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DEVELOPMENT OF AN ACCURATE NUMERICAL METHODOLOGY FOR PREDICTING THE FIRST THREE MODES OF VIBRATION OF AN ELECTRIC MOTOR FIXED ON A RIGID BASE

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Abstract. *This paper proposes a numerical methodology for the prediction of the first three modes of vibration of an electric motor fixed on a rigid base. A deep literature review supported the production of four ad hoc prototypes that aided the development of the proposed approach. Tests carried out with the prototypes led to the procurement of the modal parameters used to calibrate the numerical models, as well as the Frequency Response Function (FRF) curves used to validate the numerical solution. The validated model allowed structural changes to be then promoted on the prototypes, in order to make them more robust to variations in manufacturing and assembling processes. The mentioned adjustments and structural changes were accomplished by means of a process of structural optimization using Genetic Algorithm. The solution was developed based on the commercial finite element code ANSYS. The practical results obtained in this study show that a numerical model for modal analysis of an electric motor fixed on a rigid base with errors less than 3% for the first three modes of vibration can be achieved, allowing positive structural changes to be performed in the machine design that result in the minimization of manufacturing reworks associated with the dynamic behavior of the studied motor.*

Keywords: *Modal analys, Numerical methodology, Finite elements*

1. INTRODUCTION

According to Tustin (2005), impact and vibration often accelerates the failure of industrial machinery and equipment. Consequently, minimize or control these effects may delay a premature failure. The observation of this statement is verified in the industry through increasingly stringent specifications from normative organizations of rotating electrical machines, and even by adopting criteria by large companies more severe than those predicted in standards. The obtaining of vibration levels in electric motors which meet the criteria set in standards and / or special criteria, military applications, for example, it becomes more difficult to achieve as the motor's power increases. The power's increase is associated with increased size of the electric machine, which inevitably affects the ratio between stiffness per mass and hence its natural frequencies and associated mode shapes. Moreover, the excitation source is maintained, increasing the probability to match with some natural frequency. In such cases it is essential to full understanding of the physical behavior associated with the problem and its susceptibility to intrinsic variations in the manufacturing process. There remains the question whether these changes will cause undesirable and potentially harmful / destructive effects to the motor. Based on the above it is essential for a manufacturer of electric motors the ability to predict the dynamic behavior of their product, taking into account boundary conditions and robustness of the project, still in the conceptual phase.

1.1 Critical aspects of an electric motor with respect to vibration

The assembly of an electric motor, with the exception of the stator, is relatively simple, but two motors theoretically equal hardly present the same values of natural frequencies for their respective modes of vibration. This occurs because the interfaces between parts, for example, increase the complexity of the problem by inserting a number of variables in the manufacturing process. The following analyzes the main peculiarities, from the standpoint of the manufacturer and of some authors that should be considered in the study of natural frequencies in an electric motor.

Stator - Due to its orthotropic characteristics (Gonçalves, 2012; Roivainen, 2009; Wang, 1998; Gieras et alii, 2006; Garvey et alii, 2004; Long et alii, 1998 and Delves, 1964) and the way that the coils are mounted in their slots, the stator is undoubtedly a complicating factor in the study of natural frequencies of an electric motor. According Kukula (2007) the interface between the stack slots and coils winding of the stator is a strongly non-linear contact with the stiffness strongly dependent on an insulating film which is inserted between the slots and the coils. On the other hand, the interference between the stator and frame, besides being a contact nonlinear produces a tensioning of the assembly.

Gieras et al (2006) calculates this effect from a pre-static analysis, known as preloading, to then calculate the natural frequencies. This means that the stiffness of the frame can change and, consequently, their natural frequencies.

Rotor - With the objective of simplify the numerical modal analysis of a complete motor, Kukula (2007) considered only the effects of inertia of the rotor and its components, neglecting the effects of damping and stiffness. This consideration with relation to the rotor, suggests that its influence on the physical system represented by the motor stator and end covers is limited to the addition of mass, without significant effects of stiffness and damping. This argument is based on the assembly observation of the rotor and of the bearings at the end covers. In the standard process the bearings are installed at the ends of the rotor. These bearings are the only connection between the rotor, the end covers and the rest of the assembly. Consequently, much of the mass of the rotor will be supported between the two end covers.

End Covers and Frame - The effects of the end covers or fittings between the frame and the end covers, is an issue with scarce bibliography. A pioneering work addressing this issue was developed by Cai et alli (1999) using reluctance motors. His goal was to verify the behavior of the values of the natural frequencies of the motor before and after installation of the end covers, and try to reproduce this effect in a numerical model. Despite the simplicity of the idea, the results presented by the author are interesting. Experimentally there was an increase of 25% in the natural frequencies associated with modes of second order. This behavior was expected, since the end covers are mounted with a certain interference portion and introduce greater stiffness compared to the addition of mass. The obvious conclusion indicated that the effects of fittings between end covers and frame should be considered in the physical prototype.

2. METHODOLOGY

The experiments presented in this work were performed in order to provide data to assist in evaluation, adjustment and validation of the numerical methodology. The method used was experimental modal analysis, where the modal parameters were extracted from four prototypes and parts that compose them, and extensometry to measure the field deformations during the assembly process. The object of study was an electric motor size IEC 225S/M, two poles (60 Hz/3600 rpm), 440 V, 60 hp, 320 kg and without connection box. Figure 1 shows the appearance of a complete prototype.

Figure 1 – Prototype studied.

Initially, each component was tested in the "free" condition. This condition precludes influences inherent in the manufacturing process, such as interference fitting, fixing screws (torque) or the test basis. Thus the initial setting of the numerical model was simplified and focused to geometric variations between the actual parts and models in CAD (Computer Aided Design) and the properties of the manufacturing materials. In a second stage began a gradual assembly, where, for each new component inserted into the prototype, was conducted a collection of information about the variables involved in the process and a new modal parameters extraction was performed, in both boundary conditions (free and fixed on a IEC 60034-14 rigid base). Among the variables evaluated are the tightening torque of mounting and fixing screws of the end covers and frames, the magnitude of mechanical interference between parts and deformation caused by the machining process. The concern with such details is justified by reducing doubts about the credibility of the numerical model by reducing the computational cost of simulation safely and indicates which project characteristics are sensitive to the manufacturing process.

3. MAIN EXPERIMENTAL RESULTS

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3.1 Mechanical interfere data collection from process of manufacturing

The dimensional data of each component were collected after the machining process, the regions of greatest interest were fittings between end covers and frames, the internal diameter of the frame and the outer diameter of the stator. These are regions that can change the boundary conditions of the problem in question. To facilitate the analysis of the dimensions of each part in the assembly of sets, and then associate them with experimental modal analysis results, Tab. 1 and 2 summarize the amounts of interference that each piece (theoretically) afforded in the final prototypes.

Table 1 – Mechanical interference between end cover and frame.

Prototype	Component	Medium Interference (mm)
1	Front end cover nº2	0,02
	Frame nº3	
	Back end cover nº1	-0,07
2	Front end cover nº4	0,03
	Frame nº4	
	Back end cover nº2	0,04
3	Front end cover nº5	0,06
	Frame nº1	
	Back end cover nº3	0,08
4	Front end cover nº6	0,07
	Frame nº2	
	Back end cover nº4	0,07

Table 2 – Mechanical interference between stator and frame.

Prototype	Component	Medium Interference (mm)
1	Stator nº1	0,24
	Frame nº3	
2	Stator nº2	0,44
	Frame nº4	
3	Stator nº3	0,14
	Frame nº1	
4	Stator nº4	0,15
	Frame nº2	

Note that the interference between the stator and frame of the prototype nº 2 are greater than the others, this was intentionally performed to verify the impact that this parameter have in the change of the natural frequencies of the prototype.

3.2 Extensometry

The strain gages (SG) were installed in the frame in a way to measure the radial deformation, resulting from the mounting of the stator, and axial, derived from the assembly of the end covers. Figures 2 describe the location and identification of each SG installed on the frame and on the end covers.

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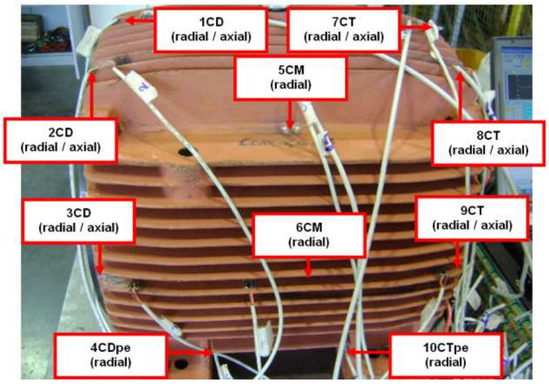
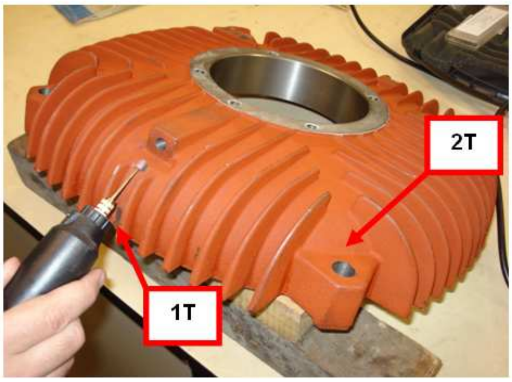


a) Frame.



b) End cover.

Figure 2 – Location and identification of the SG installed.

The results of the extensometry presents in Tab. 3 indicates that with the insertion of the stator, the region 6CM showed a magnitude of deformation approximately three times greater in the frame nº 4 when compared to the others. This behavior was expected, since the frame nº 4 had an internal diameter smaller than the others frames (more interference).

Table 3 – Results of SG 6CM after the insertion of the stator into the frame.

Component	Deformation (SG 6CM)
Frame 01 + Stator 04	0,17 %
Frame 02 + Stator 03	0,18 %
Frame 03 + Stator 01	0,22 %
Frame 04 + Stator 02	0,62 %

Taking the average of the results of SG 1T and 2T installed on the end covers, the magnitude of deformation of the back end covers was about four to five times greater relative to front end covers, as shown in Tab. 4. It can be associate this difference to the absence of fins on the back end cover (Fig. 3), making it more susceptible to deformation in the regions of interface with the frame.

Table 4 – Results of SG 1T and 2T after the installation of the end covers in the frame.

Component		Medium Deformation		Medium from SG 1T and 2T
		SG 1T	SG 2T	
Frame nº 3	Front end covers	0,002%	0,005%	0,004%
	Back end covers	0,004%	0,037%	0,021%
Frame nº 4	Front end covers	0,003%	0,017%	0,010%
	Back end covers	0,007%	0,071%	0,039%

a) Front end cover.

b) Back end cover.

Figure 3 – Differences between front and back end covers.

3.3 Experimental modal analisys – Main results

Figure 4 summarizes the percentage variation of the natural frequencies considering as reference the medium value of the natural frequency of the first four vibration modes found for samples of various components and assemblies in a

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free condition and Fig. 5 in the fixed condition on the rigid base. In these figures, can be visualized the sensitivity to assembly variations of the various components or group of components under conditions that were evaluated. The tightening torque of the screws of the end covers was 8 kgfm and for the bolts fastening the feet of frames was 10 kgfm. From these results, was possible to stratify the main test items in a more organized attempt to better understand the deviations which occurred during the manufacturing process and in the obtaining of the experimental parameters.

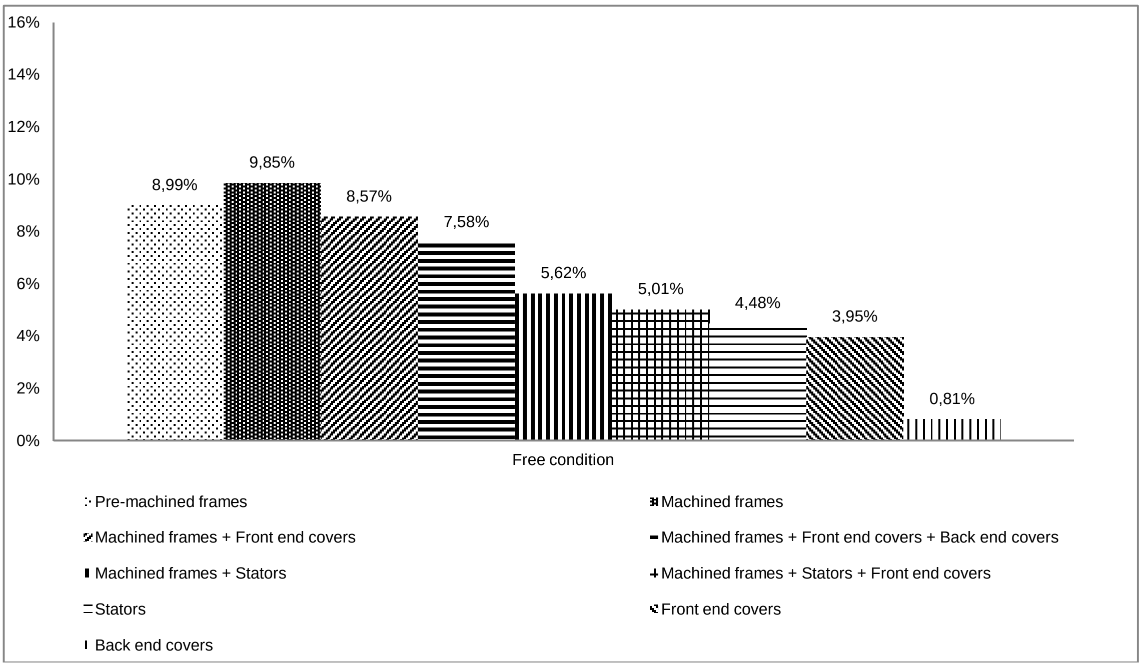

Figure 4 - Percentage variation of the experimental results of the first four natural frequencies of each component or set of components of the motor in free condition.

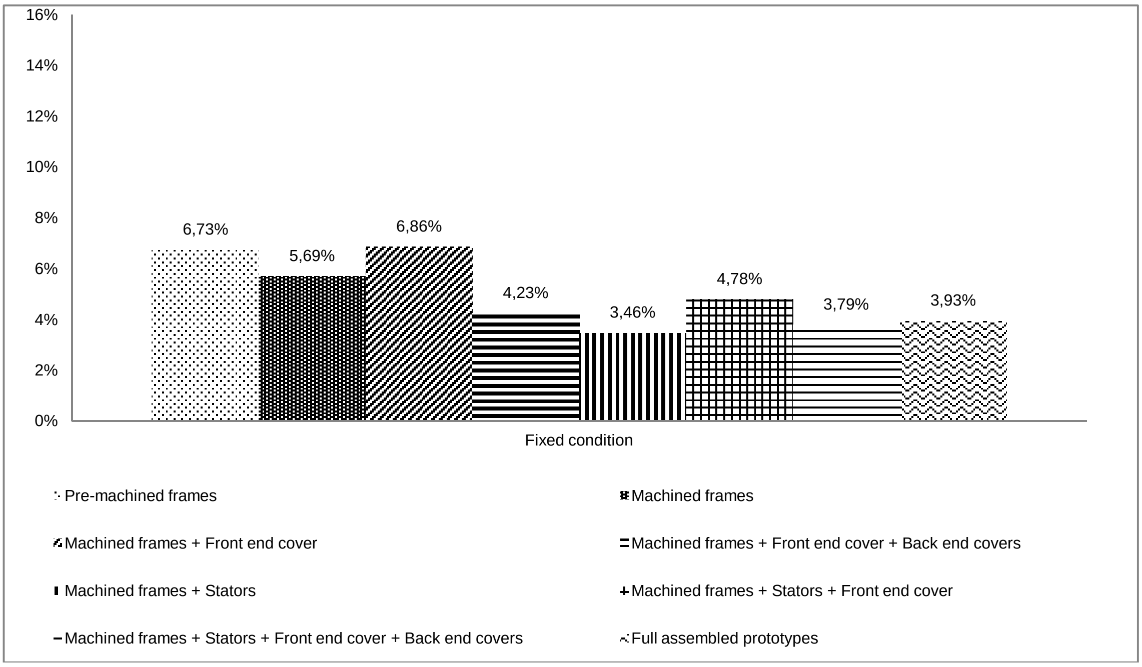

Figure 5 - Percentage variation of the experimental results of the first four natural frequencies of each component or set of components of the motor in fixed condition.

3.4 Experimental modal analysis - Machined frames with stators

Figure 6 shows the results of the natural frequencies after inserting the stators into the frames. The fact of the assembly frame nº 4 and stator nº 2 shows greater interference was not reflected in a significant increase of the values of natural frequencies when compared with the other assemblies with different interferences.

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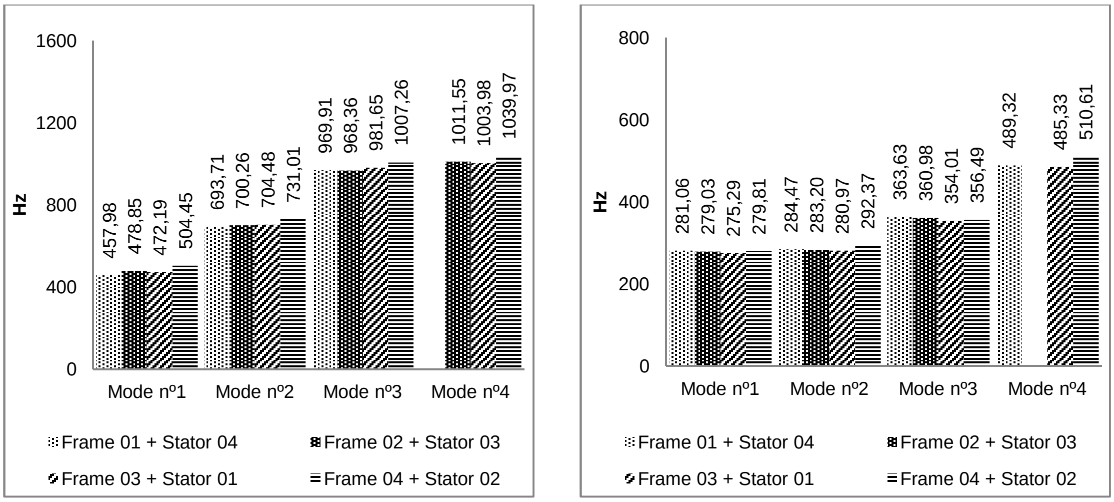

a) Free condition. b) Fixed condition.

Figure 6 - Results of the natural frequencies of the frames with stators.

3.5 Experimental modal analysis - Machined frames with stators and end covers

An important observation to be made to evaluate the results after installing both the end covers, is the cancellation of the higher interference effect of the frame nº 4 with the stator nº 2. Without the ends covers the set of components with greater magnitude of interference between the stator and frame, discreetly assumed higher values of natural frequencies for some modes, but this difference disappeared with the installation of the end covers (Fig. 7). Observing the data in Tab. 1 and comparing them to the results of Fig. 6b and Fig. 7 it can be seen that the larger interference between the frames nº 1 and nº 2 and the respective end covers gave values of natural frequencies slightly larger or near the other two mounting assemblies. Therefore, the influence of the magnitude of interference between the frame and stators in the natural frequencies of the set depend on the magnitude of the interference between the end covers and frame.

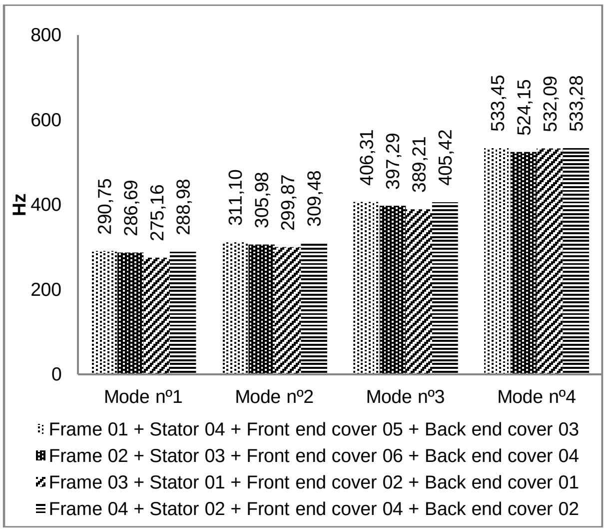

Figure 7 - Results of the natural frequencies of the frames with stators and the both end covers in the fixed condition.

3.6 Experimental modal analysis – Others components and full prototypes

In the free condition, deviations of the results of pre-machined frames were higher when compared with the fixed condition. When evaluating the data separately realizes that the greatest contributions to this difference were the results obtained with the frame nº 3. If the results of frame nº 3 were excluded, the percentage variation of the experimental results of the first four natural frequencies for this component falls from 8,99 % to 3,14 %. Evaluating the mass and dimensional data, arrived at the conclusion that the frame nº 3 had some mass difference. The relative variations from assemblies of the frames with end covers in both conditions (free and fixed) are also attached to mass difference of frame nº 3. The variations relative to the end covers, stators and the full assembled prototypes can be considered satisfactory, because they are smaller than the configurations previously evaluated. Additionally were evaluated the

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influence of the tightening torque of the fixing screws of the end covers and the frames feet, as well as the process of stress relief after frame pre-machining. The results indicated that both the torque, and the heat treatment has no major influence on the change of natural frequencies of the modes of parts and assembly sets. To full results see Gonçalves (2012).

4. NUMERICAL SIMULATION

To simulate the problem proposed in this work was used the commercial program ANSYS Workbench (WB) 2.0 - Version: 14.0.0. With the CAD model done and into the simulation environment, the first step was to simplify the geometry eliminating unnecessary details, in the second step all parts except the rotor were modeled with solid elements which have square displacement behavior. The sensitivity to changes in the mesh size was assessed using the element as parameter. The error due to this sensitivity, associated with the calculation of natural frequencies of the first four modes of vibration and the processing time was used as a condition for setting the default size of the elements to be adopted in the rest of the simulations (Fig. 8).

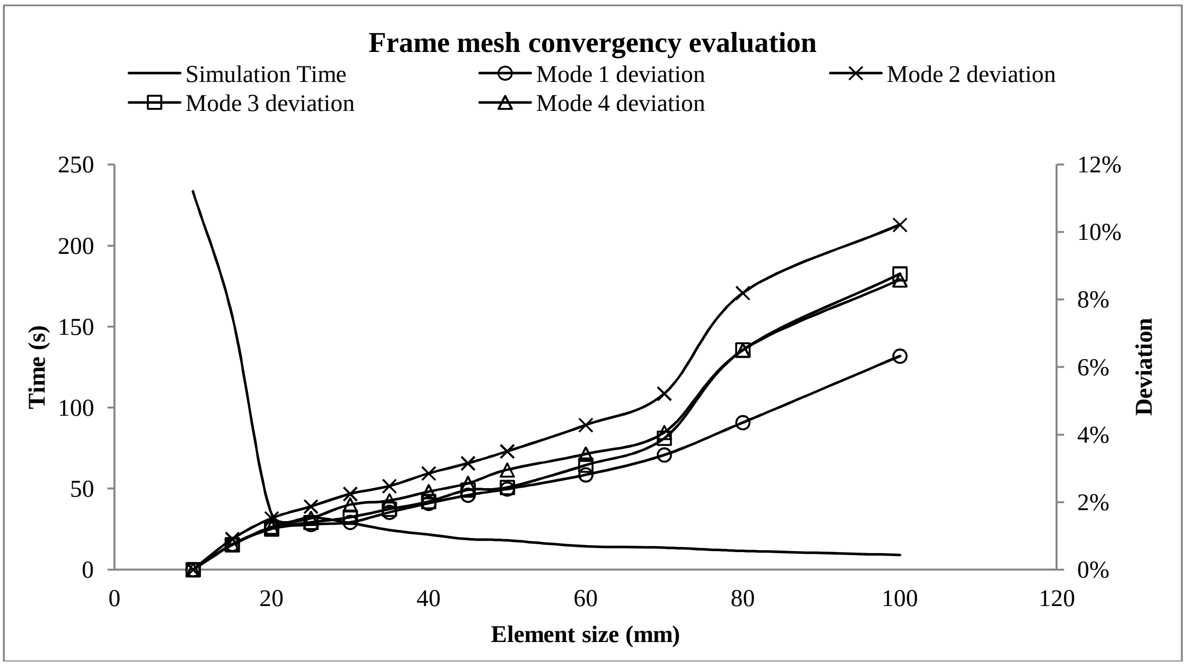
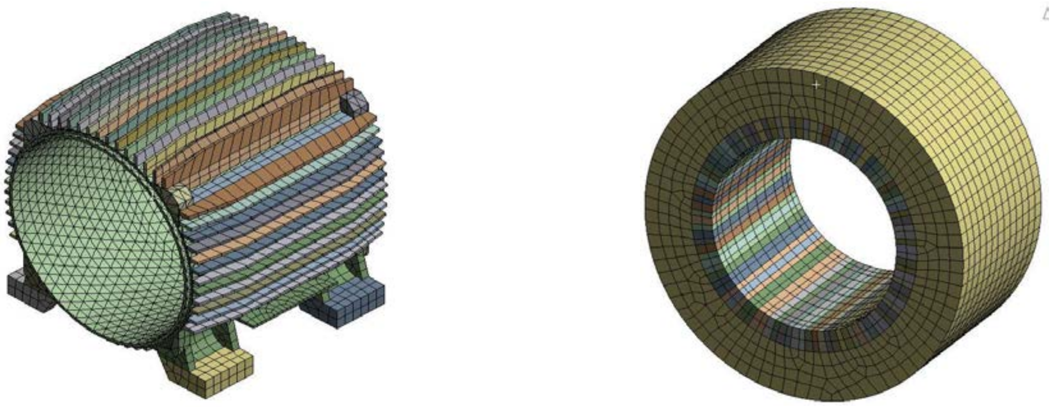


Figure 8 - Graph comparing the processing time of the diversion number of the carcass in a free condition.

The deviations were calculated with reference to the first four natural frequencies of vibration modes of the frame in the free condition obtained with the finer mesh model (element size of 10 mm). With the aid of fig. 8 was defined the element size 25 mm, because the deviations related to the four first modes have low values and the processing time is adequate. With the element size defined, was evaluated the characteristics of mesh using different generation methods. From the results of this step it was decided to use the automatically generated mesh geometry divided into several main bodies or a multi-bodies geometry (Fig. 9).



a) Motor frame model. b) Stator model.

Figure 9 – Mesh appearance using automatic multi-body meshing method.

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Based on the concepts presented in Section 1.1 relating to the rotor, it is believed from the dynamic point of view that the form of install the rotor in the motor has the effect of adding mass to the mechanical system formed by the frame, stator and end covers. This addition is proportional to the mass of the rotor. For this reason, it is believed that the stiffening effect due to the rotor assembly, is smaller and negligible. This means from numerical point of view, the rotor and all its components would be added to the model as Kukula (2007), considering only the effects of inertia forces on the end covers. Thus, the rotor was regarded as charge, proportional to its weight distributed in the two edge bearings.

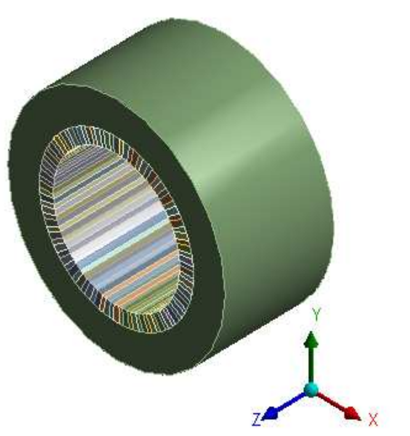
4.1 Models adjustment

At this stage the models were adjusted for each part in the free condition comparing the numerical data with their respective average experimental data based on the optimization algorithms available in the Design Explorer (DX) ANSYS. The DX is a statistical tool for parametric analysis that integrates the virtual environment of the WB. The first step was to define the parameters to be used. For the frame and end bells was used the cast iron data, to the stator was used the orthotropic data presented on Roivainen (2009) work (Fig. 10).

Properties of Outline Row 4: Coroa Estator

	A	B	C	D	E
1	Property	Value	Unit		
2	Density	7700	kg m^-3		
3	Orthotropic Elasticity				
4	Young's Modulus X direction	2,12E+11	Pa		
5	Young's Modulus Y direction	2,12E+11	Pa		
6	Young's Modulus Z direction	1,8E+09	Pa		
7	Poisson's Ratio XY	0,3			
8	Poisson's Ratio YZ	0,3			
9	Poisson's Ratio XZ	0,3			
10	Shear Modulus XY	8,2E+10	Pa		
11	Shear Modulus YZ	6,3E+10	Pa		
12	Shear Modulus XZ	6,3E+10	Pa		

a)



b)

Figure 10 – Stator orthotropic data initially used into DX.

The next step consists on correlate all these parameters to identify the model sensibility for each parameter change. With this procedure was possible to choose the most adequate parameter and exclude that ones with lower importance, minimizing the variable number and the computational costs. The tool used by DX is called Parameter Correlation and the correlation chosen was Spearman type.

From the correlation result it was determined that the most significant parameter for the calculation of natural frequencies in the cast iron is the elastic modulus. With respect to the stator, the results indicate that the coil material, as proposed by Roivainen (2009) exerts low influence on the natural frequency values of the respective modes, and may be used without any modification from original values. Already the data from orthotropic material model representing the stator stack indicated with relation to the natural frequencies of the respective modes that:

1. The Poisson coefficients have low influence;
2. The elastic modulus associated with axial compression must be low (5E9 N/m²), but a variation of ± 10 % from this value also exerts low influence;
3. The elastic modulus, and the shear modulus associated with the circumferential deformations are the main parameters that exerts influence.

The most significant parameters were subdivided by a DX statistical tool named DOE (Design of Experiments) to develop an efficiently series of simulations that represent possible solutions of the optimization and adjustment of numerical models in question. This development was done by the method Central Composite Design (CCD). The points identified in this process were then calculated and interpolated to create a response surface that is the solution of the combinations of the results produced by the DOE and solved by ANSYS. The interpolation method used was Kriding. The response surface provides approximate values of the output parameters at any point of the space created by the CCD without performing a complete solution (ANSYS, 2012). The final adjustment of the models was conceived from the response surface with the help of the DX tool Goal Drive Optimization (GDO), using genetic algorithm named Multi-Objective Genetic Algorithm (MOGA).

The basic idea behind this algorithm is similar to Darwin's theory of evolution, where natural selection is produced from a group of individuals (in this case the response surface) oriented to achieve a certain goal. This selection is made

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by three basic operations: selection, crossover and mutation, and generates the increase of the population of most suitable candidates until the stop criterion is reached (Fermiano, 2009). The stopping criterion used was the lowest value of the objective function with up to twenty iterations of an initial number of a hundred samples. In this case, the objective function used was the quadratic penalty. This calculation used the experimental data as a reference. Figure 11 shows an example of how the values of the natural frequencies of the first three modes have been entered in this step and how were presented the candidate parameter sets for the solution.

	P14 - DS_Raio_Interno_.Part	P15 - DS_Reforco_Axial_.Part	P16 - DS_Raio_Externo_.Part	P17 - Total Deformation 7 Reported Frequency (Hz)	P18 - Total Deformation 8 Reported Frequency (Hz)	P19 - Total Deformation 9 Reported Frequency (Hz)
Optimization Domain						
Lower Bound	45	37	5			
Upper Bound	54	40	15			
Optimization Objectives						
Objective	No Objective	No Objective	No Objective	Seek Target	Seek Target	Seek Target
Target Value	Medium experimental data →			385,58	513,78	895,5
Importance				Default	Higher	Higher
Constraint Handling						
Candidate Points						
Candidate A	53,429	37,256	5,9561	391,78	504,84	941,84
Candidate B	53,915	38,452	5,764	386,03	500,03	928,55
Candidate C	50,783	37,766	7,1724	387,32	501,58	942,7

Figure 11 – Result example of the adjust process presented.

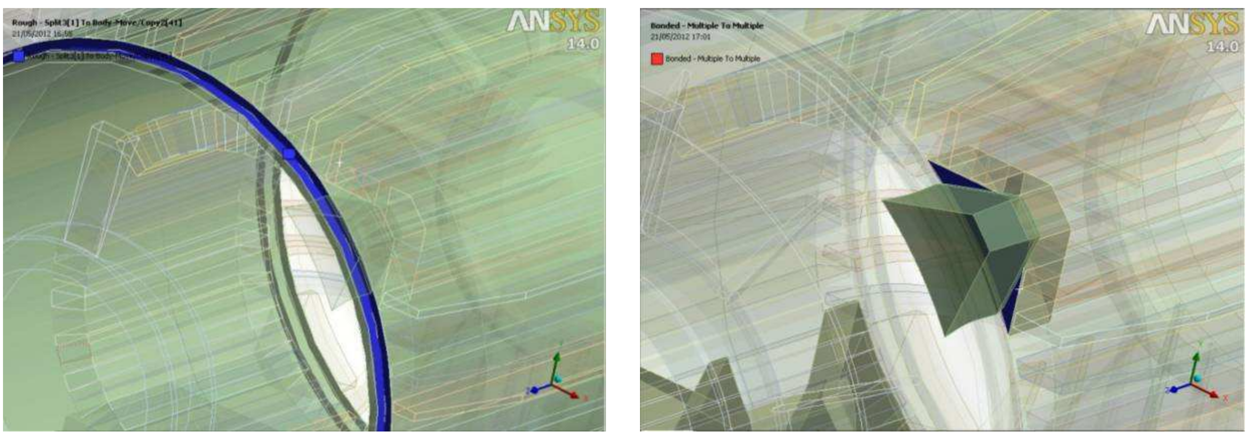
The full model adjustment result, in the fixed condition, can be conferred in Tab. 4.

Table 4 – Full numerical model adjustment result.

	Medium experimental data (Hz)	Numerical deviation
Mode nº1	276,17	2,06%
Mode nº2	317,01	-3,38%
Mode nº3	409,66	-1,87%

4.2 Contact and boundary conditions

The fixed condition was defined as Kukula (2007), zero displacement in all Cartesian directions on the lower surface of frame feet. The contact of the end covers and stator on the frame was added into the model as a pre-tension. From this definition the contacts between end covers and frame were defined as Cai et al (1999), where the contact areas near the screws fixing the end covers presented identical movements ("bonded"), while the remaining areas of contact could vibrate independently (Fig.12). In WB this definition is equivalent to using contacts that allow relative motion between the surfaces in the normal direction of separation, transferring tangential and compressive stresses in its entirety.



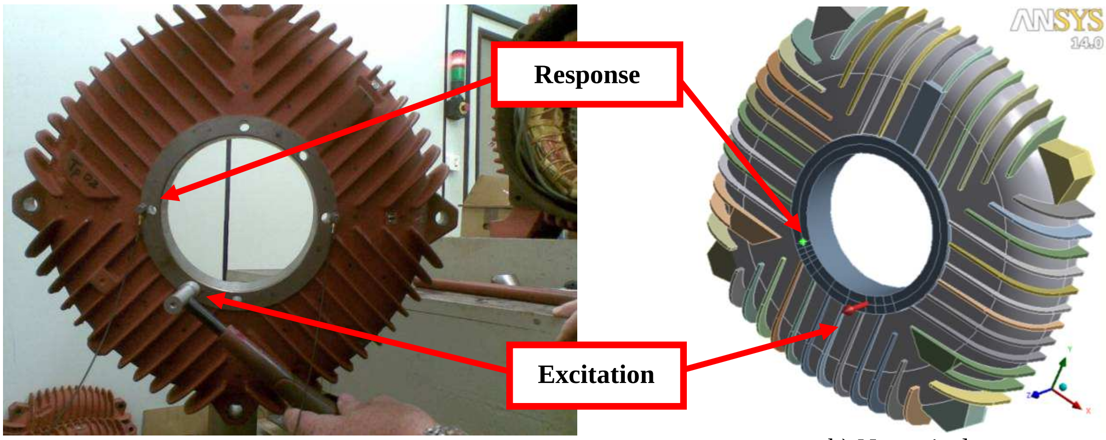
a) Contacts regions between end covers and frame. b) Contacts regions for fixing screws of the end covers.

Figure 12 – Localization details of the contacts between end covers and frame.

For comparison was performed a numerical modal analysis using the contacts configuration as defined by Cai et al (1999) and with all the contacts "bonded". With contacts "bonded" the natural frequency for the first mode of the frame with the front end cover in the fixed condition showed a value 14 % higher than the medium experimental value. Using the contacts similar to those used by Cai et al (1999) the numerical deviation with respect to medium experiment was below 4 %.

4.3 Numerical models validation

The final validation of numerical models was made by comparing the experimental and numerical FRF curves (Frequency Response Function) of each part in the free condition and the main sets of mounting in the fixed condition. The numerical FRF curves were extracted at the same points of their experimental peers and the method used was the modal superposition. The magnitude of the excitation force as well the damping values were the same used and extracted from the experimental tests. Figure 13 shows an example of the location of excitation and response to obtain the FRF curves.



a) Experimental. b) Numerical.

Figure 13 – Location example of the excitation and response points to obtain the FRF curves in the front end cover.

Figure 14 presents the FRF curve comparison for the full assembly prototype fixed on a rigid base.

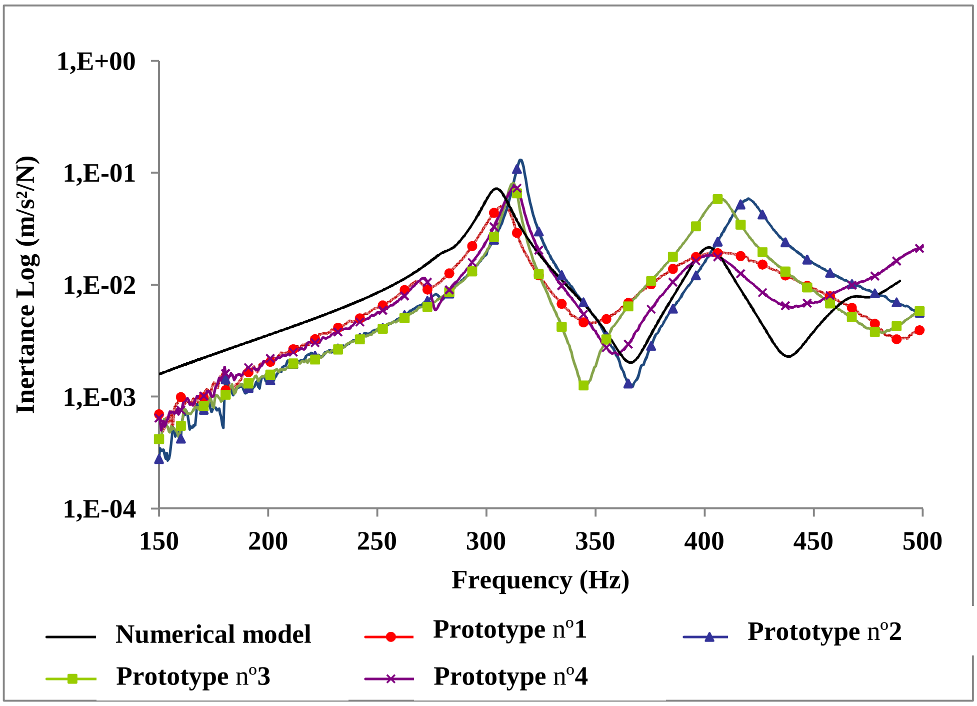


Figure 14 – Numerical and experimental FRF curves for the full prototypes mounted on a fixed base – Mean damping factor = 1,87%.

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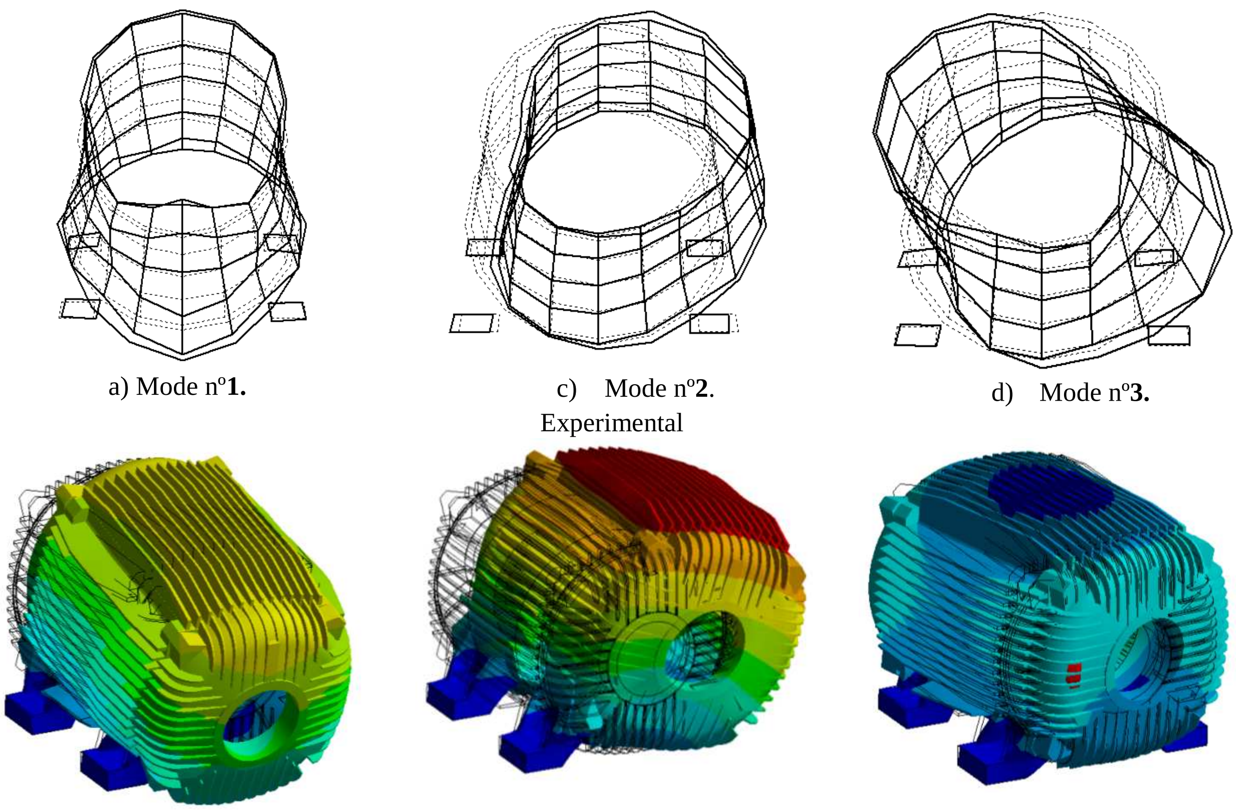
Although the peak representing the third mode exhibit a greater difference compared to the experimental ones, the curves show good relation. Such behavior is attributed to the simplification of the stator. Disregarding all the complexity of coils, insulating films and stack, seems to have directly influenced the accuracy of the numerical model, particularly with respect to the third mode, but comparing to the results of Tab. 4, it can be concluded that the final numerical model using the parameters that produce the best results is adequate for the purposes proposed in this work.

Figure 15 shows the first three modes of vibration, in the fixed condition, for the motor studied in this work.



a) Mode nº**1.** c) Mode nº**2**. d) Mode nº**3.**

Experimental

e) Mode nº**1.** b) Mode nº**2.** c) Mode nº**3.**

Numerical

Figure 15 – First three experimental and numerical modes of vibration of the studied electric motor fixed on a rigid base.

5. CONCLUSION

With the exception of the stator, the adjustment results and validation of models showed low sensitivity ratio using data and factory tolerances, indicating that variations in the production process should not result in significant differences in the frequency domain. With respect to the stator, the results showed that the method used to fit the model is appropriate and that the experimental determination of orthotropic characteristics is of fundamental importance for the accuracy of the numerical model. The modulus of elasticity found associated with axial compression differs from that found by the authors studied (Roivainen, and Delves 2009, 1964). It is believed that this difference is due to peculiarities of manufacture, as the pressing force of the lamination stack and the impregnation method.

From these results it is evident that the numerical solution is a powerful tool for the prediction of the first three modes of vibration of a structure as complex as an electric motor, but is very dependent on the orthotropic characteristics of the stator and contacts that represent the fit between end covers and the frame. The following steps must be followed for a proper numerical modeling of the motor according to the results found:

1. Determine the properties of the materials used in the stator by experimental validation, considering it an orthotropic system;
2. Using the known values of the properties of cast iron for frames and end covers;
3. Simplifying correctly the geometry models that compose the motor;
4. Using elements of appropriate size to form the mesh;
5. Assemble each component of the numerical model using bonded contacts in the regions of the fixing screws of the end covers. Between end covers and frame and between stator and frame use contact that transmits the tangential

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and compressive stress in its entirety, allowing relative movement between the surfaces in the normal sense of separation;

6. Restrict the movement of the lower surface of the model feet of the frame so as to eliminate any movement in three directions of the Cartesian axis and thus reproduce the condition "fixed on a rigid base";

7. The effect of the rotor should be considered as a pre-stress model in Item 5 before performing the numerical modal analysis;

8. Perform the numerical modal analysis.

6. ACKNOWLEDGMENTS

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