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NUMERICAL MODELING OF ACOUSTIC WAVES IN 2D-FREQUENCY DOMAINS

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Abstract. *Acoustic wave modeling has a wide range of applications such as the manufacturing of flow meters, biometric sensors, detectors for flaws in metal structures and the mapping of oil reservoirs. Due to the complexity of real media analysis and the discovery of deeper reservoirs, the search for more efficient computational models is becoming more necessary. This paper aims to present a 2D-frequency domain numerical scheme for mapping geological structures. The simulation of acoustic waves is done via a finite difference method (2nd order in space) with two different absorbing conditions (PML, ABC). The characteristics of each simulation for the chosen artificial boundary condition are discussed. Results are generated and discussed for homogeneous and heterogeneous domains. Optimizations are proposed for future analysis.*

Keywords: *Acoustic waves, finite difference method, absorbing boundary condition, seismic modeling*

1. INTRODUCTION

Thanks to the development of processing computers ever faster allied numerical modeling techniques more sophisticated it is possible to do more efficient modeling of acoustic waves in geophysical and its consequent applicability to more subtle problems, for example, a more detailed mapping of oilfields. Many studies have attempted to develop improvements in the algorithms using traditional techniques of computational numerical models as finite differences, finite elements and numerical integrations.

To simulate the propagation of seismic waves, finite difference is chosen since this methodology makes possible a complete response wave field. The finite difference method replaces the partial derivatives of equations by finite difference expressions, which transforms continuous equations on a discrete grid of equally spaced points (nodes) on the physical domain where the parameters of these equations are now defined locally (Pratt, 1990).

One of the problems commonly found in computationally modeling problems with infinite domains is the rise of seismic reflections from the artificial limitation of the computational domain (Sommerfeld, 1949). To simulate the propagation of waves in these areas is necessary to define absorbing boundary conditions to truncate the computational domain in order to minimize reflections artificially created. From the 70s to the present day, boundary conditions techniques have been studied in order to avoid the side effects raised by inappropriate radiation conditions inherent of the computational domain limitation. Clayton and Engquist (1977) introduced the Absorbing Boundary Condition (ABC) technique, where a unidirectional wave is applied to the boundary region. However, this model was efficient only for waves with angles of normal incidence or almost normal. Cerjan, *et al.* (1985) proposed the concept of a buffer zone, where the propagated waves have its amplitude gradually reduced over the absorbing layer. Berenger (1994) and Collino and Tsogka (2001) developed a Perfect Matched Layer (PML) method, an absorption method with an independent angle of incidence. This work pretends to compare the performance of ABC and PML traditional techniques in 2-D simulations using finite difference in frequency domain (2^{nd} order in space) in homogeneous and heterogeneous hypothetical domains.

2. FORMULATIONS

The propagation of acoustic waves in the frequency domain to 2D domains is governed by the scalar Helmholtz equation:

$$- k^2 p = \frac{\partial^2 p}{\partial x^2} + \frac{\partial^2 p}{\partial y^2} + S \quad (1)$$

where $p(x,y)$ is the acoustic pressure for a given wavenumber k, where k is the ratio of the angular frequency ω $(=2\pi f)$ and the propagation velocity c of the wave in the medium and S is the source of the pressure field in this case given by the second derivate of Gaussian distribution as defined by Cunha (1997):

$$S = \frac{2\, f^2}{\pi^2\, f_c^3}\, exp\left(\frac{-f^2}{\pi\, f_c^2}\right) \quad (2)$$

where f is the frequency selected, fc $(= f_{CUT}/(3\sqrt{\pi}))$ is a parameter related to the cutoff frequency f_{CUT} (=60 Hz in all work).

For the absorbing boundary conditions used in this study, the first to be applied is the ABC method employed by Clayton and Engquist (1977):

$$\frac{\partial p}{\partial n} - jkp = 0 \quad (3)$$

where n is the normal direction to the contour whose derivative must be replaced by its corresponding derivative in the perpendicular direction to the edge and j is considered a complex number.

The second technique is to use the Berenger's PML method (1994) described by Eq. (4):

$$- k^2 p = \frac{1}{\alpha}\, \nabla^2 p + S \quad (4)$$

where α is the absorption coefficient given by Hustedt, *et al*. (2004):

$$\alpha = 1 + j\, \frac{c_{PML}}{\omega}\, cos\left(\frac{\pi}{2}\, \frac{x(i)}{L}\right) \quad (5)$$

where c_{PML} is a constant chosen such that spurious contour reflections are minimal, $x(i)$ is the point on the PML layer and L is the total length of the absorption layer.

3. NUMERICAL MODELING

The Equation (1) is solved numerically by replacing the partial derivatives by 2nd order central finite difference expression,

$$- k^2\, p_{m,n} = \frac{p_{m-1,n} - 2p_{m,n} + p_{m+1,n}}{\Delta x^2} + \frac{p_{m,n-1} - 2p_{m,n} + p_{m,n+1}}{\Delta y^2} + S_{m,n} \quad (6)$$

where the indices m and n represent the references of discrete points. A 2D rectangular mesh with constant spacing in horizontal (Δx) and vertical (Δy) directions with positive y oriented down was used.

For the application of the ABC boundary condition, Equation (3) is solved using 1st order finite progressive difference,

$$\frac{-p_{m,n} + \overline{p_{m,n}}}{\Delta h} - jkp_{m,n} = 0 \quad (7)$$

where $\overline{p_{m,n}}$ is the pressure value measured in the normal direction at the node adjacent to node evaluated $p_{m,n}$ and Δh is the spacing between nodes in the same direction.

For the application of the PML boundary condition, Eq. (4) is solved in the same way that Eq. (1) by 2nd order finite central difference:

$$- k^2\, p_{m,n} = \frac{1}{\alpha}\frac{p_{m-1,n} - 2p_{m,n} + p_{m+1,n}}{\Delta x^2} + \frac{1}{\alpha}\frac{p_{m,n-1} - 2p_{m,n} + p_{m,n+1}}{\Delta y^2} + S_{m,n} \quad (8)$$

To solve the problem, Eq. (6) can be written in terms of a matrix differential operator, $\boldsymbol{M}$, that includes all variations of velocities and coefficients of the pressure field $\boldsymbol{P}$ and the source term $\boldsymbol{S}$,

$$\boldsymbol{M\, P} = \boldsymbol{S} \quad (9)$$

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where the matrix ***M*** is sparse, nonsymmetrical and complex. To solve the Eq. (9), was used the Gaussian's elimination with partial pivoting method, as standard used by the software. The entire matrix ***M*** is described by Ajo-Franklin (2005).

In all cases analyzed (ABC and PML), the dimensions are 600 m for both x and y directions with a 5 m equally spacing nodes, resulting in a 120 squared mesh points. The absorbing boundary has 10 points, resulting in 50 m. The location of the source is arbitrated at $(x,y) = (90,90)$ as show the Fig. 1. The hashed blue line is the cut section that will serve to compare the ABC and PML seismograms more ahead. For the PML boundary condition, $c_{PML} = 1000$. All simulations were processed in a processor i5-2500K 3.30GHz CPU and with 16GB of RAM using MATLAB R2011a 64-bit.

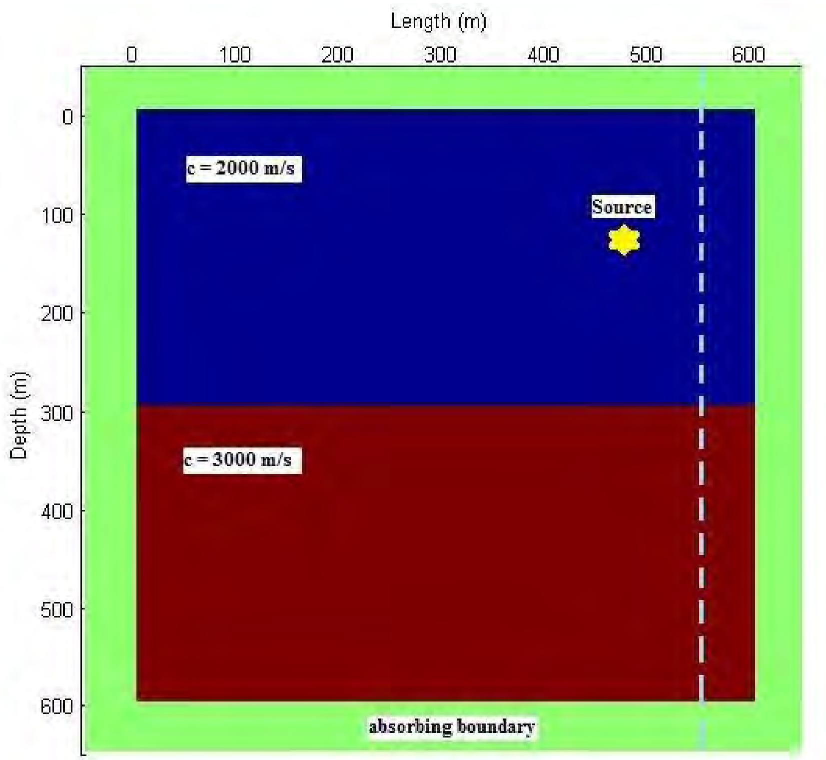

Figure 1. Two layer model with PML absorbing scheme.

4. RESULTS

Figure 2 shows the distribution of the amplitudes of a homogeneous medium with the wave propagation velocity c equal to 2000 m/s and for a selected frequency of 30 Hz using the boundary condition ABC (a), PML (b) and a side view comparing the two methods (c):

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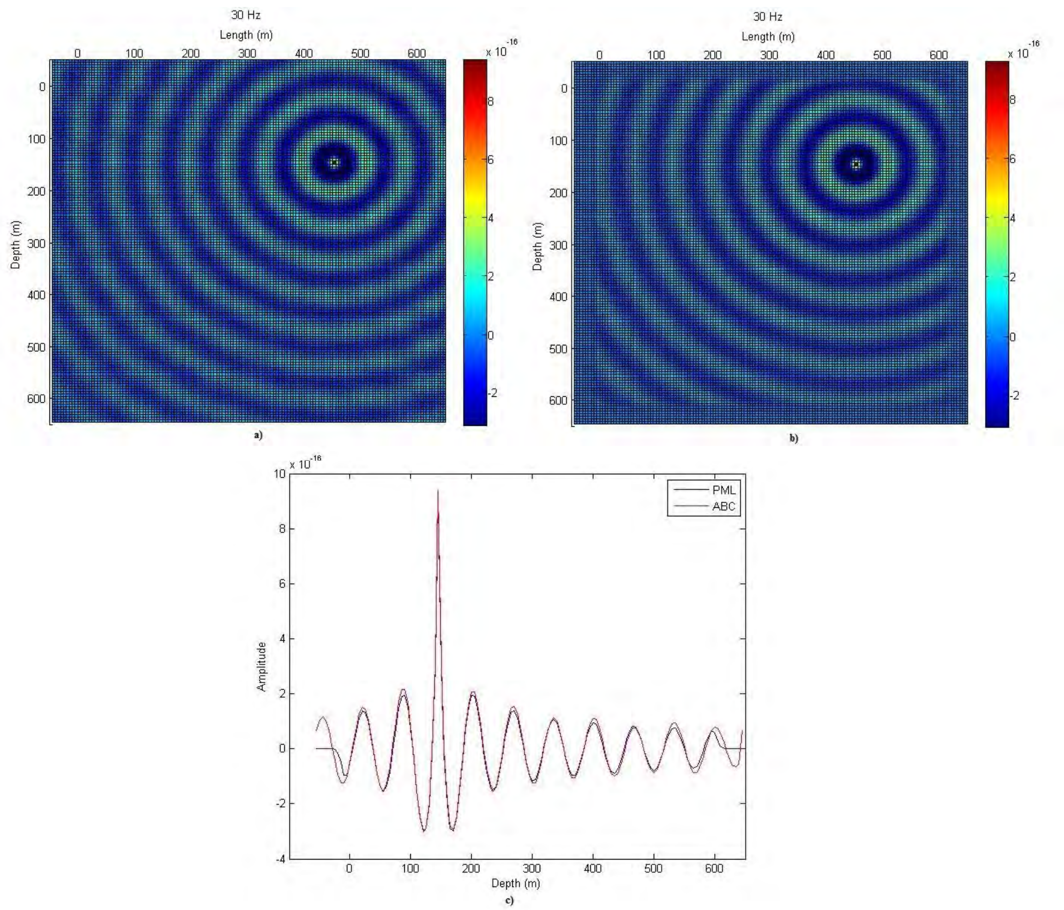

Figure 2. Propagation of waves in the frequency domain: (a) ABC, (b) PML and (c) comparison between the two conditions in a cut section along the depth passing through the source position.

Figure 3 shows the distribution of the amplitudes of the two media with wave propagation velocity *c* equal to 2000 m/s in upper medium and equal to 3000 m/s in lower medium, with a selected frequency of 30 Hz using the boundary condition ABC (a), PML (b) and a cut section comparing the two methods (c).

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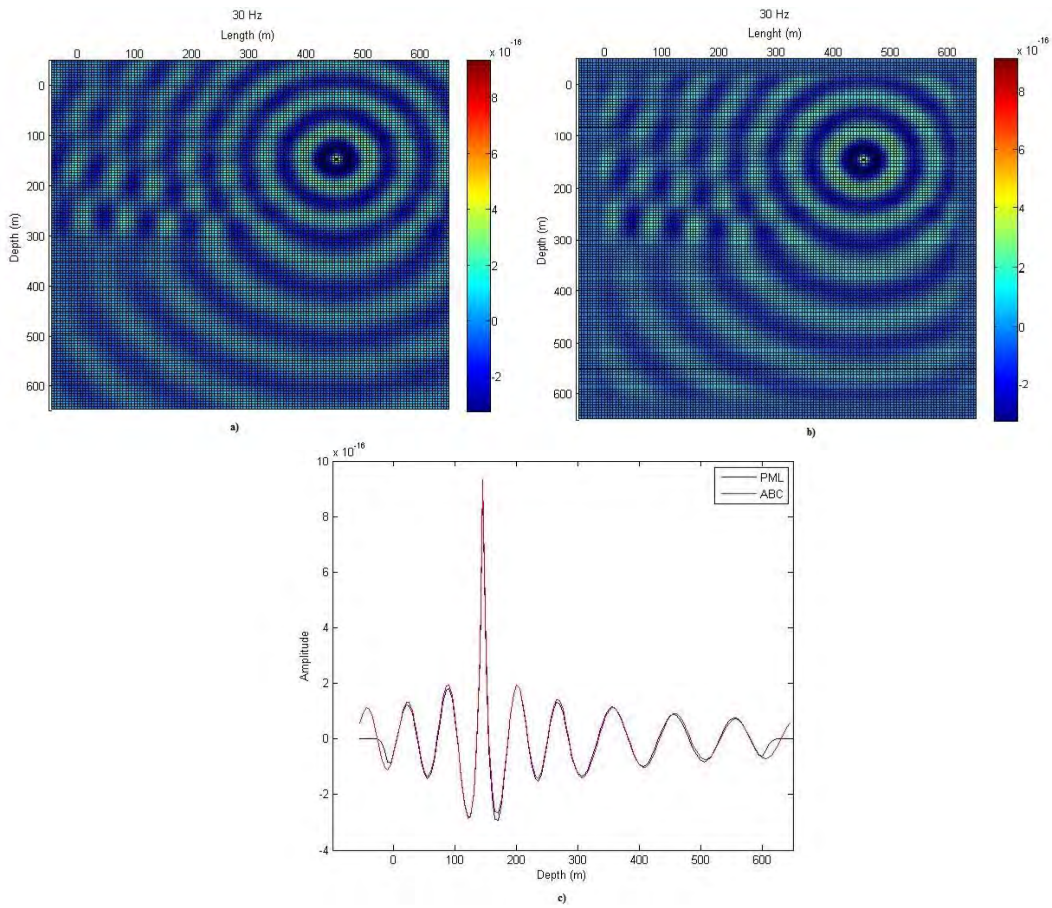

Figure 3. Propagation of waves in the frequency domain in heterogeneous medium: (a) ABC, (b) PML and (c) comparison between the two conditions in a cut section along the depth passing through the source position.

Figure 4 illustrates the time domain seismograms for ABC (a), PML (b), a comparison between these two conditions in a cut section chosen to best see the differences (*x=550m*) and a zoom in section showed by the black rectangle (d).

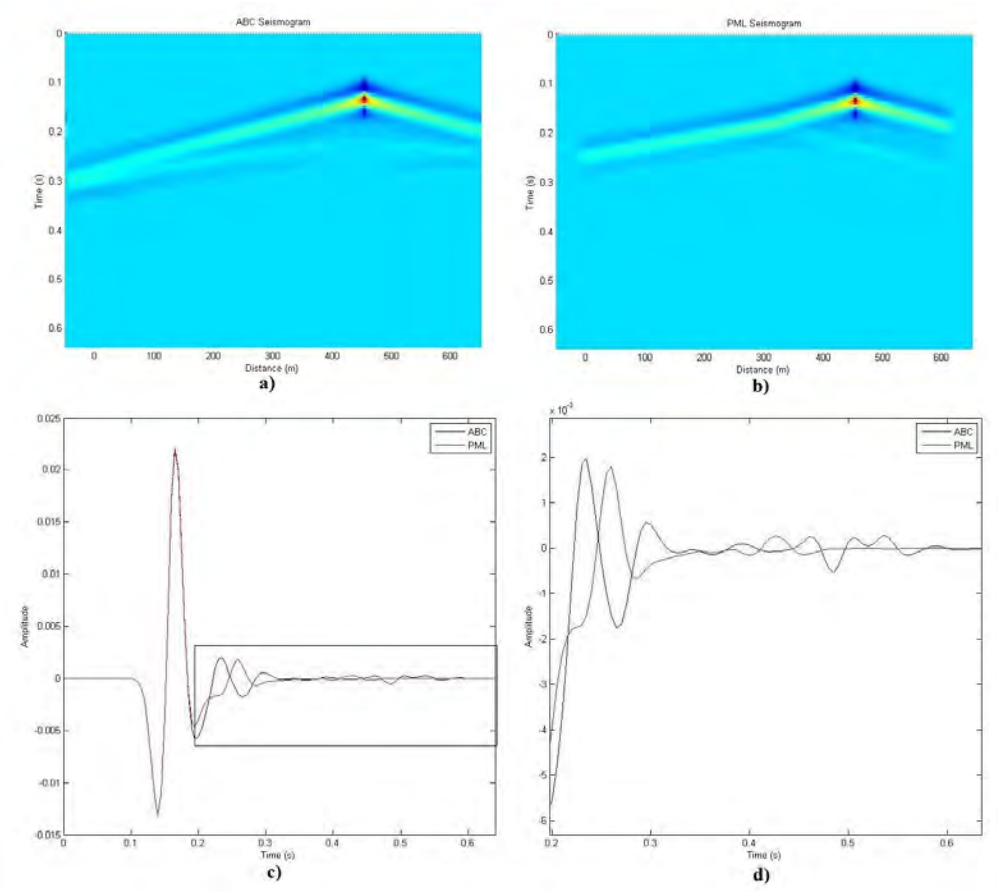

Figure 4. Time domains seismograms in heterogeneous medium: (a) ABC, (b) PML, (c) comparison between the two conditions in a cut section and (d) a zoom in section.

5. CONCLUSIONS

The method of numerical modeling by 2D finite differences was successfully implemented for ABC and PML boundary conditions for homogeneous and heterogeneous media. Small differences are noted between these methods while PML provides better absorption for the c_{PML} chosen. Further tests are needed to improve the absorption by adjusting the c_{PML} constant in PML technique.

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