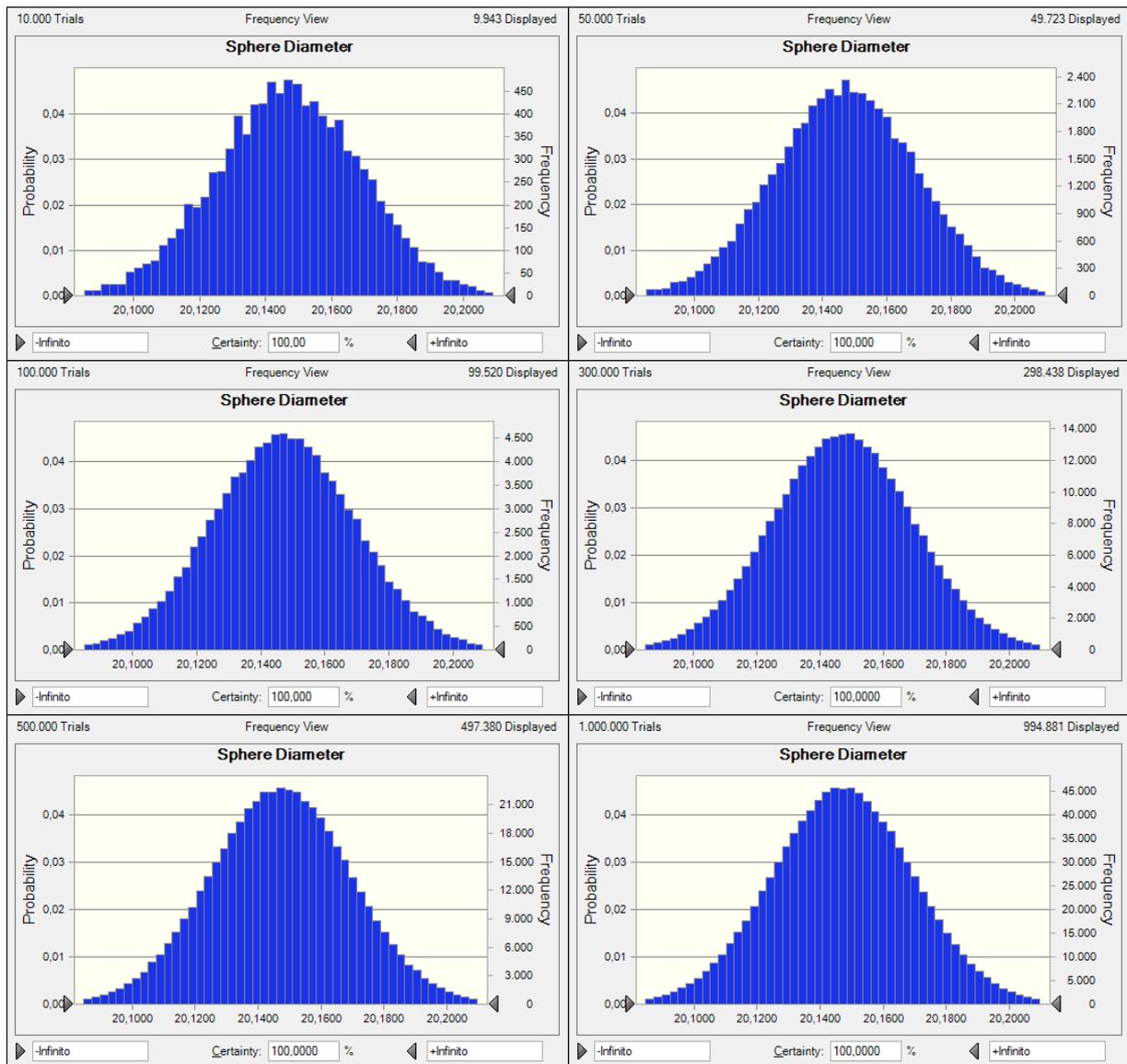
Figure 2 – Histograms of the output variable for each simulation.

Table 3. Number of iterations and measurement uncertainty.

Characteristic	Number of iterations	Asymmetry coefficient	Expanded Uncertainty (mm)
Hole Diameter	100,000	-0.0035	0.004
Circularity Deviation	200,000	-0.0034	0.006
Flatness Deviation	200,000	-0.0019	0.004
Distance between two points	300,000	0.0049	0.006
Sphere Diameter	300,000	-0.0032	0.045

Figure 3 shows that in all cases the shape of the output distribution can be considered normal. The asymmetry coefficient is close to zero and assumes values of 0.0113; -0.0095; -0.0039; 0.0002 and 0.0026, respectively. The most time consuming step, during the simulation using Excel, was the one relative to the determination of the number of elements present in each class (frequency) to build the histogram. In this work conditions, considering 300,000 iterations were consumed on average 47 seconds. It is noteworthy that the simulation was performed using a HP 420 notebook.

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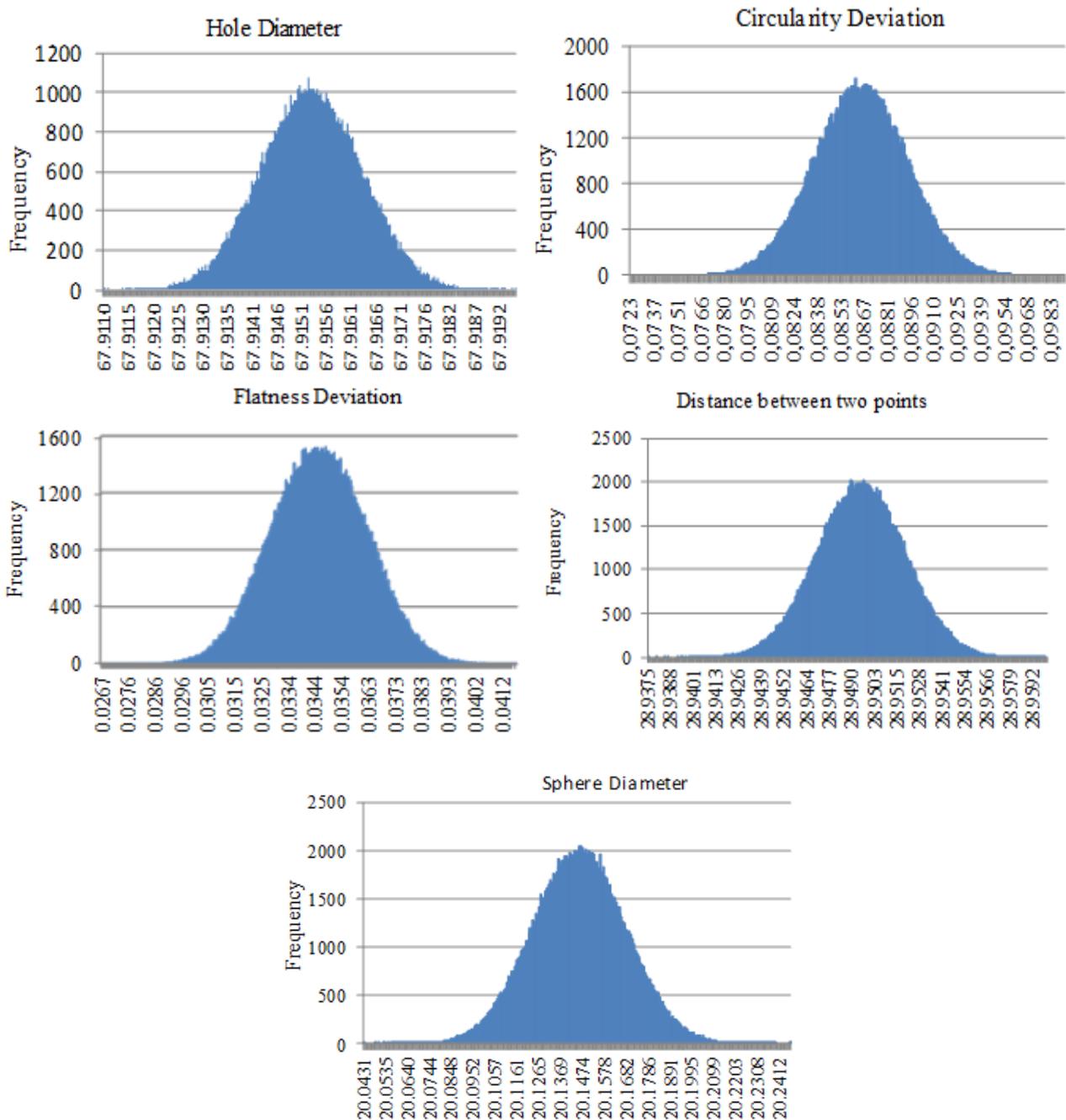Three-dimensional uncertainty evaluation applying Monte Carlo method

Figure 3 – Histograms of the output variable for all characteristics, using Excel.

The expanded uncertainty values associated with the measurement, obtained using Excel are shown in Figure 4. For comparison purposes are shown also the uncertainty values obtained when using the program Oracle Crystal Ball V.11.1.1.1.00.
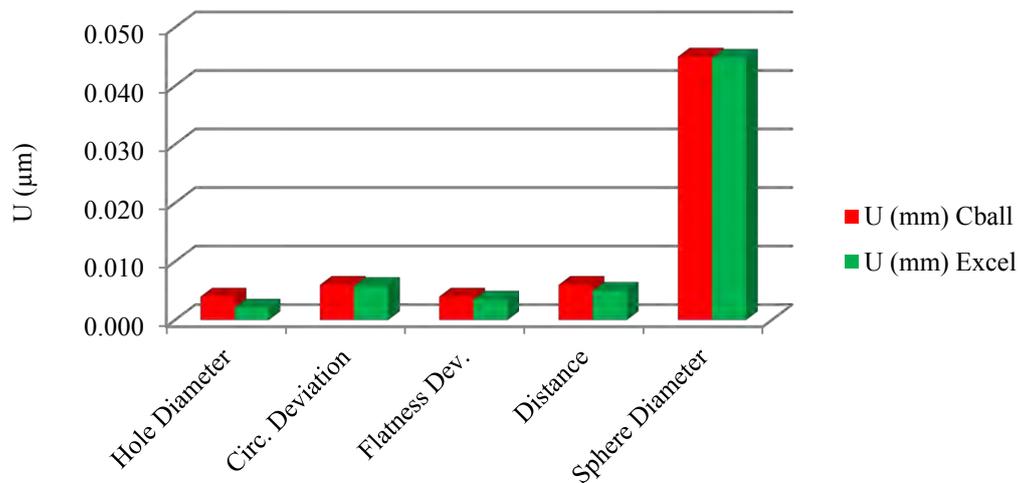
Figure 4 – Expanded uncertainty (95.45%).

It is observed in Figure 6 that the expanded uncertainty values obtained using the Excel application are equal to or slightly smaller than those found by the program Oracle Crystal Ball V.11.1.1.1.00 for all geometries evaluated. The largest difference was found for the hole diameter being of 0.001 mm.

4. CONCLUSION

To evaluate the uncertainty associated with measurements in a CMM, in this work conditions, 300,000 iterations were enough to obtain reliable results. The time spent in the simulation was approximately 20 seconds with the program Oracle Crystal Ball V.11.1.1.1.00.

Microsoft Excel 2007 was found to be appropriate to perform the Monte Carlo simulation in this work conditions, providing expanded uncertainty values similar to those obtained when using the program Oracle Crystal Ball V.11.1.1.1.00. The most time consuming step using Excel was the determination of the number of elements present in each class (frequency), that was found to be on average 47 seconds.

Simulations considering more than 300,000 iterations don't bring any further benefit. It only increases the time spent on the simulation and the computational effort.

5. ACKNOWLEDGEMENTS

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